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Issues in Ancient Philosophy

ANCIENT LOGIC, LANGUAGE, AND METAPHYSICS

SELECTED ESSAYS BY MARIO MIGNUCCI

Edited by
Andrea Falcon and Pierdaniele Giaretta

ROUTLEDGE



Ancient Logic, Language, and Metaphysics

The late Mario Mignucci was one of the most authoritative, original, and influential scholars in the area of ancient philosophy, especially ancient logic. Collected here for the first time are sixteen of his most important essays on *Ancient Logic, Language, and Metaphysics*.

These essays show a perceptive historian and a skillful logician philosophically engaged with issues that are still at the very heart of history and philosophy of logic, such as the nature of predication, identity, and modality. As well as essays found in disparate publications, often not easily available online, the volume includes an article on Plato and the relatives translated into English for the first time and an unpublished paper on *De interpretatione* 7.

Mignucci thinks rigorously and writes clearly. He brings the deep knowledge of a scholar and the precision of a logician to bear on some of the trickiest topics in ancient philosophy. This collection deserves the close attention of anyone concerned with logic, language, and metaphysics, whether in ancient or contemporary philosophy.

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Ancient Logic, Language, and Metaphysics

Selected Essays by Mario Mignucci

**Edited by Andrea Falcon and
Pierdaniele Giaretta**

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Foreword

Mario Mignucci was Professor of Philosophy at the University of Padua and Research Fellow of the Philosophy Department, King's College London. Upon his death, he left an extensive body of essays, as well as several books, dealing with all aspects of ancient logic, including its reception in antiquity and beyond.

We believe that his essays are still of great interest to scholars working in ancient philosophy and the history of logic. His articles are guided by an interest in issues that are still topical in philosophy of logic such as the nature of predication, identity, and modality. Mignucci deals with these issues in a way that shows not only command of the most technical aspects of formal logic but also historical consciousness. He was among the first historians to understand the importance of studying the developments of logic until the end of antiquity.

This volume is a collection of sixteen essays organized around the following five topics: (1) inference and syllogism; (2) identity, predication, and quantification; (3) modality, time, and future contingents; (4) paradoxes; (5) relatives. These are the issues that most concerned Mignucci over the years. He returned to them in a number of publications. We have selected those articles that seem to represent his most considered position. With one exception ("Relatives in Plato"), all the essays reprinted here were originally written in English. One ("On Universals and Particulars: Aristotle, *On Interpretation* 7") was intended for publication in the proceedings of the 13th Symposium Aristotelicum. Since those proceedings were never published, we present it here for the first time.

What follows is a brief summary of the contents of the five sections in which the volume is organized.

Inference and syllogism: this section contains three essays. The first is a detailed study of the definition of an Aristotelian syllogism. Mignucci argues that there is correspondence between the definition and the class of deductive arguments treated in Aristotle's syllogistic. The second addresses "exposition," a method of syllogism validation used by Aristotle in his *Prior Analytics*. The third turns to Stoic syllogistic and offers a comprehensive study of Stoic proof theory. Taken together, these articles provide a fairly comprehensive picture of ancient theories of syllogism and inference.

Identity, predication, and quantification: the four essays collected in this section are all concerned with Aristotle's logic. In the first, Mignucci warns interpreters

not to impose anachronistic set-theoretical notions on Aristotle's doctrine of predication. Instead, he proposes to take seriously the idea that subject/predication relations are explained by Aristotle in terms of part/whole relations. The second is one of the best-known pieces of this collection. It deals with Aristotle's concept of identity and his apparent denial of the principle of the indiscernibility of identicals. It also explores the treatment of the same doctrine in the Greek commentators. The third is concerned with Aristotle's views on the topic of contingent identity. The fourth was Mignucci's contribution to the never published proceedings of the 13th Symposium Aristotelicum devoted to the treatise *On Interpretation*. Starting from a brisk commentary on Chapter 7, Mignucci opens up his discussion by touching upon issues related to quantification, the square of opposition, and the problem of the existential import of propositions.

Modality, time, and future contingents: this section contains four essays. The first is an attempt to read into Aristotle's logic a syntactic definition of possibility. The second tries to reconcile the post-Aristotelian conception of indefinitely-tensed sentences with God's foreknowledge and providence. The last two articles deal with the late antique reception of the discussion of future contingency in *On Interpretation* 9. They focus on the interpretation offered by Ammonius and Boethius. There are a few overlappings in content and language in these two articles because the treatment of future contingency is the same in these two commentators. Still, given the importance of the topic, we have decided to publish both articles anyway.

Paradoxes: the Sorites and the Liar were discovered in antiquity. These paradoxes are a perennial source of philosophical perplexity. In the two articles that we reprint in this section, Mignucci offers a reconstruction of the ancient discussion of these paradoxes.

Relatives: the three essays reprinted in this section cover the most important discussion held in antiquity on the so-called relatives (predicates involving a relation to something or someone such as being double or being a slave). Taken together, these essays constitute a short monograph on this too often neglected but important topic.

Andrea Falcon prepared the typescript and translated the essay on the relatives in Plato. Pierdaniele Giaretta helped with the logical formalizations.

We would like to thank Katherine (Kay) Rollans and Francesco Gallina for having read the whole typescript, and Francesco Gallina also for his helpful comments. We are grateful to Paolo Fait for his input on the selection of the articles.

This collection of essays is published with the permission and assistance of Mario Mignucci's wife and children.

Andrea Falcon
Pierdaniele Giaretta

Acknowledgments

With the exception of the article on the treatise *On Interpretation* 7, all the chapters of this book have been previously published as follows. We are grateful to the publishers for permission to reprint them here.

Syllogism and Deduction

Le Style de la pensée. Recueil de textes en hommage à Jacques Brunschwig, sous la direction de P. Pellegrin et M. Canto-Sperber. Les Belles Lettres, Paris 2002: 245–266.

Expository Proof in Aristotle's Syllogistic

Oxford Studies in Ancient Philosophy. Supplementary Volume: *Aristotle and the Later Tradition*, edited by H. Blumenthal and H. Robinson. Clarendon Press, Oxford 1991: 9–28.

The Stoic *themata*

Dialektiker und Stoiker: zur Logik der Stoa und ihrer Vorläufer, herausgegeben von K. Döring und Th. Ebert. Franz Steiner Verlag, Stuttgart 1993: 217–238.

Remarks on Aristotle's Theory of Predication

Beiträge zur antiken Philosophie. Festschrift für Wolfgang Kullmann, herausgegeben von H. Ch. Günther und A. Rengakos. Franz Steiner Verlag 1997: 145–151.

Puzzles about Identity: Aristotle and His Greek Commentators

Aristoteles Werk und Wirkung. Paul Moraux gewidmet. Volume I: *Aristoteles und seine Schule*, herausgegeben von J. Wiesner. Walter de Gruyter, Berlin and New York 1985: 57–97.

Aristotle's Topics and Contingent Identity

Kann man heute noch etwas anfangen mit Aristoteles? Herausgegeben von Th. Buchheim, H. Flashar, R. A. H. King. Felix Meiner Verlag, Hamburg 2003: 39–59.

x *Acknowledgments*

Aristotle's Conception of the Modal Operators

Interpretation und Argument, herausgegeben von H. Linneweber-Lammerskiten und G. Mohr. Königshausen & Neumann, Würzburg 2002: 75–92.

Logic and Omniscience: Alexander of Aphrodisias and Proclus

Oxford Studies in Ancient Philosophy 3 (1985): 219–246.

Ammonius on Future Contingent Propositions

Rationality in Greek Thought, edited by M. Frede and G. Striker. Clarendon Press, Oxford 1996: 279–310.

Truth and Modality in Late Antiquity: Boethius and Future Contingent Proposition

Le teorie della modalità. Atti del convegno internazionale di Storia della Logica, San Gimignano 5–8 dicembre 1989. CLUEB, Bologna 1998: 47–78.

The Stoic Analysis of the Sorites

Proceedings of the Aristotelian Society 93 (1993): 231–245.

The Liar Paradox and the Stoics

Topics in Stoic Philosophy, edited by K. Ierodiakonou. Clarendon Press, Oxford 1999: 54–70.

Platone e i relativi

Elenchos 9 (1988): 259–284.

Aristotle's Definitions of Relatives in *Categories* 7

Phronesis 31 (1986): 101–127.

The Stoic Notion of Relatives

Matter and Metaphysics. Proceedings of the Fourth Symposium Hellenisticum, edited by M. Mignucci and J. Barnes. Bibliopolis, Napoli 1988: 131–217.

Abbreviations

<i>DG</i>	<i>Doxographi graeci</i> collegit recensuit prolegomenis indicibusque instruxit H. Diels, Walter de Gruyter, Berlin 1965 ⁴ .
<i>DM</i>	<i>Die Megariker</i> . Kommentierte Sammlung der Testimonien herausgegeben von K. Döring. Grüner, Amsterdam 1972.
<i>FDS</i>	<i>Die Fragmente zur Dialektik der Stoiker</i> . Neue Sammlung der Texte mit deutscher Übersetzung herausgegeben von K. Hülser. 4 vols. Frommann & Holzboog, Stuttgart 1978–1987.
<i>LS</i>	<i>The Hellenistic Philosophers</i> , edited by A. A. Long and D. N. Sedley. 2 vols. Cambridge University Press, Cambridge 1987.
<i>LSJ</i>	<i>A Greek-English Lexicon</i> . Ninth edition with revised supplement, edited by H. G. Liddell, H. Scott, H. S. Jones, R. McKenzie, P. G. W. Glare. Oxford University Press, Oxford 1996.
<i>ROT</i>	<i>The Complete Works of Aristotle</i> . The Revised Oxford Translation edited by Jonathan Barnes. 2 vols. Princeton University Press, Princeton 1984.
<i>SVF</i>	<i>Stoicorum veterum fragmenta</i> collegit H. F. von Arnim. 4 vols. Teubner, Stuttgart 1964 ² .

Conventions

< . . . >: angle brackets enclose additions to the transmitted Greek text deriving from parallel sources or editorial conjectures. In the translation, they are used to mark amplifications.

{ . . . }: curly brackets enclose words or phrases added by the editors of the volume (Andrea Falcon and Pierdaniele Giaretta).

Part I

Inference and syllogism



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1 Syllogism and deduction in Aristotle's logic

I

It is common among scholars nowadays to render the Greek “συλλογισμός” as “deduction” and to interpret the connected notion accordingly. To my knowledge, this usage originated in 1981 with Jonathan Barnes's influential article “Proof and Syllogism”¹ and has been accepted by the majority of interpreters.²

Of course, I am not concerned here with a simple question of translation but with the logically more interesting question of whether Aristotle really tries to capture the notion of deduction with “συλλογισμός” in such a way that it would be misleading to render the Greek with “syllogism” – a term which, by itself, refers to the particular kind of deduction theorized in the *Prior Analytics*.

Let us examine the view of the defendants of the deduction-translation and start by quoting the official definition of συλλογισμός as it appears in the *Prior Analytics*:

(A) A syllogism (συλλογισμός)³ is a discourse in which, certain things having been posited, something other than the things laid down follows of necessity (ἕτερον τι τῶν κειμένων) in virtue of the fact that they are there (τῷ ταῦτα εἶναι). By “in virtue of the fact that they are there” I mean that it follows because of them (τὸ διὰ ταῦτα συμβαίνειν), and by it “follows because of them” that no external term is required for the production of the necessity.

(Aristotle, *Prior Analytics* 1.1, 24b18–22)

In order to interpret this definition Barnes distinguishes three different terms: “inference,” “deduction,” and “syllogism.” In his view, an inference is an ordered pair of items, with the first one being a set of premises, let us say Π , and the second one a conclusion σ , such that σ follows necessarily from Π . A deduction is a similar pair in which two conditions must be satisfied, namely

(i) σ follows necessarily from Π .

and

(ii) σ holds (if it holds) because each element of Π holds.

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Finally, a “syllogism” is a deduction in which Π has two members.⁴ It is easy to see that every syllogism is a deduction and every deduction an inference but not vice versa. Having given these definitions, Barnes claims, without any further delay, that

(B) Aristotle’s word for “Deduction” is “συλλογισμός”; for “Inference” he will sometimes use τὸ ἀναγκαῖον [. . .]; he has no word for “Syllogism,” but he can express the notion periphrastically.⁵

Barnes does not explain in so many words why the definition given in text (A) should correspond to his characterization of “deduction.” We can fill the gap by supposing that, in his view, condition (i) corresponds to ἕτερον τι τῶν κειμένων ἐξ ἀνάγκης συμβαίνει, “something other than the things laid down follows of necessity,” whereas condition (ii) has τῷ ταῦτα εἶναι, “in virtue of the fact that they are there,” as its counterpart. Therefore, an Aristotelian συλλογισμός is a deduction and not a syllogism.

The problem with this interpretation is to understand what these distinctions mean. Let us begin by considering the definition of deduction according to clause (ii), which makes a deduction a special kind of inference. I take the term “holds” in this clause to mean, or at least to imply, “is true.”⁶ A possible paraphrase of (ii) is:

(ii*) if Π is true, then σ is true because of the truth of each element of Π .

This way of putting things entails that in those cases in which σ , the conclusion, is false, the condition is automatically satisfied, and to get a deduction it would be sufficient for σ to follow necessarily from Π . This looks strange because, in Barnes’s view, condition (ii) is intended to distinguish deduction from a simple inference, whereas in fact it distinguishes only deductions with true premises and conclusions from the corresponding inferences. Consider, for instance, an argument such as

- (1) every geometrical figure is a figure
 every triangle is a geometrical figure
 every isosceles is a triangle

 every isosceles is a geometrical figure.

As we will see in a moment, according to Aristotle the conclusion of (1) does not follow from its premises “because they are there,” that is, in Barnes’s interpretation, (1) does not satisfy condition (ii*).⁷ Therefore, (1) can neither be classified as a deduction nor, alternatively, as a συλλογισμός. Take now:

- (2) every natural number is odd
 every animal is a natural number
 every book is an animal

 every book is a natural number.

Here the conclusion is clearly false. Therefore, condition (ii*) is vacuously satisfied so that (2) is a deduction and a συλλογισμός, although one should not find it too difficult to recognize that (1) and (2) are instances of the same type of argument.

More importantly, Aristotle clearly accepts some arguments in which the conclusions are true and their premises false as συλλογισμοί. For instance, a syllogism such as

- (3) every man is a quadruped
 every horse is a man

 every horse is a quadruped

is a first-figure syllogism and nobody could cast doubts on its validity.⁸ But are we allowed to say that its conclusion, “every horse is a quadruped,” is true because of the truth of its premises? No: They are plainly false.

Finally, the parenthetical “if it holds” in clause (ii) is not part of the Aristotelian definition in any of its versions.⁹ To be faithful to the Greek, one should omit the parenthetical, and so the clause becomes:

- (ii**) σ is true because the elements of Π are true.

But it is clear that in an Aristotelian syllogism it is not necessary for the conclusion to be true. Actually, this is why the parenthetical has been added by Barnes. So the only way to interpret (ii) without the parenthetical is as a conditional, namely

- (ii***) if the elements of Π are true, then σ is also true.

Condition (ii***) is weaker than the Tarskian definition of logical consequence, because no modal requirement is implied by it.¹⁰ But because of its weakness, it is difficult to believe that it expresses a requirement that is not already implied by condition (i). May we think that σ follows necessarily from Π and that there may be a case in which truth is not preserved, namely that all the elements of Π are true and σ is false? What sort of following is this, and how can we conceive of a sound inference that is not always truth preserving? If (ii***) is what is supposed to distinguish inference from deduction, one is led to conclude that there is no difference at all between the two.¹¹ As we will see in a moment, this is exactly Aristotle’s position, at least in my interpretation.

II

I do not think that it is very useful to continue to consider Barnes’s approach to the Aristotelian definition of συλλογισμός, and we must try to start afresh. Let us

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return to text (A). It is quite obvious that two distinct conditions play a role in the definition of a συλλογισμός. The first is Barnes's condition (1), that is,

- (i) σ follows necessarily from Π

and however mysterious the notion of “following of necessity” may be, let us take it as primitive. The second condition needs more investigation. It is expressed by the clause:

- (iii) $\tau\tilde{\omega}\ \tau\alpha\tilde{\upsilon}\tau\alpha\ \epsilon\tilde{\iota}\nu\alpha\iota$.

What does Aristotle mean by (iii)? A first problem concerns its translation. It is clear that “ $\tau\alpha\tilde{\upsilon}\tau\alpha$ ” is acting as the subject of the sentence,¹² as is confirmed by *Prior Analytics* 1.4, 26a2–5, where (iii) appears. So the problem concerns the way in which “ $\epsilon\tilde{\iota}\nu\alpha\iota$ ” ought to be understood. Although it surely has an existential sense, we cannot take it to imply that in a συλλογισμός the conclusion follows from its premises because they express real states of affairs. What “ $\epsilon\tilde{\iota}\nu\alpha\iota$ ” probably means is “being there.” Therefore, a possible paraphrase of (iii) is:

- (iii*) because the elements of Π are there

where “are there” means nothing more than “are at the beginning of the deductive process.” Because the premises are posited or laid down, the conclusion follows necessarily from them.

Yet the question of the meaning of (iii) remains. An obvious way to answer it is to look at the way in which Aristotle explains the expression “ $\tau\tilde{\omega}\ \tau\alpha\tilde{\upsilon}\tau\alpha\ \epsilon\tilde{\iota}\nu\alpha\iota$.” In text (A) Aristotle explicitly glosses (iii) by saying that it entails that “no external term is required for the production of the necessity.” We may take “term” (ὅρος) here either as synonymous with “proposition,” as is sometimes the case with Aristotle,¹³ or as a term predicatively related to the terms that constitute the premises of the conclusion σ . In both cases, the meaning of (iii) seems to be that the elements of Π are sufficient to link σ to Π necessarily.

By taking things this way, one might think that (iii) does not constitute a new condition with respect to (i), but only to be explanatory of this clause. If the elements of Π are not sufficient to establish σ , then σ does not follow necessarily from Π , and, vice versa, if σ does not follow necessarily from Π , then the elements of Π are not sufficient to establish σ .¹⁴ But we can read the passage differently. Consider, for instance, an argument such as

- (4) $\frac{\text{every man is mortal}}{\text{every Athenian is mortal.}}$

There is a sense in which one might claim that in (4) the conclusion follows necessarily from its premise. However, it is not *because* of its premise that the conclusion

follows. One is ready to agree that every Athenian is mortal, having previously agreed that every man is mortal, simply because one has tacitly admitted that every Athenian is a man. But it is only when this implicit premise is laid down that we can claim that “every Athenian is mortal” follows necessarily from its premises “because they are there.” In this way (4) is not a συλλογισμός because it fulfills condition (i) but not condition (iii). In other words, the point introduced by (iii) is that in an Aristotelian συλλογισμός no elliptical set of premises is admitted.

This interpretation is confirmed by other passages in which the clause τῷ ταῦτα εἶναι plays a role in discriminating arguments. Consider the following text:

(C) There are five criticisms of an argument in itself. [. . .] Again, <a further case occurs>: if a syllogism (συλλογισμός) comes about with certain premises taken away. For sometimes more premises are assumed than those necessary, so that it is not in virtue of the fact that they are there that the syllogism comes about (οὐ τῷ ταῦτ' εἶναι γίγνεται ὁ συλλογισμός).

(Aristotle, *Topics* 8.11, 161b19 and 28–30;
R. Smith translation, modified)

Here, condition (iii) is said to be violated not because the elements of Π are insufficient to establish σ , but because they may be redundant. Take Π to be a proper subset of Φ and suppose that σ follows necessarily from Π . It is obvious that σ follows necessarily from Φ as well, but at the same time, it is also clear that Aristotle would be entitled to claim that the following of σ from Φ does not fulfill condition (iii). Actually, if we suppose that σ follows from Π *because of* Π and that Π is a proper subset of Φ , there is a reason to claim that it is not because of Φ , strictly speaking, that σ follows from Φ , because Φ contains redundant items.

The same idea is stated in connection with the so-called “fallacy of the false cause.” As Aristotle explains, this fallacy occurs when an impossible proposition χ is derived from a set of premises ρ_1, \dots, ρ_n and one claims that it is because of ρ_i that χ is false, when in fact ρ_i is redundant in the deduction of χ .¹⁵ This background explains the rather cryptic statement in the *Sophistic Refutations* about this kind of fallacy:

(D) Refutations that depend on the assumption of the starting point and upon positing as the cause what is not the cause, are clear through the definition. For the conclusion ought to follow in virtue of the fact that the premises are there (δεῖ γὰρ τὸ συμπέρασμα τῷ ταῦτα εἶναι συμβαίνειν), and this does not happen with the fallacies of the non-cause.

(Aristotle, *Sophistical Refutations* 6, 168b22–25)

The passage does not mean that a refutation fulfills condition (iii) if the premises are causes of the conclusion. The point is more complicated and refers to the case in which a conclusion χ *does* follow from premises ρ_1, \dots, ρ_n but in such a way that it is not because of ρ_1, \dots, ρ_n that this happens. In this case, if χ is impossible and at least one of the premises is redundant, let us say ρ_i , we can claim that

it is not because of this set of premises that the false conclusion is derived and therefore deny that we are compelled to assert the negation of ρ_i as the conclusion of the argument.

Texts (C) and (D) show that we must take the explanatory gloss of “τῷ ταῦτα εἶναι” in text (A) – that is, “no external term is required for the production of the necessity” – *cum grano salis*. If we refer only to this passage, we should conclude that condition (iii) is a requirement that all the premises necessary to the conclusion be explicitly stated. Texts (C) and (D) add a further point, namely that there should be no redundancy among the premises of a συλλογισμός. Therefore, the explanatory gloss in text (A) cannot be taken as a strict definition of “τῷ ταῦτα εἶναι” despite the peremptory way in which it is introduced, but rather as a sort of elucidation of what imposing condition (iii) on an argument implies for its premises.

III

An obvious conclusion emerges from the review of the relevant passages in which our expression “τῷ ταῦτα εἶναι” appears. Condition (iii) is clearly distinct and independent from condition (i), at least in the sense that we can have arguments that fulfill condition (i) but not condition (iii). Moreover, if condition (iii) is read contextually, it introduces the constraint that the premises of a συλλογισμός are not only nonelliptical but also nonredundant with respect to the conclusion, or, to put together their being nonelliptical and nonredundant, they are *adequately pertinent* to the conclusion.¹⁶

I would like to set aside the question of whether, and to what extent, the notion of adequate pertinence is related to the modern notion of relevance in a deduction.¹⁷ What is more interesting for the matter at hand is to ask how the idea of adequate pertinence is used by Aristotle in discriminating arguments. Let us consider the following passage, which is embedded in Aristotle’s discussion of the way in which an argument can be reduced to a syllogism in one of the figures:

(E) First, then, one must try to pick out the two premises of the syllogism (συλλογισμός) (for it is easier to divide into larger parts than smaller ones, and composite things are larger than what they are made from); next, one must see which is universal and which is particular; and, if both have not been assumed, we must ourselves posit the other premise. For sometimes people who put forward a universal premise do not assume the premise included in it, either in writing or in discussion. Or, they put forward these premises but leave out those through which they are inferred, and instead ask for other useless things. One must therefore see whether something superfluous has been assumed, and whether one of the necessary premises has been left out; and the one should be posited and the other taken away, until the two premises are reached. For without these, it is not possible to reduce arguments that have been asked in this way.

(Aristotle, *Prior Analytics* 1.32, 47a10–22;
R. Smith translation, modified)

It is pretty clear that situations both of missing and redundant premises are hinted at here. When we have to put an argument in the syllogistic form, we have first to select the two premises on which the conclusion depends and then check whether they are in the appropriate syllogistic relation. Consider the case of a premise that is missing, as in our example (4). Argument (4) cannot be reduced until the missing premise is supplied. When we have added it, then we get a συλλογισμός, and a syllogism in the first figure, by putting “every Athenian is a man” under “every man is mortal.” Similarly, in the case of redundancy, we obtain a συλλογισμός only when we have removed the superfluous premises. This means that an argument such as (4), or one with redundant premises, can neither be classed as a συλλογισμός nor be put into a syllogistic figure until the missing premises are supplied or the redundant ones eliminated.

In an argument such as (4) the introduction of the missing premise coincides with its reduction to a syllogistic figure. When we add “every Athenian is a man” to “every man is mortal,” we immediately get a first-figure syllogism. But Aristotle is well aware that sometimes things are not so easy:

(F) It is easy to see what is missing from some arguments, but others escape notice and appear to syllogize (δοκοῦσι συλλογίζεσθαι) because something necessary follows from what has been laid down, as for instance if it were assumed that a substance is not destroyed by the destruction of what is not a substance and that if the things out of which something is composed are destroyed, then what is made from them is also destroyed. For, if these premises have been posited, it is necessary for a part of a substance to be a substance; yet it has not been syllogized (συλλελόγισται) through the propositions assumed (διὰ τῶν εἰλημμένων), but rather premises have been left out. Again, if it is necessary for something to be an animal if something is a man, and for something to be a substance if something is an animal, then it is necessary for something to be a substance if something is a man,¹⁸ but it has not yet been syllogized (συλλελόγισται); for the premises are not related as we have said.

(Aristotle, *Prior Analytics* 1.32, 47a22–31;
R. Smith translation, modified)

Here we have two examples. Let us consider only the first, which is the more complicated. By means of it the claim is made that the proposition

(5) every part of a substance is a substance

follows necessarily from the two premises

(6) if what is not a substance is removed from a substance, the substance is not destroyed

and

(7) when a part of a composite is removed from it, the composite is destroyed.

Let us call this argument the “Incomplete Argument.” If we put ourselves in the frame of first-order logic and use natural deduction, we can easily verify that (5) is a logical consequence of (6) and (7). What we need is to add a series of logical rules and nothing else.¹⁹ It is more difficult to understand how we can put this derivation in a syllogistic form. Aristotle does not tell us how to do it, and I suspect that he would have a lot of trouble getting a syllogistic derivation of (5) by implementing premises (6) and (7). Even if we grant this possibility for the sake of the argument, it should be clear that to get a syllogistic deduction of (5) from premises including (6) and (7) we need not only logical premises but also premises relating to substances and parts. But this is another story. What is relevant here is that the Incomplete Argument is not a *συλλογισμός*. Aristotle states that this sort of argument appears to syllogize (*δοκοῦσι συλλογίζεσθαι*), and he implies that it does not in fact syllogize. He goes on to explain why we are mistakenly led to consider these arguments as *συλλογισμοί*:

(G) We are misled in cases like these by the fact that something necessarily follows from what has been laid down, because a syllogism (*συλλογισμός*) is also necessary. But necessary is wider than syllogism (*συλλογισμός*): for every syllogism (*συλλογισμός*) is necessary, but not everything necessary is a syllogism (*συλλογισμός*). Consequently, if something follows certain things having been posited, one should not try straight-off to reduce it. Instead, one must first get the two premises and next divide them in this way into terms, and that term which is stated in both premises must be put as the middle, for the middle must occur in both of them in all of the figures.

(Aristotle, *Prior Analytics* 1.32, 47a31–40; R. Smith translation, modified)

The point is clear. An argument such as the Incomplete one is not, properly speaking, a *συλλογισμός*. We are inclined to consider it a *συλλογισμός* only because its conclusion follows necessarily from the premises. But this condition is not sufficient by itself to characterize it as a *συλλογισμός*: There are processes in which a proposition follows necessarily from others, but nevertheless they are not *συλλογισμοί*. Why? My answer is that they are not *συλλογισμοί* because they do not satisfy condition (iii) of the definition of *συλλογισμός*, although they fulfill condition (i). Because in the derivation of (5) from (6) and (7) some premises are missing, (5) does not follow from (6) and (7) “because they are there.”²⁰

One might react to this analysis by pointing out that if in an argument such as the Incomplete Argument the addition of the missing premises is made in such a way that it produces a syllogism in one of the figures, the reduction itself consists in these additions. But Aristotle seems to distinguish two different steps in the process of reduction: a first one in which we have to add the appropriate premises to the Incomplete Argument and a second one in which, after having added the premises, we accomplish the reduction.

This objection does not look very strong to me. Aristotle is here considering a dialectical context in which arguments are produced by one opponent on the basis of statements granted by the other opponent. The simplest possible situation could

be as follows. X asks Y: “Do you grant that every man is mortal?” Y answers: “Yes.” Then X asks again: “Do you grant that every Athenian is a man?” Suppose that Y’s answer is again: “Yes.” Then X is allowed to conclude: “Therefore, every Athenian is mortal.” The argument contained in this dialog can easily be put in a syllogistic figure. We have first to select the two premises from which the conclusion follows: “every man is mortal” and “every Athenian is a man.” Having checked that they share a common term, “man,” that this term is universally predicated of “Athenian,” and that “mortal” is universally predicated of “man,” we can conclude that we have a syllogism in the first figure. Sometimes things may be more complicated. It may be that the selection of the middle term is not as easy as in our trivial case because the term is complex and it can be taken in different ways. It may be that the original premises or the added one must be rephrased in a linguistically different form to fit the pattern of the figure exactly.²¹ Or it may be that two synonymous terms are chosen to indicate the middle term. To get a formal syllogism, we need to eliminate one of the two synonymous expressions.²² All this is part of the reduction. Therefore, the reduction is a different operation from the implementation of the premises in an argument such as the Incomplete Argument, and I stick to my interpretation of texts (E), (F), and (G).

Should we maintain that an argument like the Incomplete Argument or a correct argument in which a redundant premise such as (1) or (2) is introduced is not a deduction? Of course, one might claim that the answer depends on what we mean by “deduction.” In the context of formal logic, it is quite easy to define what a deduction is. If we take as our reference point a standard first-order calculus, it is quite clear that Aristotelian *συλλογισμοί* do not coincide with deductions. The Incomplete Argument is surely a deduction in first-order logic, but according to Aristotle, it is not a *συλλογισμός* because its premises are elliptical. That (1) and (2) are real deductions in the formal sense is even more obvious. If we take “deduction” in an informal way to mean something like “correct argument,” where there is no precise definition of “correctness,” only a more or less strong intuition can be associated with the use of this term. However, it is difficult to believe that by appealing to any intuitive sense of “deduction” one would deny that the Incomplete Argument is a deduction. After all, the Incomplete Argument is a correct argument. If we formalize it, we are able to get a sound derivation in first-order logic, and what is sound in first-order logic is surely correct from an intuitive point of view.

Let us sum up the results of our analysis. On the one hand, there are sound arguments that are not *συλλογισμοί* because they do not fulfill condition (iii) of the definition of *συλλογισμός*. They cannot be put as such in a syllogistic form. On the other hand, deductions that satisfy conditions (i) and (iii) of the definition of *συλλογισμός* can be arranged in a syllogistic form. Only for these deductions is the possibility of a reduction to one of the figures assured. I believe this to be a faithful paraphrase of Aristotle’s claims in texts (E), (F), and (G).

If we add to all this the thesis that redundant and elliptical arguments are deductions – a plausible thesis considering the examples set out by Aristotle – more than one conclusion can be drawn. The most obvious is that not every deduction

is a συλλογισμός. For instance, the Incomplete Argument is a deduction but not a συλλογισμός. The same holds for redundant arguments such as (1) and (2).

What is less plausible is to conclude that there is a corresponding deduction for every συλλογισμός, namely that there is an Aristotelian deduction that satisfies (i) and (iii). In some cases, such a deduction may be obtained from the initial deduction by adding or eliminating one or more premises. This corresponds to the claim that the Incomplete Argument can be formalized as an Aristotelian deduction by adding some premises. One might believe that this view is unlikely, but in the end its degree of plausibility depends on the focus of our interest. Take, once more, the Incomplete Argument. If we want to derive its conclusion by keeping nonlogical assumptions to a minimum, then we need a logic at least as powerful as first-order logic. But it may be that we are not worried about the number of nonlogical assumptions we are compelled to make. In this case I think that it would be possible, at least to a certain extent, to represent the derivation of (5) from a set of premises that include (6) and (7) through a chain of Aristotelian deductions. However, I am not committed to this claim in defending my interpretation.

A third and, at least *prima facie* even more debatable, conclusion that emerges from our analysis is that every Aristotelian deduction, namely every συλλογισμός that satisfies conditions (i) and (iii) of the definition, is such that it can be expressed in one of the three figures. We must dedicate some time to investigating this claim, which will bring us to grips with a difficult and controversial chapter of the *Prior Analytics*.

IV

Prior Analytics 1.23 begins with the following statement:

(H) It is clear from what has been said, then, that the syllogisms (συλλογισμοί) in these figures are both made perfect through the universal syllogisms (συλλογισμῶν) in the first figure and reduced to them. It will now be evident that this applies to every syllogism (συλλογισμός) in general,²³ when every one has been proven to come about through some one of these figures.

(Aristotle, *Prior Analytics* 1.23, 40b17–22;
R. Smith translation, modified)

What does the claim that every συλλογισμός must “come about through some one of these figures” mean? In a recent paper, Jonathan Barnes has pointed out the indeterminacy of this statement. After a subtle analysis of the texts, he is inclined to think that what Aristotle means here is that every συλλογισμός has the form of a syllogism (or a chain of syllogisms) in one of the figures.²⁴ The problem for Barnes is that he takes “συλλογισμός” as meaning “deduction,” so that in his interpretation Aristotle’s claim would amount to stating that every deduction takes place in one of the three figures.

This interpretation is unconvincing. Claiming that every deduction is a formal syllogism does not account for what Aristotle himself says about putting informal

arguments in a formal way, as is explained in texts (E), (F), and (G). For the sake of argument, however, we can avoid considering here this further complication and drop any reference to the linguistic arrangement of an informal συλλογισμός, which is required to get the status of a formal syllogism. What one should instead point out is that the thesis “every deduction is a syllogism or a chain of syllogisms” is quite difficult to accept, and Aristotle himself does not seem to believe it. He holds that there are arguments that can only partially be put in a syllogistic form, namely in one of the figures. These are the so-called “συλλογισμοὶ ἐξ ὑποθέσεως” and reductions *ad absurdum*. Consider, for instance, the συλλογισμοί from a hypothesis. They may consist in the assumption of a conditional, such as “if P then Q ” and a proof of P . On the basis of this assumption, we can claim that the proof of P holds also as a proof of Q . According to Aristotle, the proof of P is syllogistic, and the final conclusion Q is reached not syllogistically but in virtue of the agreement about the conditional.²⁵ In other words, the basic structure of the συλλογισμός from a hypothesis that Aristotle considers is as follows:

1	M	A
2	N	A
3	P	1, 2 Syllogism
4	$P \rightarrow Q$	A
5	Q	3, 4 MPP.

The symbolism is obvious: “ M ,” “ N ,” “ P ,” and “ Q ” stand for propositions. P is deduced by means of a syllogism in one of the figures, and we derive (5) from lines (3) and (4) by *Modus Ponens*, as we would say nowadays. According to Aristotle, the step from (3) to (5) cannot be acquired by a formal syllogism, being based upon a hypothesis. Barnes, consistent with his decision of translating “συλλογισμός” with “deduction,” is compelled to conclude that the movement from (3) to (5) is not a deduction.²⁶ Does this not sound a little too odd? And what sort of idiosyncratic notion of deduction should we attribute to Aristotle? If we drop the claim that “συλλογισμός” is the same as “deduction,” we can abandon the implausible view that the application of *Modus Ponens* does not hold as a deductive step. Aristotle simply claims that every deduction that satisfies conditions (i) and (iii) is a syllogism in one of the figures and that there are nonetheless deductive steps that cannot be configured syllogistically.

We are now in a position to return to text (H). Aristotle is there speaking of συλλογισμοί, and it is reasonable to take this word in the sense defined in text (A). Therefore, his point is that deductions that satisfy conditions (i) and (iii) are syllogisms in one of the figures or, perhaps better, are such that they can be put in one of the figures – maybe after a convenient linguistic rearrangement (but we have decided to avoid this further complication). It is not the whole range of deductions that is not supposed to be structured into syllogisms but only those deductions that satisfy conditions (i) and (iii).

We do not seem to be in any better position than with Barnes’s interpretation. There we had to deal with the whole corpus of deductions; here the claim

is reduced to those deductions that are neither elliptical nor redundant. But is it reasonable to think that every nonelliptical and nonredundant deduction is a syllogism? Take *Modus Ponens* and consider again steps (3) to (5) of the scheme of deduction we considered earlier. Premises “ $P \rightarrow Q$ ” and “ P ” are neither elliptical nor redundant with respect to the conclusion “ Q ,” which necessarily follows from them. Therefore, one might conclude that *Modus Ponens* is a deduction that satisfies (i) and (iii) and, hence, a συλλογισμός. But as we have seen, according to Aristotle, *Modus Ponens* does not have a syllogistic form. We end up with an exception to Aristotle’s main thesis, and to avoid it, we may have to go back to Barnes’s interpretation and deny that *Modus Ponens* is a deduction.

I think that there is a better way around this difficulty. It consists in reconsidering once more the meaning of clause (iii) – namely, the expression “τῷ ταῦτα εἶναι.” If we simply think that its role is to single out those deductions in which the premises are adequately pertinent, we will not be able to understand Aristotle’s claim, which looks inconsistent with his own statements. Clause (iii) must have a stronger meaning by which a smaller set of deductions is discriminated because, for instance, *Modus Ponens* cannot be included in it. To understand what Aristotle means by (iii) we have to underline the connection he establishes between “τῷ ταῦτα εἶναι” and “διὰ ταῦτα συμβαίνειν,” “following because of them” in the explanatory gloss of text (A). By saying that σ follows necessarily from π_1, \dots, π_n διὰ ταῦτα, namely because of π_1, \dots, π_n , a causal or explanatory role that π_1, \dots, π_n have with respect to σ is suggested. In other words, to get a συλλογισμός not only must σ follow from π_1, \dots, π_n but also π_1, \dots, π_n must to some extent represent the reason for the following. It is not sufficient that we are entitled to put σ after π_1, \dots, π_n ; we need also to give a reason why it is so. Consider now the kind of premises we have to deal with in a συλλογισμός according to Aristotle:

(I) Consequently, a syllogistic (συλλογιστική) premise in general will be either the affirmation or the negation of one thing about another in the way we have said. It will be demonstrative if it is true and has been obtained by means of the initial assumptions; a dialectical premise, on the other hand, for the questioner is the positing of a contradiction as a question, and when one is syllogizing (συλλογιζομένῳ) it is the assumption of what is apparent and commonly accepted, as was said in the *Topics*.

(Aristotle, *Prior Analytics* 1.1, 21a28–12; R. Smith translation, modified)

The point is clearly stated. Neither truth nor having been accepted by an opponent is a relevant characteristic of the premises that come into a συλλογισμός. The cognitive content of the propositions involved in a συλλογισμός does not play a role in establishing the conclusion. (That is why, as we have seen, Aristotle is able to accept deductions in which the premises are false as correct syllogisms.) What matters for the premises of a συλλογισμός is the affirmative or negative character of the predicative relations involved in it “in the way we have said” (τὸν εἰρημένον τρόπον). This last expression refers to what Aristotle stated at the beginning of the chapter, where he divided affirmative and negative propositions, according

to their quantity, into “universal,” “particular,” and “indefinite.” Therefore, what is relevant in the premises of a συλλογισμός is their quality and quantity. It is because premises are quantified and qualified in specific ways that the conclusion follows. If the definition of συλλογισμός includes the idea that what is defined is a deduction in which σ follows from π_1, \dots, π_n in virtue of their being affirmative or negative, universal or particular, or indefinite, one can easily explain why an application of *Modus Ponens* cannot be classified as a συλλογισμός. Q ’s following from “ $P \rightarrow Q$ ” and “ P ” is not in virtue of P and Q being affirmative or negative, universal or particular, nor is it required to deduce that the propositions involved have a predicative structure. Although this is a deduction and a real one, we are not entitled to call it a συλλογισμός.

On the other hand, the theorem stating that every συλλογισμός “comes about in one of these figures” retains its significance and is not reduced to a trivial truism. Aristotle’s thinking may be something like this. Suppose that we have a deduction in which the conclusion is established in virtue of its premises being affirmative or negative and universally or particularly (or indefinitely) quantified, and suppose that none of these premises are missing and none are redundant. Are we entitled to claim that this deduction is a syllogism in one of the figures? The question is far from being banal, and I am not at all sure that Aristotle’s affirmative answer is wrong. His attempt to demonstrate his claim is far from being overtly incorrect, as it would seem if we attributed to him the absurd view that every deduction takes a syllogistic form.

The picture that emerges from this analysis is that Aristotle’s definition of συλλογισμός in fact includes only those deductions that we usually call “syllogisms.” Aristotle seems to be aware of other forms of deduction, which in his view are as compelling as his own syllogisms, although he seems to be unable to deal with them. He does not seem to recognize the logical form that lies behind them. They work, but he is not able to tell us why. It will be a great merit of the Stoics to analyze some of them from a logical point of view. But one logician cannot do everything, and we can confidently say that Aristotle has done enough for the development of this discipline.²⁷

Appendix

With regard to the first example of text (F), let us suppose that “ $\Sigma(x)$ ” stands for “ x is a substance,” “ $\neg\Sigma(x)$ ” for “ x is not a substance” (or maybe “ x is a nonsubstance”), “ $D(x,y)$ ” for “ y is destroyed by removing x from y ,” and finally “ $x \leq y$ ” for “ x is a part of y .” What we have to prove is

$$1 \quad \forall x \forall y (\neg\Sigma(x) \wedge \Sigma(y) \rightarrow \neg D(x,y)), \forall x \forall y (x \leq y \rightarrow D(x,y)) \vdash \forall x \forall y (x \leq y \wedge \Sigma(y) \rightarrow \Sigma(x)).$$

Proof

1	(1) $\forall x \forall y (\neg\Sigma(x) \wedge \Sigma(y) \rightarrow \neg D(x,y))$	A
2	(2) $\forall x \forall y (x \leq y \rightarrow D(x,y))$	A
3	(3) $a \leq b \wedge \Sigma(b)$	A
4	(4) $\neg\Sigma(a)$	A
1	(5) $\forall y (\neg\Sigma(a) \wedge \Sigma(y) \rightarrow \neg D(a,y))$	1 UE
1	(6) $\neg\Sigma(a) \wedge \Sigma(b) \rightarrow \neg D(a,b)$	5 UE
2	(7) $\forall y (a \leq y \rightarrow D(a,y))$	2 UE
2	(8) $a \leq b \rightarrow D(a,b)$	7 UE
3	(9) $a \leq b$	3 \wedge E
3	(10) $\Sigma(b)$	3 \wedge E
2, 3	(11) $D(a,b)$	8, 9 MPP
2, 3	(12) $\neg\neg D(a,b)$	11 DN
1, 2, 3	(13) $\neg(\neg\Sigma(a) \wedge \Sigma(b))$	6, 12 MTT
3, 4	(14) $\neg\Sigma(a) \wedge \Sigma(b)$	4, 10 \wedge I
1, 2, 3, 4	(15) $\neg(\neg\Sigma(a) \wedge \Sigma(b))$ $\wedge (\neg\Sigma(a) \wedge \Sigma(b))$	13, 14 \wedge I
1, 2, 3	(16) $\neg\neg \Sigma(a)$	4, 15 RAA
1, 2, 3	(17) $\Sigma(a)$	16 DN
1, 2	(18) $a \leq b \wedge \Sigma(b) \rightarrow \Sigma(a)$	3, 17 CP
1, 2	(19) $\forall y (a \leq y \wedge \Sigma(y) \rightarrow \Sigma(a))$	18 UI
1, 2	(20) $\forall x \forall y (x \leq y \wedge \Sigma(y) \rightarrow \Sigma(x))$	19 UI

Q.e.d.

It is pretty clear from this formalization that the premises to be added in the argument in order to reach the conclusion are only logical premises, with no further assumption concerning substances or destruction or parts of substances being required.

The formalization of the second example considered by Aristotle in text (F) depends on how we translate εἰ ἀνθρώπου ὄντος ἀνάγκη ζῶον καὶ ζώου οὐσίαν, ἀνθρώπου ὄντος ἀνάγκη οὐσίαν εἶναι (47a28–30). If we translate this sentence as we have done earlier (“if it is necessary for something to be an animal if something is a man, and for something to be a substance if something is an animal, then it

is necessary for something to be a substance if something is a man”), then what must be proven is:

$$2 \quad \exists x A(x) \rightarrow \exists x Z(x), \exists x Z(x) \rightarrow \exists x \Sigma(x) \vdash \exists x A(x) \rightarrow \exists x \Sigma(x)$$

Proof

1	(1) $\exists x A(x) \rightarrow \exists x Z(x)$	A
2	(2) $\exists x Z(x) \rightarrow \exists x \Sigma(x)$	A
3	(3) $\exists x A(x)$	A
1, 3	(4) $\exists x Z(x)$	1, 3 MPP
1, 2, 3	(5) $\exists x \Sigma(x)$	2, 4 MPP
1, 2	(6) $\exists x A(x) \rightarrow \exists x \Sigma(x)$	3, 5 CP

Q.e.d.

Another way of formalizing the sentences of the argument in the translation we have adopted would be by supplying a determinate individual as the missing subject of “ἄνθρωπου ὄντος” and the like. Therefore, the first premise might take the following form:

$$A(a) \rightarrow Z(a).$$

The other premises would be formalized accordingly. The basic structure of the proof does not change. More or less the same happens if we prefer the other translation (“if it is necessary for an animal to be if a man is, and a substance if an animal is, then it is necessary for a substance to be a man”). The formalization depends on the way we take “being” in propositions such as “a man is.” If we take it as a predicate, which may be expressed by “ $E(x)$,” then the first premise of the argument would be either

$$i \quad \exists x (E(x) \wedge A(x)) \rightarrow \exists x (E(x) \wedge Z(x))$$

or

$$ii^* \quad E(a) \wedge A(a) \rightarrow E(a) \wedge Z(a)$$

Alternatively we may prefer to take “being” as a second-order predicate. Consequently, the first premise becomes:

$$\exists x (x = a \wedge A(x)) \rightarrow \exists x (x = a \wedge Z(x)).$$

Other possible formalizations might be proposed, but none of them seem to affect the proof in any relevant way. On the contrary, the choice of one of them could be relevant for the reduction to a syllogistic form of the argument. But that is a different story.

Notes

- 1 Barnes 1981: 17–59.
- 2 Smith 1989; Striker 1996: 203n2.
- 3 For the time being, I take “syllogism” as a transliteration rather than a translation of “συλλογισμός.”
- 4 Barnes 1981: 22–23. Barnes uses capitals for “inference,” “deduction,” and “syllogism” to indicate that they have a “semi-technical sense” (22).
- 5 Barnes 1981: 23.
- 6 This interpretation is confirmed by what Barnes says in Barnes 1980: 161–162 and 168, where he explains the clause “τῷ ταῦτα εἶναι” as follows: “if $\langle \sigma, \Pi \rangle$ is a *sullogismos* then if the members of it are true, σ follows *by virtue of their truth*” (emphasis in the original).
- 7 The form of this example is considered by Aristotle at *Prior Analytics* 2.17, 65b22–28.
- 8 Aristotle, *Prior Analytics* 2.2, 53b26–35.
- 9 Other definitions of “συλλογισμός” are given in *Topics* 1.1, 100a25–27; *Rhetoric* 1.2, 1356b15–16; *Sophistical Refutations* 1, 165a 1–2. It is easy to see that these alternative definitions do not differ in a substantial way from the characterization of the *Prior Analytics*.
- 10 Tarski has clearly stated what the intuitive counterpart of his formal definition of logical consequence is in Tarski 1983: 414–415.
- 11 I am delighted to point out that in a more recent paper Jonathan Barnes (Barnes 1990: 7–119) makes his point in a much more acceptable way when he says: “Aristotle’s definition of a συλλογισμός requires that the conclusion must not only follow from the premises – it must also come about because of the premises.”
- 12 I am grateful to Hermann Weidemann and Ulrich Nortmann, who made me aware of this point.
- 13 For instance, Aristotle, *Prior Analytics* 1.3, 72b24.
- 14 I defend this interpretation in Mignucci 1996: 47–48. An unpublished paper by Elena Gobbo on the meaning of the expression “τῷ ταῦτ’ εἶναι” kindly forwarded by the author has made me aware that my view was wrong. A written commentary by Paolo Fait on my article and further conversations with him and Andrea Falcon have confirmed the implausibility of my previous position. Elena Gobbo has also pointed out to me the passages from *Topics* 8.11, 161b19 ff. and *Sophistical Refutations* 6, 168b22–25 where the expression “τῷ ταῦτα εἶναι” is used. I will discuss these texts in due course. I am glad to say that I share her general interpretation, to which I have tried to provide further support.
- 15 Aristotle, *Prior Analytics* 2.17, 65a38 ff.
- 16 See Cavini 1991: 29–30.
- 17 See, for instance, Barnes 1980: 168 ff.; Cavini 1991: 29–31; and especially Thom 1981: 27–31.
- 18 An alternative translation could be: “if it is necessary for an animal to be if a man is, and a substance if an animal is, then it is necessary for a substance to be a man.”
- 19 The same holds for the other example mentioned by Aristotle. See Appendix.
- 20 See also Frede 1974b.
- 21 See, for instance, what Aristotle says about the different relations that ὑπαρχεῖν is supposed to cover in *Prior Analytics* 1.36, 48a40 ff.
- 22 Aristotle, *Sophistical Refutations* 6, 168a28–33.
- 23 Alternatively one could translate: “it will now be evident that this applies in general to every syllogism.”
- 24 See Barnes 1997: 153–166.
- 25 Aristotle, *Prior Analytics* 1.23, 41a21 ff.; 1.44, 50a16. In the latter passage, Aristotle explicitly claims that συλλογισμοί from a hypothesis cannot be reduced to a syllogistic figure (50a16–19), whereas in the former he seems to imply that they can be reduced

(41a37–38). This formal contradiction has been explained by Jonathan Barnes (Barnes 1997: 162–164) in a way that avoids a substantial inconsistency.

26 Barnes 1997: 164–165.

27 It is a great pleasure and honor for me to dedicate this chapter to Jacques Brunschwig whose subtle, elegant, and learned approach to ancient philosophical texts I have tried to learn for many years. I hope that he will forgive my clumsy attempt and accept it as a sign of friendship and esteem.

2 Expository proofs in Aristotle's syllogistic

I

It is well known that “ἐκθεσις” and the connected verb “ἐκτιθέναι” have several meanings in the *Corpus Aristotelicum*.¹ These words are normally rendered by “exposition” and “to expose,” and I have no reason for rejecting this translation, although it does not seem to me ideal. In the *Analytics*, our terms have at least two technical meanings. The first, not considered here, relates “ἐκθεσις” and “ἐκτιθέναι” to the operation of setting out terms in order to put informal arguments into a syllogistic form,² or to the operation by which terms are substituted for schematic letters in a syllogism.³ In their second meaning, “ἐκθεσις” and “ἐκτιθέναι” refer to a particular kind of proof used by Aristotle to prove some syllogisms and the conversion law for universal negative propositions (hereafter the “*e*-conversion law”). Here I am concerned with this second meaning.

Five passages of the *Prior Analytics* involve, directly or indirectly, the proof by exposition. I list them together with the subject of the proof:

- | | |
|-------------------|--|
| (a) 1.2, 25a14–17 | proof of the <i>e</i> -conversion |
| (b) 1.6, 28a2–26 | proof of Darapti |
| (c) 1.6, 28b14–15 | proof of Datisi and Disamis |
| (d) 1.6, 28b17–21 | proof of Bocardo |
| (e) 1.8, 30a6–14 | proof of Baroco LLL and Bocardo LLL ⁴ |

In (b), (c), and (e) ἐκθεσις or ἐκτιθέναι is explicitly mentioned, but not in (a) and (d). However, a reference to the proof by exposition is certain even in these passages because the implied arguments present obvious structural similarities with the arguments in which exposition is explicitly mentioned.

More importantly, in (b), (c), and (d) exposition is offered as an additional proof to be put alongside the standard ones, that is, the usual proofs by conversion or *reductio ad impossibile*. But in the case of (a) and (e) exposition is the only proof proposed by Aristotle, and this simple fact casts doubts upon Łukasiewicz's statement, according to which ἐκθεσις is of little importance in Aristotle's assertoric logic because of its redundancy.⁵ Because the *e*-conversion law is proven by exposition and no other method of proof is suggested, admitting or rejecting this kind

of proof *does* make a difference for the system created by Aristotle.⁶ Of course, I do not claim that reconstructing Aristotle's assertoric syllogistic without using ἔκθεσις is impossible. But in so far as our concern is a historical one, we cannot put aside Aristotle's use of ἔκθεσις simply because it is not clear what sort of proof it is, and we have to try to understand it and its relevance for Aristotle's logic.

The arguments in which ἔκθεσις is used or mentioned are so short and compressed that it is not obvious how to reconstruct their logical structure. As a result, we have to rely on some conjectures. It thus becomes easy to understand why different interpretations of "exposition" have been given recently. However, as often happens, the obscurities of Aristotle's texts, as well as the different views on logic held by the interpreters, have contributed to make the differences sharper. Maier and Ross, for instance, condemned Aristotle's use of ἔκθεσις as an illegitimate procedure because a reference to perception would be made in it, which is worthless to logic.⁷ This view was dismissed by Łukasiewicz, who proposed an interpretation according to which exposition appears to be a perfectly admissible way of proving logical theses. Today two interpretations seem to impose themselves among scholars. One is the Łukasiewicz interpretation, which has been considerably improved by Patzig.⁸ The other view is shared by a rather heterogeneous group of scholars, some of them pure logicians rather than historians of logic.⁹

I believe that this latter view is the true one, but it has not yet been defended in a convincing way against the Łukasiewicz–Patzig interpretation and corroborated by an analytical examination of the relevant texts. This is the job that I hope to accomplish now.

II

Needless to say, we have to come back to the relevant texts. Let us start with Aristotle's proof of the *e*-conversion, which is as follows:

- (A) (a) First then take AB as a universal negative proposition. (b) If A belongs to no B , then B belongs to no A . (c) For, if B does belong to some A , for example C , it will not be true that A belongs to no B ; (d) for C is one of the B s.
(Aristotle, *Prior Analytics* 1.2, 25a14–17; ROT translation)

All scholars agree that Aristotle's argument is ecthetic, even though exposition is not mentioned. The main lines of the argument are clear. Clauses (a) and (b) state the thesis Aristotle wants to prove, and we may express it by means of:

$$(EC) \quad AeB \vdash BeA.$$

As usual, we take " AxB " as a formalization of " A belongs to . . . B ," where " A " stands for the predicate, " B " for the subject, and " x " for " a " or " e " or " i " or " o ," which denote that the predicative relationship between A and B is universal affirmative or universal negative or particular affirmative or particular negative. In order to hold that (EC) is an adequate formalization of (b), one has to accept at least that (1) the

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letters appearing in it have to be taken as schematic letters, if not as variables, and that (2) the conditional by which (b) is constituted has “ \vdash ” as its formal counterpart.¹⁰ Both assumptions might be challenged, but we cannot pause here to discuss this point.¹¹ Let us, at least temporarily, admit that (EC) expresses what Aristotle says in (b). Clause (c) clearly introduces a *reductio ad impossibile*, which can reasonably be represented in the frame of a natural deduction system as follows.¹² Assume the premise of (EC) and the contradictory of its wanted conclusion:

I

1	[1]	AeB	A^{13}
2	[2]	BiA	$A.$

It is clear that if we were able to derive AiB from BiA , namely to state

2	[n]	AiB
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our conclusion would have been reached in a standard way by writing down:

	[n+1]	$AiB \rightarrow \neg AeB$	TI ¹⁴
2	[n+2]	$\neg AeB$	MPP [n], [n+1]
1, 2	[n+3]	$AeB \wedge \neg AeB$	\wedge I [1], [n+2]
1	[n+4]	$\neg BiA$	RAA [2], [n+3]
	[n+5]	$\neg BiA \rightarrow BeA$	TI
1	[n+6]	BeA	MPP [n+4], [n+5].

The critical step is, of course, the passage from BiA to AiB . We cannot use the conversion law for particular affirmative propositions here without falling into a vicious circle because the conversion law is proven by means of (EC).¹⁵ But even a superficial inspection of the text shows that Aristotle reaches AiB from BiA in an independent way. It is sufficient to notice that in (c) he refers to a term “C,” and such a reference would be incongruous if a straightforward *reductio* had taken place. It is for this reason that the majority of interpreters think that an ecthetic procedure is hinted at here.

The difficult point is to understand how ἔκθεσις works in the proof. According to Günther Patzig, the main idea that grounds a proof by exposition is the following.¹⁶ By asserting that an affirmative particular proposition holds, we claim that there is a general term such that the predicate and the subject of the proposition are true of whatever the term is true of. For instance, if we state that

some animal is biped

we are implicitly committed to admit that there is a general term “biped animal,” such that both “animal” and “biped” are true of whatever “biped animal” is true of. A way to express this idea formally is by means of the following equivalence:

$$(P_1) \quad AiB \leftrightarrow \exists C(AaC \wedge BaC).$$

The same can be easily extended to negative particular propositions so that we can state not only (P₁) but also

$$(P_2) \quad AoB \leftrightarrow \exists C(AeC \wedge BaC).$$

If (P₁) is admitted, then it is easy to derive BiA from AiB by means of the obvious assumption that conjunction is commutative. From (P₁) we obtain

$$(1) \quad AiB \rightarrow \exists C(AaC \wedge BaC)$$

and by commutativity of conjunction we can state

$$(2) \quad \exists C(AaC \wedge BaC) \rightarrow \exists C(BaC \wedge AaC).$$

A further application of (P₁) allows us to write

$$(3) \quad \exists C(BaC \wedge AaC) \rightarrow BiA.$$

By repeating the argument starting from BiA , step $[n]$ is easily obtained in our derivation (I). After having stated $[1]$ and $[2]$, take as line $[3]$

$$[3] \quad BiA \rightarrow \exists C(BaC \wedge AaC) \quad \text{TI (S) (1).}$$

Then everything goes on in a straightforward way:

2	[4]	$\exists C(BaC \wedge AaC)$	MPP [2], [3]
	[5]	$\exists C(BaC \wedge AaC) \rightarrow \exists C(AaC \wedge BaC)$	TI (S) (2)
2	[6]	$\exists C(AaC \wedge BaC)$	MPP [4], [5]
	[7]	$\exists C(AaC \wedge BaC) \rightarrow AiB$	TI (S) (3)
2	[8]	AiB	MPP [6], [7].

At this point the $[n+1]$ to $[n+6]$ steps can be added, which conclude the proof. No additional premise is needed, nor is the i -conversion law invoked, and we obtain a derivation that is perfectly safe from a formal point of view.

Patzig believes that every passage in which $\epsilon\kappa\theta\epsilon\sigma\iota\varsigma$ is involved can be interpreted in this way, namely through (P₁) or (P₂). This means that not only in the case of the e -conversion but in any application of this method, exposition is nothing more than the assertion that a subclass of the subject of a particular proposition exists. In this way, Patzig is entitled to conclude – with Łukasiewicz and against the preceding tradition – that $\epsilon\kappa\theta\epsilon\sigma\iota\varsigma$ is a rigorous method of proof that does not rely on extralogical procedures.¹⁷

III

As a first step toward an understanding of Aristotle's view, I shall try to show that Patzig's interpretation of $\epsilon\kappa\theta\epsilon\sigma\iota\varsigma$ cannot be applied to every case in which it is

supposed to be used by Aristotle. The point is easily made if we consider the passage where exposition is applied to the proof of Darapti. Aristotle says:

(B) (a) It is possible to demonstrate this <i.e., Darapti> both *per impossibile* and by exposition. (b) For if both P and R belong to every S then (c) if one of the S s, e.g., N , is taken ($\alpha\upsilon\lambda\eta\phi\theta\eta\tilde{\iota}\ \tau\iota\ \tau\tilde{\omega}\nu\ \Sigma\ \omicron\iota\omicron\nu\ \tau\tilde{o}\ N$), (d) both P and R will belong to this and thus (e) P will belong to some R .

(Aristotle, *Prior Analytics* 1.6, 28a22–26;
ROT translation, slightly modified)

According to Patzig, the argument runs as follows.¹⁸ We have to prove Darapti:

(DR) $PaS, RaS \vdash PiR$.

From (P_1) we obtain

(4) $\exists N(PaN \wedge RaN) \rightarrow PiR$.

Therefore, if we are able to prove

(5) $PaS \wedge RaS \rightarrow \exists N(PaN \wedge RaN)$,

Darapti is easily derived by transitivity from (4) and (5). But implication (5) can be stated at once, because its consequent represents the existential generalization of its antecedent. Therefore, (DR) must be asserted.

May we be satisfied with this interpretation of the proof of Darapti? An affirmative answer can hardly be given. First of all, (DR) is supposed to be an imperfect syllogism, and according to the leading interpretation proposed by Patzig himself,¹⁹ an imperfect syllogism is a syllogism that is not evident, namely one that needs to be proven in order for us to be convinced that it holds. On the other hand, (4) depends on (P_1) , and its role as an implicit axiom in the proof of the *e*-conversion law shows that it has to be considered an axiom for the whole theory of the syllogism. This fact suggests that (4) and probably (P_1) should be considered evident propositions. But why should we think that (DR) is nonevident and (4) is evident? It seems quite natural to claim that (DR) and (4) have the same epistemological status: They are both either evident or nonevident.

Besides, Patzig's reconstruction of the argument does not correspond precisely to what Aristotle says. In Patzig's interpretation, the expository method is grounded upon the assumption that there exists a general term that satisfies certain requirements. But Aristotle does not speak of the existence of something. He simply refers to "one of the S s, e.g., N ." The most natural way of interpreting this clause is by supposing that " N " is the name of one of these S s, whatever they might be.

To meet the latter remark, one might be led to modify the original interpretation offered by Patzig. One might suppose, in particular, that the core of the expository

method consists in singling out from a particular proposition a general term that is characterized by the fact that both the predicate and the subject of the proposition are true of whatever is denoted by it. In other words, one should assume, once again, that (P_1) and (P_2) hold in general, but exposition would not consist of assessing these equivalences. Rather, it would be characterized by arbitrarily singling out one of the C s that satisfy (P_1) or (P_2) . But in making this move, it is not clear how the proof of Darapti should be recast. Exposition cannot be applied to the conclusion of Darapti, as happens in Patzig's interpretation, because the term exposed must be "one of the S s," and implication (4) just offers the condition from which we can proceed to pick up "one of the S s." On the other hand, if we really single out "one of the S s" in the sense that we isolate a subset of S so that " N^* " is the name of this subset,²⁰ we obtain

(6) SaN^* .

Together with the premises of Darapti, this gives us, by means of two applications of Barbara, the propositions PaN^* and RaN^* , which depend on PaS and RaS . But in order to obtain the desired conclusion, we need something like

(DR#) $PaN^*, RaN^* \vdash PiR$.

Then the obvious question arises: Why should we admit that (DR#) is immediately valid and think that (DR) needs a proof? One might observe that, properly speaking, (DR#) is not (DR), because it contains a predicative constant that is absent from (DR). But this simply means that (DR#) is a special case of (DR). And why should we trust (DR#) if (DR) is not evident or, put in other terms, why should we look for a proof of (DR) if we simply admit (DR#)? Needless to say, there is no logical impossibility in taking an instance of a general principle as reliable without being committed to accepting the general law itself. But the point is that there is no reason to accept (DR#) if (DR) is thought to need proof. (DR#) does not seem to possess special features that make it more reliable than (DR). Therefore, if (DR) is not a primitive statement of the system, (DR#) cannot be thought to be in this privileged situation.

These remarks cast some shadows on Patzig's interpretation of the *e*-conversion proof as well. As we have seen, equivalence (P_1) is invoked to justify both (1) and (3). Now, (3) is logically equivalent to Darapti and differs from it because of the existential quantification.²¹ It is this tiny difference that enables Patzig's interpretation to avoid the charge of circularity. If (3) were the same as Darapti, we would get a vicious circle, because Darapti would be used in the proof of the *e*-conversion law and the latter is a premise in the proof of the *a*-conversion,²² whereas the *a*-conversion is utilized in the proof of Darapti.²³ Although Patzig's interpretation cannot be accused of circularity from a strictly formal point of view, it is still hardly plausible that Aristotle accepted (3) as unquestionable and looked for a proof of (DR). In his view, such a proof would imply a *petitio principii*, because the premise from which it starts is by no means more reliable than its conclusion.²⁴

IV

After these critical remarks one might be led to reject Patzig's interpretation as well as the quasi-Patzig version. But things are not so straightforward. There is a passage that, at least *prima facie*, supports Patzig's interpretation of ἔκθεσις. As Aristotle moves from categorical to modal syllogisms, he points out that we cannot prove Baroco LLL and Bocardo LLL in the same manner by which the corresponding categorical forms are proven, that is, by *reductio ad impossibile*.²⁵ The reason for this is probably that in order to prove Baroco LLL and Bocardo LLL by *reductio* we need mixed syllogisms with one possible and one necessary premise, and these syllogisms are not proven to be valid prior to the time of the proof of Baroco LLL and Bocardo LLL.²⁶ In these cases, Aristotle claims that we have to proceed by exposition. Aristotle says:

(C) (a) But in the middle figure, when the universal premise is affirmative and the particular negative, and again in the third figure when the universal is affirmative and the particular negative, the demonstration will not be the same, (b) but it is necessary to expose that to which the predicate particularly does not belong and (c) make the syllogism in reference to this (κατὰ τούτου ποιεῖν τὸν συλλογισμόν). (d) For the syllogism will be necessary with such terms (ἔσται γὰρ ἀναγκαῖος ἐπὶ τούτων). (e) But if the syllogism is necessary (ἐστὶ ἀναγκαῖος) in respect of the exposed term, it will also be necessary in respect of some of the other terms, because the exposed term is exactly one of the others. (f) Each of the syllogisms is in the appropriate figure (γίνεται δὲ τῶν συλλογισμῶν ἐν τῷ οἰκείῳ σχήματι).

(Aristotle, *Prior Analytics* 1.8, 30a6–14;
ROT translation, slightly modified)

The meaning of the passage is clear in its main lines. In clauses (a) and (b) the general point is made that Baroco LLL and Bocardo LLL are proven by exposition, and in (c)–(f) the argument is developed. By using exposition, new syllogisms can be proposed that have a necessary conclusion. Because the terms exposed have a special relation to the terms originally given, the inference can be made that the conclusions of the syllogisms to be proven are also necessary. Finally, (f) states that the syllogisms are in the appropriate figures. We shall later discuss the possible meaning of this sentence.

If we adopt Patzig's view, a relatively clear explanation of our passage can be given. Consider Baroco LLL:

(BR_L) $LAaB, LAoC \vdash LBoC$.

If we follow Patzig's proof by ἔκθεσις, (BR_L) implies that we have to apply a modal version of (P₂) to it. Patzig does not give us any idea of what such a modal version of (P₂) might look like. He declares himself uninterested in modal logic and drops any reference to modalities in his analysis of the proof.²⁷ In order to

simplify our discussion, we can follow him and consider the nonmodalized version of Baroco:

(BR) $AaB, AoC \vdash BoC$

to which (P₂) can be applied. There is more than one way in which the proof of (BR) can be reconstructed. A first demonstration can easily be put as follows:

II

1	[1]	AaB	A
2	[2]	AoC	A
	[3]	$AoC \rightarrow \exists D(AeD \wedge CaD)$	TI (S) (P ₂) ²⁸
2	[4]	$\exists D(AeD \wedge CaD)$	MPP [2], [3]
5	[5]	$AeD^* \wedge CaD^*$	A
5	[6]	AeD^*	\wedge E [5]
5	[7]	CaD^*	\wedge E [5]
1, 5	[8]	BeD^*	SI (S) Camestres [1], [6]
1, 5	[9]	BoC	SI (S) Felapton [7], [8]
1, 2	[10]	BoC	EE* [4], [5], [9]. ²⁹

To perform this proof we need not only an elimination rule for the existential quantifier on general terms but for Camestres and Felapton as well. This is actually the reconstruction of the proof that Patzig more or less has in mind, and *prima facie* it appears to match the text rather well.

On closer inspection, however, a difficulty emerges. Patzig takes clause (f) as referring to the syllogisms by means of which the proofs by ἔκθεσις are carried out, and he thinks that Aristotle says that these syllogisms are in the same figures as the syllogisms to be proven. Therefore, the expository proof of Baroco is made by means of a syllogism in the second figure, and the proof of Bocardo with the help of a syllogism in the third figure. And this is true in Patzig's interpretation, because the proof of Baroco is carried out by means of Camestres and the proof of Bocardo by means of Felapton. But as we have seen, in the proof of Baroco not only Camestres but also Felapton is used, which is not in the second but in the third figure, and the same can be repeated for the proof of Bocardo. We must use not only Felapton, which is in the third figure, but also Barbara, a first-figure syllogism. If Aristotle had in mind the sort of proof that Patzig attributes to him, why should he have claimed that the syllogisms used in the proofs are in the same figures as the syllogisms to be proven?

One might answer this objection by reconstructing Aristotle's argument in a slightly different way. We simply repeat the argument up to and including step [8], and we put instead of [9]:

1, 5	[9*]	$BeD^* \wedge CaD^*$	\wedge I [7], [8]
1, 5	[10*]	$\exists D(BeD \wedge CaD)$	EI [9*]

	[11]	$\exists D(BeD \wedge CaD) \rightarrow BoC$	TI (S) (P ₂)
1, 5	[12]	BoC	MPP [10*], [11]
1, 2	[13]	BoC	EE* [4], [5], [12].

By exploiting the full force of (P₂) we are able to avoid recourse to Felapton, and in this way the objection to the Patzig interpretation is answered.

Unfortunately, the same maneuver cannot be repeated in the case of Bocardo

(BC) $AoC, BaC \vdash AoB$

which is considered by Aristotle in its modal version. In light of Patzig's interpretation, its ecthetic proof can be reconstructed as follows:

III

	[1]	AoC	A
	[2]	BaC	A
	[3]	$AoC \rightarrow \exists D(AeD \wedge CaD)$	TI (S) (P ₂)
1	[4]	$\exists D(AeD \wedge CaD)$	MPP [1], [3]
5	[5]	$AeD^* \wedge CaD^*$	A
5	[6]	AeD^*	\wedge E [5]
5	[7]	CaD^*	\wedge E [5]
2, 5	[8]	BaD^*	SI (S) Barbara [2], [7]
2, 5	[9]	AoB	SI (S), Felapton [6], [8]
1, 2	[10]	AoB	EE* [4], [5], [9].

It is easy to see that in this case resorting to Barbara cannot be avoided even if we use (P₂) in its full force and Barbara is not in the same figure as Bocardo. We must therefore give up Patzig's general interpretation of ἔκθεσις, or at least his understanding of (f).

V

Because of the weakness of Patzig's position we have to turn to the other interpretation of ἔκθεσις that has found some supporters recently. The idea is simple and straightforward. By "exposing a term" Aristotle means the operation that in modern logical language we are accustomed to calling "existential instantiation."³⁰ To explain the point, let us reconsider the proof of the *e*-conversion law. As we have seen, the crucial point in the proof is constituted by the deduction of AiB from BiA . To complete the deduction we do not appeal to (P₁), as in Patzig's interpretation, but rather we reflect on BiA . By stating BiA , we are committed to saying that there is at least one individual that is both *B* and *A*. Exposing a term with respect to a particular proposition, according to this interpretation, consists in singling out an arbitrary individual that is both *A* and *B*. Following Aristotle, let us suppose that this individual is *c*. In this way, we can easily reach AiB , and the proof of (EC) can be carried out along the following lines:

IV

1	[1]	AeB	A
2	[2]	BiA	A
3	[3]	$A(c) \wedge B(c)$	A
3	[4]	$A(c)$	$\wedge E$ [3]
3	[5]	$B(c)$	$\wedge E$ [3]
3	[6]	$B(c) \wedge A(c)$	$\wedge I$ [4], [5]
3	[7]	AiB	EI [7] ³¹
2	[8]	AiB	EE [2], [3], [7].

At this point we can continue with the final part of the *e*-conversion proof we have already sketched in derivation (I).

The argument is formally correct and involves a treatment of existentially quantified propositions that correspond exactly to the existential instantiation of modern logic. The difference with respect both to Patzig's and to the quasi-Patzig interpretation is also clear. In Patzig's view, the existence of a subclass of a general term was supposed to be in question, and in the quasi-Patzig interpretation a general term was singled out. In this interpretation, an individual term is picked out, and in this way the main objection against Patzig's position is avoided. The exposed term is a well-determined individual term, and in this way Aristotle's move of introducing a new term in the expository proof is explained much better than on Patzig's interpretation. On the other hand, by considering an individual term we are not involved in the difficulties of the *petitio principia*, which can be leveled against both the Patzig and the quasi-Patzig interpretation of the proof of Darapti.

Notwithstanding these advantages, the view we are considering is far from being completely reassuring. Two objections must be examined. First of all, it is not at all perspicuous how the expository proof of Darapti can be developed if we suppose that exposition is something similar to the modern existential instantiation. As is said in text (B), the premises of Darapti, namely PaS and RaS , are given and an n is considered, which is supposed to be one of the S s to which both P and R belong. Then the problem arises. Existential instantiation applies to particularly, but not universally, quantified propositions, and the premises of Darapti are both universal.

One could try to solve this difficulty by supposing that a full reconstruction of Aristotle's argument for Darapti must be formulated along the following lines. Suppose, as the interpretation demands, that the exposed term n mentioned by Aristotle stands for an individual. As is well known, if we interpret the premises of Darapti by means of normal first-order predicate logic, we have to assume an extra premise in order to obtain the conclusion, and this premise is

(EP) $\exists xS(x)$.

It is with respect to this premise that the exposition can be carried out. We have the following derivation:

V

1	[1]	PaS	A
2	[2]	RaS	A
3	[3]	$\exists xS(x)$	A
4	[4]	$S(n)$	A
1	[5]	$S(n) \rightarrow P(n)$	UE [1] ³²
1, 4	[6]	$P(n)$	MPP [4], [5]
2	[7]	$S(n) \rightarrow R(n)$	UE [2]
2, 4	[8]	$R(n)$	MPP [4], [7]
1, 2, 4	[9]	$R(n) \wedge P(n)$	\wedge I [6], [7]
1, 2, 4	[10]	PiR	EI [9]
1, 2, 3	[11]	PiR	EE [3], [4], [10].

In this way, Darapti is derived under the assumption that (EP) holds. By taking this proof as an expansion of the Aristotelian one, it might be claimed that ἔκθεσις is applied when an individual that makes $\exists xS(x)$ true is selected. In this way, we obtain the usual existential instantiation that was at work in the proof of the *e*-conversion law.

Although this reconstruction of the Aristotelian argument is perfectly safe from a logical point of view, it is open to an easy objection. (EP) is not an explicit premise of Darapti. We only need it because we put ourselves in a perspective that differs from that embraced by Aristotle. Because we recognize the possibility that nondenoting general terms may appear in the premises of a syllogism, we have to add an extra premise to the premises of Darapti to get its conclusion. While Aristotle seems to have overlooked this possibility, if we restrict ourselves to denoting terms, we do not need (EP) to prove Darapti. Therefore, it is hardly plausible that the exposition implied by Aristotle here consists of an existential instantiation over (EP), which is not part of the premises of the Aristotelian Darapti. If we take this objection seriously, we may conclude that exposition has nothing to do with existential instantiation in this case. We might guess instead that it must be rather compared with what is nowadays called “universal instantiation.” If this is the case, ἔκθεσις would refer to the transition not from line [3] to line [4], but from [1] to [5] and from [2] to [7], and it should be regarded as a rather ambiguous term covering different logical processes that one might well prefer to keep distinct.

I do not think that the objection we are considering is as effective as it first appears. As is clear, when giving his account of Darapti – and in general also of assertoric and modal syllogisms – Aristotle implicitly limits himself to a consideration of denoting terms. But one could say that because he has implicitly made such a restriction, he is allowed to consider one of these (let it be n), which is both P and R . In other words, because there are S s, one is entitled to single out one of these, namely n , which is both P and R , because it has been supposed that every S is P and R . Of course, when we fully formalize his argument, we are compelled to posit (EP) as an explicit premise, that is, to make precise the informal assumption about denoting general terms. But if the preliminary assumption is made

that every term constituting the propositions of Darapti is a denoting term, we are allowed to introduce [3] not as an assumption but as a theorem and substitute

$$[3^*] \quad \exists xS(x) \quad \text{TI (EP)}$$

for [3]. Consequently, line [11] should be replaced with

$$1, 2 \quad [11^*] \quad PiR \quad \text{EE } [3^*], [4], [10]$$

which corresponds exactly to (DR). In this way, we can also explain why Aristotle does not put $\exists xS(x)$ among the premises of Darapti, because it is not a premise in the same sense in which PaS and RaS are. Whereas PaS and RaS are assumptions, in his perspective $\exists xS(x)$ is not. In fact, Aristotle's argument for Darapti, as it is expounded in text (B), corresponds quite well to this reconstruction, which is based on interpreting ἔκθεσις as existential instantiation. Clause (c) states that one of the Ss , namely n , must be selected, and this corresponds to the assumption [4], which must be taken with reference to axiom [3*]. Clause (d) is affirmed as a consequence of what is said in (b) and (c), and this move finds an exact parallel in statement [9], which is said to depend on assumptions 1, 2, and 4. Finally, (e) is the informal equivalent of the existential generalization by which [10] is inferred from [9].

VI

The hypothesis that ἔκθεσις corresponds to existential instantiation must be confronted with text (C) – that is, the ecthetic proof of Baroco LLL and Bocardo LLL – before we can be certain of its reliability. The problems raised by this issue are not easily solved. If we take exposition as existential instantiation, a reasonable way to recast Aristotle's proof goes as follows. To simplify it, we can drop the reference to modal operators as before and reformulate Baroco LLL into Baroco so that its proof begins in the following way:

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1	[1]	AaB	A
2	[2]	AoC	A
3	[3]	$C(n) \wedge \neg A(n)$	A
3	[4]	$C(n)$	$\wedge E [3]$
3	[5]	$\neg A(n)$	$\wedge E [3]$.

At this point we could go on by applying the UE-rule to [1] in order to get $B(n) \rightarrow A(n)$; then by an application of *Modus Tollens* to it $\neg B(n)$ can be obtained. But in (c) Aristotle speaks of a syllogism that is directed to n , whereas *Modus Tollens* and the UE-rule are not syllogisms. To stay closer to Aristotle's text, we could use

$$(\text{MTT}^*) \quad AaB, \neg A(n) \vdash \neg B(n)$$

in order to reach $\neg B(n)$. The derivation can easily be completed as follows:

1, 3	[6]	$\neg B(n)$	SI (MTT*) [1], [5]
1, 3	[7]	$C(n) \wedge \neg B(n)$	\wedge I [4], [5]
1, 3	[8]	BoC	EI [7]
1, 2	[9]	BoC	EE [2], [3], [8].

The proof is sound, but this is not sufficient for us to accept it. We must ask whether it corresponds to the Aristotelian one. One might say “no.” Aristotle asserts in (b) that a term must be exposed, and in (c) he adds that we have to “make the syllogism in reference to” such a term. Now in the proof of Baroco that we have sketched there is, properly speaking, no syllogism by means of which a conclusion about n , the exposed term, is drawn. To obtain line [6] we have used (MTT*), and (MTT*) is no syllogism at all. If our reconstruction of the proof were correct, we would have to conclude that (MTT*) is called a “syllogism” by Aristotle, and this is hard to believe.

One might try to reply to this objection in two ways. First, one might say that “syllogism” in clause (c) – and in (d), it should be added – has the generic meaning of inference, as sometimes happens in Aristotle.³³ And, of course, (MTT*) is an inference but not a syllogism. This sort of reply is immediately blocked by the obvious remark that in clause (f) an allusion is made to the syllogisms used in the expository proof of Baroco LLL and Bocardo LLL, and they are said to be in the appropriate figure, which means that here “syllogism” has to be taken in the strict sense.

The other attempt to avoid the objection consists in denying that Aristotle clearly distinguishes between Camestres and (MTT*) by assuming that a singular proposition must be treated as a universal one. In this way, we might consider a proper name as a predicate and expand $A(n)$ into AaN (where “ N ” is the predicate corresponding to the proper name “ n ”), so that (MTT*) would be transformed into

$$(MTT_s) \quad AaB, AeN \vdash BeN$$

which is in fact an instance of Camestres. This would also explain Aristotle’s statement in (f) that the syllogism by which Baroco is proven, *namely* Camestres, is in the same figure as Baroco itself.³⁴

Although the idea of considering singular propositions as universal is not new in the history of logic, there is no sure ground for attributing it to Aristotle. But there is a more specific reason that leads one to reject this line of reasoning. It is sufficient to consider the proof of Bocardo LLL, which we shall simplify into a proof for Bocardo. As before, to make our reconstruction more faithful to Aristotle’s way of developing the proof, we assume

$$(MPP^*) \quad AaB, B(n) \vdash A(n)$$

which parallels (MTT*). By taking exposition as existential instantiation, the derivation is as follows:

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1	[1]	AoC	A
2	[2]	BaC	A
3	[3]	$C(n) \wedge \neg A(n)$	A
3	[4]	$C(n)$	$\wedge E [3]$
3	[5]	$\neg A(n)$	$\wedge E [3]$
2, 3	[6]	$B(n)$	SI (S) (MPP*) [2], [4]
2, 3	[7]	$B(n) \wedge \neg A(n)$	$\wedge I [5], [6]$
2, 3	[8]	AoB	EI [7]
1, 2	[9]	AoB	EE [1], [3], [8].

The proof runs perfectly parallel to the previous one, with the difference that in the former (MTT*) was used, whereas here (MPP*) is employed. If we transform (MPP*) in the way we have transformed (MTT*) into (MTT_s), we obtain:

(MPP_s) $AaB, BaN \vdash AaN$.

This inference is a special case of Barbara for the same reason that (MTT_s) was taken to be an instance of Camestres. But if so, the point made in (f) is no longer satisfied, because Barbara is not in the same figure as Bocardo.

To try to escape these difficulties, let us first sum up the results of our discussion. Our attempts at finding an answer to the objection about the adequacy of our reconstruction of the proof of Baroco LLL and Bocardo LLL have been in the end unsuccessful because of the interpretation given to clause (f). We have taken τῶν συλλογισμῶν ἑκάτερος to refer to the syllogisms used in the proof of Baroco LLL and Bocardo LLL. Because these syllogisms are said to be in the appropriate figure, they must be *sensu stricto* syllogisms and in the same figures as the syllogisms of which they demonstrate the validity.³⁵ Thus, we are on the horns of a dilemma. Either we must say that the reconstruction of the proofs, which was based on (MTT*) and (MPP*), does not fit Aristotle's words, because (MTT*) and (MPP*) are not *sensu stricto* syllogisms; or, if we admit that (MTT*) and (MPP*) are instances of syllogisms, then we cannot fulfill the condition that these syllogisms must be in the same figure as the syllogisms to be proven.

At this point the question must be raised whether the interpretation of clause (f) we have given is the only possible one. One might suppose that τῶν συλλογισμῶν ἑκάτερος refers not to the syllogisms used in the proofs but to Baroco LLL and Bocardo LLL themselves. To explain the point we have to examine the meaning that “συλλογισμός” has all along in text (C). Consider what Aristotle says at 30a10–11: ἔσται γὰρ ἀναγκαῖος ἐπὶ τούτων (see (d)). The word “συλλογισμός” is clearly understood here, and this word cannot properly refer to the whole inference. What is said to be necessary is not the inference, but its conclusion. If so, “κατὰ τούτου ποιεῖν τὸν συλλογισμόν” at 30a10 must be taken to mean “draw the conclusion with reference to this,” that is, derive the conclusion by taking the exposed term n as the term of which it is said in the conclusion that it is both C and $\neg B$. The same is confirmed by what is said in (e): Because the conclusion of

the inference is necessary with respect to the exposed term, that is, because it is necessary that both $C(n)$ and $\neg B(n)$, then we can infer that there is at least one individual that is both C and $\neg B$. In this case, clause (f) adds that this conclusion is drawn in its own figure, and we may take this to mean that the conclusion depends on the original antecedent, which is in the second figure in the case of Baroco and in the third figure in the case of Bocardo. In this sense, clause (f) would be hinting at the relevant fact that the conclusion of Baroco, BoC , which in line [8] of the proof is said to depend on assumptions 1 and 3, can be said to depend on the proper premises of the syllogism, as is stated in line [9] of the derivation. If this interpretation of (f) is accepted, we can plainly reconstruct the ecthetic proofs of Baroco LLL and Bocardo LLL in the way described earlier and maintain that “exposition” refers to one and the same logical operation: existential instantiation.

Notes

- 1 Cf. Bonitz 1955², s.v. “ἐκτιθέναι,” 231b50 ff. See also Maier 1969–1970³: II.2, 141–143.
- 2 See, for instance, Aristotle, *Prior Analytics* 1.34, 48a1, 25; 1.39, 49b6.
- 3 For instance, Aristotle, *Prior Analytics* 1.10, 30b31–32; 2.4, 57a35. See also Ross 1949: 311; Patzig 1969: 167–168.
- 4 As usual, “LLL” refers to the modal condition of the premises and conclusion of a syllogism and means that they are all necessary.
- 5 Łukasiewicz 1957²: 59. This view is endorsed in Patzig 1969: 166–167.
- 6 Cf. Albrecht and Hanisch 1970: 46.
- 7 Cf. Maier 1969–1970³: II.2, 144–149; Ross 1949: 32.
- 8 Łukasiewicz 1957²: 59; Patzig 1969: 166–180. Similar are Berka 1958: 150–151; Rose 1968: 36; Blanché 1970: 54n1; Rescher 1974: 3–4, 10–12; Wieland 1966: 36–37. Granger 1976: 132 seems to be not very far from Patzig’s basic idea. Kneale and Kneale 1978⁷: 77 are more cautious, but in the end they give up offering a general interpretation of ἐκθεσις.
- 9 Beth 1959: 191–192; Beth 1971: 44; Hintikka 1973b: 237–240; Lear 1980; Smith 1982: 113–127. See also Thom 1981: 164–174.
- 10 For the distinction between schematic letters and variables see Quine 1974: 141–142.
- 11 Assumption (i) has been challenged to some extent by Hadgopoulos 1975: 133; Frede 1974b: 18–19; *contra* Patzig 1988: 272–273. Assumption (2) should be rejected by supporters of the view held by Łukasiewicz 1957²: 1–3; Patzig 1969: 11–14, according to which Aristotelian syllogisms must be formalized as conditionals.
- 12 Corcoran 1974: 85–132 has convincingly shown that the best formalization of Aristotle’s *reductio ad impossibile* is by means of a natural deduction system.
- 13 For this notation, as well as others used in this chapter, I refer the reader to Lemmon 1965.
- 14 The theorem hinted at here is based on the obvious equivalence $AiB \leftrightarrow AeB$.
- 15 Cf. Aristotle, *Prior Analytics* 1.2, 25a20–22.
- 16 Cf. Patzig 1969: 171 ff.
- 17 Cf. Patzig 1969: 169–170; Łukasiewicz 1957²: 60–62.
- 18 Cf. Patzig 1969: 174.
- 19 Cf. Patzig 1969: 51 ff.
- 20 I write “N*” with an asterisk against “N” to stress that “N*” stands for a constant, not for a schematic letter.
- 21 As is easy to see, the other main difference between (DR) and (3) is that (3) has the form of a conditional, while (DR) is presented as a sequent. But (DR) can easily be recast in

the form of a conditional or (3) in the form of a sequent. Therefore, it is not by pointing at this difference that one might hope to escape from the difficulty we are considering.

22 Cf. Aristotle, *Prior Analytics* 1.2, 25a17–19.

23 Cf. Aristotle, *Prior Analytics* 1.6, 28a17–22.

24 A similar criticism of the Łukasiewicz-Patzig view can be found in Smith 1982: 118. Adopting Patzig's revised interpretation is no help in avoiding these difficulties.

25 Cf. Aristotle, *Prior Analytics* 1.8, 30a3 ff.; 1.5, 27a36–b1.

26 To be precise, in order to prove Baroco LLL, we need a Barbara syllogism with a possible major and a necessary minor premise, and Aristotle never considers such a syllogism. The mixed syllogisms he analyzes have two-sided possible premises, whereas here one-sided possible premises are required.

27 Cf. Patzig 1969: 166 and 176–177.

28 Properly speaking, [3] is an immediate consequence of (P₂).

29 By “EE*” we refer to the elimination rule of the existential quantifier for general terms.

30 Cf. Quine 1974: 162–163.

31 We assume that $\exists x(B(x) \wedge A(x))$ is logically equivalent to AiB , and this allows us to get AiB from $B(c) \wedge A(c)$ by applying the introduction rule for the existential quantifier.

32 AaB must in general be considered as defined by $\forall x(B(x) \rightarrow A(x))$, and this fact allows us to infer $S(n) \rightarrow P(n)$ from PaS by an application of the elimination rule for the universal quantifier.

33 For instance, the definition of syllogism at the beginning of the *Prior Analytics* (1.1, 24b18–22) refers to the generic meaning of the word, and it does not apply to the special inferences of which Aristotle offers a theory in the following chapters.

34 Aristotle admits that *Modus Ponens* and *Modus Tollens* cannot be reduced to syllogisms in the proper sense (cf. *Prior Analytics* 1.23, 41a23 ff.; 1.44, 50a16 ff.). In principle, this remark does not necessarily rule out the possibility that (MTT*) could be viewed as a special case of Camestres.

35 This is the interpretation of (f) proposed by Patzig 1969: 176–177.

3 The Stoic *themata*

I

One of the classical problems of Aristotle's syllogistic is how to interpret the reduction of imperfect to perfect syllogisms. Second- and third-figure syllogisms are imperfect, that is, not self-evident syllogisms, and they are shown to be valid by linking them to the perfect ones, namely the syllogisms of the first figure.¹ This link is normally interpreted as a deduction of imperfect syllogisms from perfect ones by means of some implicit rules of inference and some previously stated theorems mainly concerning the conversions of propositions. Aristotle does not make explicit the arguments by which imperfect syllogisms are derived from perfect syllogisms, and scholars have proposed different ways to formalize these deductions.² What is common to all these views is that they impose something on the text, that is, a formalization of an informal reasoning that is only sketched.

It is the Stoics who produced a sort of formalization of a metalinguistic theory of deduction, although the Aristotelians, as usual, claimed that Aristotle had already thought of everything and that the Stoics merely copied him by adding a few useless details.³ The Stoics presented five types of arguments, which they called ἀναπόδεικτοι, “indemonstrables.” Among them were a version of *Modus Ponens*, a version of *Modus Tollens*, and three other elementary inferences of conjunction and disjunction. I am not here concerned with the problems of a general definition of indemonstrable arguments. For my purpose, it is better to consider a passage from Diogenes Laertius in which indemonstrable inferences are distinguished from other inferences that are said to reduce to the indemonstrable ones by means of one or more *themata*. The text runs as follows:

(A) Syllogistic arguments are either indemonstrable arguments or arguments reducible to indemonstrables through one or more *themata*.

(Diogenes Laertius 7.78)

As we will see, a *thema* is a sort of metalogical rule that, as Diogenes Laertius explains, applies to arguments in order to reduce them to indemonstrables.⁴ Other authors call this reduction an “analysis” (ἀνάλυσις) of the arguments to be reduced, and the corresponding operation is expressed by the verb ἀναλύειν.⁵

Sextus (*Against the Professors* 8.229) uses “ἀνάλυσις” to describe the operation by which nonsimple arguments are shown to be compounded from simple, namely indemonstrable, arguments. It is natural to think that the analysis of which Sextus speaks refers to the same procedure that Diogenes calls a reduction, even though Sextus does not consider what seems to be characteristic of the Diogenian reductions, namely *themata*. Instead, Sextus mentions what he calls a “dialectical theorem” as playing a relevant role in the analysis of nonsimple arguments:

(B) One must know that there is a dialectical theorem (θεώρημα διαλεκτικόν) handed down for the analysis of syllogisms, namely this: when we have the premises implying a certain conclusion, we virtually (δυνάμει) have in them also that conclusion, even though it is not explicitly stated.

(Sextus Empiricus, *Against the Professors* 8.231)

It is difficult to understand from this quotation what the dialectical theorem is meant to be, but the context makes it clear that the main point concerns a virtual power of the premises with respect to their conclusions, which can be understood as follows. Suppose that premises A_1, \dots, A_n yield B as their conclusion. Because B is virtually contained in them, it can be omitted when its premises are present. The application of the theorem that Sextus makes immediately afterward (*Against the Professors* 8.232–233) confirms this interpretation. If we have two inferences of the form

$$(1) \quad A \rightarrow (A \rightarrow B), A \vdash A \rightarrow B$$

and

$$(2) \quad A \rightarrow B, A \vdash B$$

we are allowed to infer

$$(3) \quad A \rightarrow (A \rightarrow B), A \vdash B$$

because the conclusion of (1) is the same as one of the premises of (2). Therefore, we can substitute its premises, that is, $A \rightarrow (A \rightarrow B), A$, for $A \rightarrow B$, because the premises virtually contain the conclusion.

If this interpretation of the Sextan passage is accepted, we can compare his dialectical theorem with another rule that we find, this time not in Sextus but in Alexander of Aphrodisias. Alexander is commenting on the Aristotelian claim (*Prior Analytics* 1.25, 41b36 ff.) that a syllogism cannot have more than two premises and one conclusion. Therefore, if we have a deductive chain where more than two premises are involved, we must split it into a series of syllogisms, all of which contain the thesis of two premises and one conclusion. One of the cases Alexander discusses⁶ is that in which a conclusion K is obtained from the premises A and B according to the inference

$$(4) \quad A, B \vdash K$$

and A is the conclusion of premises D, E , whereas B is the conclusion of premises F, G according to the inferences

$$(5) \quad D, E \vdash A$$

and

$$(6) \quad F, G \vdash B$$

In this case one can state

$$(7) \quad D, E, F, G \vdash K$$

and this inference, as Alexander puts it,

(C) is not simple but composite. For it is necessary that what is implied by some premises is implied also by the premises that imply the premises, since the propositions implied are virtually ($\deltaυνάμει$) in the propositions that imply them.

(Alexander of Aphrodisias, *On Aristotle's Prior Analytics* 274.11–14)

The point is that (7) is a composite inference and can be seen as a composition of the three standard syllogisms (4), (5), and (6) because the premises have the property of virtually containing their conclusions. It is the same as Sextus's explanation, and this confirms our claim about the analogy between the two texts. This observation is interesting, especially if we look at the passage following what we have just quoted. Alexander continues by giving an example, and then he states:

(D) In what he says here <Aristotle> sketches for us more clearly what is called the synthetic theorem ($\sigmaυνθετικὸν \thetaεώρημα$),⁷ of which he himself is the discoverer. Its formulation is as follows: When something is implied by some premises, and what is implied together with one or more premises implies something, then the premises that imply it together with the one or more premises with which it implies that imply the same conclusion. For <Aristotle> says that the premises that imply A and B from which, say, C is derived, imply also what is implied by $A B$, that is C .

(Alexander of Aphrodisias, *On Aristotle's Prior Analytics* 274.19–25)⁸

The Greek is probably clearer than my translation. At any rate, the meaning of the theorem is pretty obvious. It can be expressed formally in the following way:

$$(T3) \quad \frac{\Delta, A \vdash B \quad \Theta \vdash A}{\Delta, \Theta \vdash B}$$

Capital Greek letters stand for sets of premises. I represented the synthetic theorem in this way to stress its similarity to what today is called “the cut rule” in sequent logic.⁹

Two preliminary points must be made to confirm our interpretation of the synthetic theorem, namely:

- (i) We must show that (T3) can do the job that Alexander assigns to his synthetic theorem; and
- (ii) Since the analogy between the passages in Sextus and in Alexander is obvious, we are entitled to conclude that the synthetic theorem is the precise formulation of the idea expressed by the dialectical theorem of Sextus. Therefore, (T3) must also do the job that Sextus assigns to his own theorem.

As far as Alexander is concerned, if (7) is split into (4), (5), and (6), then by means of (T3) we can link (7) to (4)–(6) as a conclusion to a set of premises. In the style of sequent logic, the deduction can be formulated in the following way:

I

- | | |
|-----|--|
| (a) | $\frac{D, E \vdash A \quad A, B \vdash K}{\quad}$ |
| (b) | $\frac{F, G \vdash B \quad D, E, B \vdash K}{\quad}$ |
| (c) | $D, E, F, G \vdash K$ |

The meaning of this deduction should be clear: (5) and (4) constitute line (a). By an application of (T3) to them, we eliminate premise A and reach the second formula of (b). To it and (6) we once more apply (T3), and so we “cut” B and obtain (c), namely (7).

If we turn to Sextus we get a similarly pleasing result. Sextus shows how two nonsimple inferences can be analyzed, that is, connected to one or more simple indemonstrables from which they are formed. Let us consider in detail his first example, which concerns (3). To carry out his analysis Sextus (*Against the Professors* 8.231–233) considers two instances of the first indemonstrable, namely (1) and (2), and he claims that (3) is reached from them by means of the dialectical theorem, that is, Alexander’s synthetic theorem, which we have interpreted as (T3). In the spirit of our formalization, we can easily reconstruct his analysis by means of the schema

- | | |
|----|---|
| II | $\frac{A \rightarrow (A \rightarrow B), A \vdash A \rightarrow B \quad A \rightarrow B, A \vdash B}{\quad}$ |
| | $A \rightarrow (A \rightarrow B), A, A \vdash B$ |

where the conclusion of (II) is obtained by applying (T3), which allows us to remove premise $A \rightarrow B$ in (2), replacing it with the antecedent of (1), namely $A \rightarrow (A \rightarrow B), A$. Of course, the conclusion of (II) is not yet inference (3) because in (II) there

is a repetition of premise A , which does not appear in (3). Suppose that we have a means of deleting one of the occurrences of A . Then we get (3).¹⁰

If this interpretation is accepted, we have a clear idea of what the operation of analyzing a nonsimple argument means. It consists in finding some premises, which must be instances of the indemonstrables, from which the argument can be derived by means of the dialectical theorem and, as we will soon see, other theorems. We can imagine that an analysis of an argument is successful if by reversing it we get a proof starting from simple indemonstrables.

II

We can establish a preliminary result: We may rely on (T3) as a faithful interpretation of the synthetic theorem that corresponds to Sextus's dialectical theorem. This point is important because Alexander's synthetic theorem is linked to the third Stoic *thema*, and an understanding of the former should throw some light on the latter. Let us consider the passage where the link between the two is stated. Once more we must turn to Alexander, who says:

(E) Such a composite inference (ἡ τοιαύτη σύνθεσις)¹¹ is generated by means of what recent thinkers call the third *thema*, which is dependent on the synthetic theorem that we have already stated. In fact the formulation of the former, as we have said, is: When something is implied by some premises, and what is implied together with one or more premises implies something, then the premises that imply it together with the one or more premises with which it implies that imply the same conclusion. And the formulation of the so-called third *thema* is as follows: When by two premises a third proposition is implied and extra premises are assumed from which one of them is inferred, the same conclusion is implied by the remaining premise and the extra premises from which the other premise is inferred.

(Alexander of Aphrodisias, *On Aristotle's Prior Analytics* 278.6–14)¹²

The point is clear. The Stoic third *thema*, according to Alexander, has the following formal representation:

$$(T3_A) \quad \frac{A_1, A_2 \vdash B \qquad \Delta \vdash A_i}{\Delta, A_j \vdash B}$$

where $i, j = 1, 2$ and $i \neq j$. (T3_A) is weaker than (T3), because it requires that the cut can be applied to exactly two inferences, one of which has only two premises.

A formulation of the third *thema*, which is preserved by Simplicius, is even weaker than (T3_A). Simplicius says:

(F) An analysis of the argument that takes the conclusion and adds to it another premise yields a conclusion through what the Stoics call the third

thema, and its formulation according to the old philosophers is as follows: When by two premises a third proposition is implied, and the implied proposition together with an extra one implies something, then the same conclusion is implied by the first two premises and the extra one.

(Simplicius, *On Aristotle's On the Heavens* 236.33–237.4)

Whereas in Alexander's report no condition was imposed on the premises of one of the inferences to which the cut was applied, here the requirement is introduced that both inferences must contain exactly two premises. The formal representation of Simplicius's version is therefore

$$(T3_s) \quad \frac{A_1, A_2 \vdash B \quad B, C \vdash D}{A_1, A_2, C \vdash D}$$

which differs from (T3_A) because it has $B, C \vdash D$, whereas (T3_A) has $\Delta \vdash A_i$.

All this looks very strange. There is no reason at all to introduce constraints on the number of the premises such as those that are present in (T3_A) and (T3_s). If we take Alexander and Simplicius seriously, we must admit that according to the Stoics the third *thema* may not be applied to two inferences, both of which have more than two premises. Alexander's version of the third *thema*, which some scholars think may be nearer to the Stoic formulation, is particularly odd.¹³ According to (T3_A) a deduction such as

$$\text{III} \quad \frac{A, B \vdash F \quad C, D, E \vdash A}{B, C, D, E \vdash F}$$

would be perfectly legitimate, whereas

$$\text{IV} \quad \frac{A, B, C \vdash F \quad D, E \vdash A}{B, C, D, E \vdash F}$$

cannot be justified by the third *thema*; and this is hard to believe. If it is reasonable to suppose that (III) could form a part of a deduction, then it is difficult to find a reason for denying that (IV) could.

One might try to meet this objection by supposing that the Stoics had other rules apart from the third *thema* that were introduced to strengthen it and generalize the application of the cut. A passage in Alexander seems to support this view. While discussing the relation between the Stoic *themata* and the synthetic theorem, he points out that:

(G) οἱ περὶ Ἀριστοτέλη, taking utility as their measure, formulated the synthetic theorem in the way that utility demanded; but the Stoics, who took

it from them, divided it up and made from it what they call the second, the third, and the fourth *themata* – paying no attention to utility, they zealously discussed everything which can possibly be said in such a study, even if it is useless.

(Alexander of Aphrodisias, *On Aristotle's Prior Analytics* 284.12–17)

The expression “οἱ περὶ Ἀριστοτέλη” may allude to Aristotle himself,¹⁴ and this is confirmed by text [D] where it is explicitly stated that Aristotle was the inventor of the synthetic theorem. One might then think that the Stoics for some reason divided the synthetic theorem inherited from Aristotle into three different rules, by weakening its original force. That would explain why the third *thema* is a restricted formulation of the synthetic theorem.

I find this picture quite strange and difficult to believe. First of all, the beginning of Alexander's story is a mere fiction. There is no proof that Aristotle found the synthetic theorem. I do not know any text in which this rule is stated, and the passage that Alexander (*On Aristotle's Prior Analytics* 274.19 ff.) offers as evidence, namely *Prior Analytics* 1.25, 42a1, contains no formulation of it at all. It may be that Aristotle in his discussion of composite inferences uses the synthetic theorem, but he never mentions it. To be charitable, we may say that Alexander confuses between use and mention – a confusion analogous to the mistake made by many interpreters who attributed the discovery of the syllogism to Plato on the ground that he sometimes propounds arguments in which syllogisms are possibly used or that are reducible to syllogisms.¹⁵

There is a second point that casts doubts on Alexander's report. Suppose that the Stoics really had divided the synthetic theorem into three different rules. They must have done so to cover the different cases in which inferences with a different number of premises may be involved. Whatever the identification of the three *themata* may be, it is difficult to reconcile the testimonies of Alexander and Simplicius. If we trust Alexander, then (T3_A) is the third *thema* and Simplicius is wrong in taking (T3_S) to be the third *thema*; or else we trust Simplicius and reject Alexander's evidence. One might meet this point by claiming that what Alexander really means in his report of the third Stoic *thema* is nothing more than what Simplicius says and that the difference depends only on Alexander's careless formulation of the *thema*.¹⁶ But if the difference between the third and the remaining *themata* consists precisely in the number of the premises in the inferences covered by the rules, it is difficult to believe that Alexander is careless or ambiguous here.

Besides, how can we represent the other *themata* within Alexander's picture? On the one hand, it is reasonable to suppose that the three *themata* together should have, at most, the same deductive power as the synthetic theorem, because they have been generated by splitting it up. On the other hand, it would be difficult to think that they differ from one another because one is a more general formulation of the other. If we keep in mind these requirements, one way of representing the three *themata* might be this. One might think that (T3_S) corresponds to one *thema*, with the other two being variations of (T3_A) and (T3).

Instead of (T3_A) one might take

$$(T3_{A*}) \quad \frac{A_1, A_2 \vdash B \quad C_1, \dots, C_n \vdash A_i}{C_1, \dots, C_n, A_j \vdash B} \quad (n > 2)$$

($i, j = 1, 2$ and $i \neq j$); and instead of (T3)

$$(T3^*) \quad \frac{A_1, \dots, A_n \vdash B \quad C_1, \dots, C_m \vdash A_i}{C_1, \dots, C_m, A_1, \dots, A_{i-1}, A_{i+1}, \dots, A_n \vdash B} \quad (n, m > 2)$$

($i = 1 \dots n$).¹⁷ It is clear that (T3_S), (T3_{A*}), and (T3*) do not form a series of increasing generality. Although (T3_S) is a special case of (T3_A) and (T3), and (T3_A) a special case of (T3), the requirements on the number of the premises ensure that no such relation of inclusion holds among (T3_S), (T3_{A*}), and (T3*). In this case, we really do have a splitting up of the synthetic theorem into three different rules that have different functions.

But it is difficult to understand why the case of inferences with more than two premises should be opposed to the case of inferences with two premises. If there is a reason to make a separate case for the cut related to inferences with two premises, why not consider separately the case of inferences with three premises, the case of inferences with four premises, and so on, singling out a different cut rule for each number of premises?

A last, and more striking, point can be raised against a literal interpretation of Alexander's words. To make it, it is useful to consider another passage where Alexander mentions the second *thema* as one example, among others, of a useless Stoic theorem. He says:

(H) Therefore, it is clear that these things of which Aristotle did not speak, and which are considered by the recent logicians, since they are useless for the theory of demonstration, are left aside by him because of their uselessness and not because of his ignorance. Of this sort are duplicated or tautologous arguments or the argument of indefinite matter (ἄπειρος ὕλη) and in general what is called by the recent thinkers the second *thema*.

(Alexander of Aphrodisias, *On Aristotle's Prior Analytics* 164.27–31)

To understand the passage correctly, it must be pointed out that when Alexander blames the Stoics for developing useless inferences, he does not have in mind what a modern logician would mean by this charge. From a modern perspective we say that a logical system contains, for instance, useless axioms or rules when they can be proven not to be independent or, in general, when they do not increase the deductive power of the system. This is not what Alexander alludes to in his criticism. He considers an inference or a rule of inference useless if its conclusion does not contain new information beyond what the premises already offer.¹⁸ Thus,

the reason why Alexander says in text (H) that the second *thema* is useless is not that it is a special case of either another *thema* or the synthetic theorem. What he probably means is that the second *thema* is useless because it allows us to justify inferences that are epistemologically irrelevant.

If we consider text (G) with this remark in mind, a feature appears that is, *prima facie*, odd. Alexander says that the formulation of the synthetic theorem is commensurate with its application to useful inferences, and this suggests the idea that this rule was not meant to yield useless deductions. Then he adds that generating useless inferences is the result of the Stoic splitting the synthetic theorem into the second, third, and fourth *themata*. So, on the one hand, by taking the splitting literally, one would be led to think that the three *themata* together had, at most, the same deductive force as the synthetic theorem; on the other hand, the three Stoic *themata* were able to yield useless inferences, and this seems to imply that they were deductively more powerful than the synthetic theorem that did not produce useless inferences.

To get out of this difficulty I propose that the splitting of the synthetic theorem into the three *themata*, to which Alexander alludes, should not be construed in a proper technical sense. We are not compelled to infer from his words that the three *themata* are special cases of the synthetic theorem. The point he is making may be simpler, namely that the Peripatetics used only the synthetic theorem in order to reduce complex to simple inferences, whereas the Stoics needed three rules to do the same job.

Against this proposal, one might easily reply that Alexander should have said that the Stoics split the synthetic theorem into four and not three rules, because this is the number of the rules they accepted. Therefore, one might insist, the dependence of the second, third, and fourth *thema* on the synthetic theorem must be taken seriously, and we must interpret the three Stoic rules as somewhat weaker versions of the cut rule expressed in its full generality by the synthetic theorem. To answer this objection it can be pointed out that the first Stoic *thema*, as we will see in a moment, corresponds to what the Peripatetics called “syllogistic conversion,” which they used to reduce syllogisms from one figure to another or from one mood to another in a way that Aristotle had already established (*Prior Analytics* 2.8–10).¹⁹ Therefore, the first *thema* did not manifest itself as a novelty with respect to Aristotle’s teaching. The problem was constituted by the three remaining *themata* to which Alexander could contrast only the synthetic theorem. In his attempt to show that the new logicians were not at all as original as they probably claimed to be, he says that they took the synthetic theorem from Aristotle and split it into three different rules so that they were able to generate useless inferences. Aristotle, one might add in the spirit of text (H), did not produce these new rules not because he did not know them, but because he was not interested in what they generate.

III

The third *thema* is not the only rule we know that the Stoics used in the analysis of arguments. Apuleius, or better Pseudo-Apuleius, offers us a version of the first *thema* that runs as follows:

(I) There is another argument that is common to all indemonstrables. It is called “*per impossibile*” and is named by the Stoics first arrangement (*prima constitutio*) or first exposition (*primum expositum*).²⁰ They define it as follows: If by two premises a third proposition is implied, one of the two premises with the contrary of the conclusion implies the contrary of the other premise. The old philosophers defined it as follows: If the conclusion of an inference is drawn back, then by assuming one of the two premises the remaining one is taken away.

(Ps-Apuleius, *On Interpretation* 191.5–12)

The kind of rule to which Apuleius refers is clear, although its precise formulation is not completely transparent. What he says when he reports the Stoic definition of the first *thema* is that, given an inference with two premises and one conclusion, we are allowed to state an inference in which the contradictory of one of its premises is deduced from the contradictory of the conclusion and the remaining premise. This interpretation is based on the view that “*contrarium*” here means contradictory. By adopting the style of formalization we have already used, the rule can be expressed

$$(T1_{A1}) \quad \frac{A_1, A_2 \vdash B}{A_i, Ct(B) \vdash Ct(A_j)}$$

where, as usual, $i, j = 1, 2$ and $i \neq j$. “*Ct(X)*” denotes the opposite or the contradictory of X , which is A if X has the form $\neg A$ and $\neg A$ if X is A . This way of stating the rule is paralleled by the formulation proposed by the *veteres*, namely the “old philosophers,” the Peripatetics.²¹ If the verb “*tollere*” is interpreted as a metaphor for the operation of negating, the rule may be reconstructed as follows: If an inference with two premises and one conclusion is given, then the inference can be stated in which the negation of one premise is deduced from the negation of the conclusion and the other premise. Then we have, formally,

$$(T1_{A2}) \quad \frac{A_1, A_2 \vdash B}{A_i, \neg B \vdash \neg A_j}$$

where $i, j = 1, 2$ and $i \neq j$. It is easy to see that (T1_{A2}) is less general than (T1_{A1}), because it covers the same range as (T1_{A2}) only on the condition that the Principle of Double Negation (PDN) holds.²² This interpretation looks *prima facie* plausible, but a few lines later Apuleius explains that the Stoics held a view that was different from that of the other “dialecticians” who admitted the rule. He says:

(J) Not without reason the dialecticians maintained that that form of argument is true in which the opposite of the conclusion with one of the two premises takes away the remaining premise. But the Stoics think that the conclusion

is rejected or one of the premises is taken away only by placing a negative particle before them, as for instance “every/not-every,” “some/not-some.” The old philosophers instead think that the conclusion is rejected or one of the premises is taken away by means of two propositions, and therefore in two ways, as for instance: “every/not-every/some.”

(Ps-Apuleius, *On Interpretation* 191.19–25)²³

According to this passage it would be more natural to take (T1_{A2}) as a formulation of the first *thema* and (T1_{A1}) as the Peripatetic rule, and this view is confirmed by the way in which Alexander of Aphrodisias (*On Aristotle's Prior Analytics* 29.10–12) formulates what he calls syllogistic conversion (ἐπὶ συλλογισμῶν ἀντιστροφή).²⁴ Probably Apuleius was confused or was relying on a confused tradition. It may be that the Stoics adopted some version of (T1_{A2}) instead of (T1_{A1}) as their rule of deduction. If they did so, they must have allowed an implicit use of (PDN) in their reductions of nonsimple syllogisms. Another possibility might be that the difference between the Stoics and the Peripatetics depends not on the assumption of (T1_{A2}) or (T1_{A1}), but on the way in which the notion of “opposite” was taken. One might suppose that the contrariety of which Apuleius speaks with respect to the Stoics is strict contrariety, namely contradiction, which we have formalized by “Ct(X).” On the other hand, the notion of opposition, which is in question in the Peripatetic way of expressing syllogistic conversion, might be taken in a wider sense as denoting any sort of incompatibility. In fact, the standard word that Alexander uses to refer to this relation is “ἀντικείμενα,”²⁵ and we know that “ἀντικείμενα” in the Peripatetic terminology is used to refer to more than just two contradictory members.²⁶ If we stipulate that a proposition *A* is the opposite of another proposition *B* if *A* implies the negation of *B* – $Op(X) = Y$ if $Y \rightarrow \neg X$ – then we may reformulate the Peripatetic rule as follows:

$$(T1_{A3}) \quad \frac{A_1, A_2 \vdash B}{A_i, Op(B) \vdash Op(A_j)}$$

where $i, j = 1, 2$ and $i \neq j$. (T1_{A3}) is, of course, more general than (T1_{A1}) and (T1_{A2}).²⁷

It is difficult to draw any conclusion from all this. One might take seriously Apuleius's way of formulating the first *thema*. From this perspective the following hypothesis might be proposed: Having retained the common view that the Peripatetic version of the conversion rule is more general than the first *thema*, but having lost the explanation for this, Apuleius or his source had recourse to the well-known strict position of the Stoics about negation in order to explain how the Stoic rule was less general than the Peripatetic one. Thus, the syllogistic conversion of the Peripatetics would be something like (T1_{A3}), and the Stoic first *thema* might be (T1_{A1}). But one might take the whole story in the opposite way and suppose that the difference in the way in which negation was conceived played a role in solidifying the first *thema* and the syllogistic conversion as different rules. In this

case one must admit that the Stoic first *thema* is better represented by (T1_{A2}) than by (T1_{A1}), which corresponds to the Peripatetic formulation. Thus, we should conclude that the way in which Apuleius formulates the first *thema* is incorrect. The reason for this might be that in his general formulation of the first *thema* Apuleius was influenced by the Peripatetic way of explaining the syllogistic conversion.

I am slightly more in favor of this latter hypothesis, because another Peripatetic influence on Apuleius's formulation of the first Stoic *thema* can perhaps be detected. Clearly, Apuleius's first *thema* is restricted to inferences with just two premises. A more general way of stating the rule would be by dropping this condition:

$$(T1) \quad \frac{\Delta, A \vdash B}{\Delta, \neg B \vdash \neg A}$$

Because in stating the rule Apuleius had in mind the Aristotelian syllogisms, which by definition have just two premises, and because the Peripatetic rule was developed with reference to the problem of transforming one syllogism into another, it may be that Apuleius formulated the first *thema* from the perspective of the application of the Peripatetic rule for syllogisms. This hypothesis finds a vague confirmation in Galen. He explicitly states (*Introduction to Logic* [*Inst. Log.*] VI 5-6) two versions of the conversion rule according to whether it is applied to a two-premised or a many-premised inference. One might then suppose that this enlarged version of the rule is the result of the influence of Stoic on Peripatetic logic. In fact the Stoics admitted the possibility of many-premised inferences, and surely at least some of these were thought to be reducible to the indemonstrables. To do this, a generalized version of the first *thema* is required. Thus, let's us imagine that the first Stoic *thema* corresponds to (T1). We will also suppose that sometimes an implicit use of (PDN) is made in connection with an application of (T1).²⁸

At least one remark is in order here. Inferences with just one premise can easily be derived by means of (T1_{A2}). For example, this deduction is perfectly safe

$$\begin{array}{l} V \\ (a) \quad \frac{\neg A \rightarrow A, \neg A \vdash A}{\neg A, \neg A \vdash \neg(\neg A \rightarrow A)} \\ (b) \quad \frac{\neg A, \neg A \vdash \neg(\neg A \rightarrow A)}{\neg A \vdash \neg(\neg A \rightarrow A)} \\ (c) \quad \frac{\neg A \vdash \neg(\neg A \rightarrow A)}{\neg A \rightarrow A \vdash A} \end{array}$$

Deduction (V) is straightforward: (a) is an instance of the first indemonstrable; (b) is obtained from (a) by means of the first *thema*, possibly (T1_{A2}) or (T1); (c) is the result of deleting the iterated premise in (b), that is, it is obtained by contraction; a

further application of the first *thema* to (c) and (PDN) yields (d). We know already that the Stoics admitted contraction either as a special or as a tacit rule. Therefore, (c) can be derived safely from (b), and consequently we may state (d), which, as everyone can see, has only one premise.²⁹ We know from our sources that there was a controversy about the so-called “μονολλήματοι λόγοι,” single-premised arguments. Antipater admitted them, whereas they were rejected by Chrysippus.³⁰ Because the *themata* were developed by Chrysippus and he is supposed to have rejected single-premised arguments, we must believe that an inference schema such as (d) should not be counted as a μονολλήματος λόγος, if we credit Chrysippus with a consistent position. According to Alexander (*On Aristotle's Topics* 8.18), an example of a single-premised argument rejected by Chrysippus is the following:

- (8) It is day
 Therefore, it is light.

We may suppose, then, that the arguments that were called “μονολλήματοι” were not only arguments with one premise, but rather arguments that have a single premise and are also correct for an extralogical reason. This is the case with (8), which is neither an instance of a logical law nor of a logically valid inference. If this interpretation is accepted, the rejection of (d) does not follow from Chrysippus's rejections of inferences such as (8).³¹

IV

We know that the Stoics used at least two more rules for the reduction of non-simple arguments, namely the second and fourth *themata*. Unfortunately, no formulation of them has been preserved, and a great deal of speculation has taken place in the scholarship. Before entering the battlefield, let us collect the meager evidence at our disposal.

The most interesting passage is found in Galen, and it is unfortunately very obscure. Galen says:

(K) Now you can find many people expertly trained in the analysis of syllogisms διὰ δύο τροπικῶν or of tautologous arguments (οἱ ἀδιαφόρως πειραίνοντες) or of any others of this kind that employ the first and the second *thema*, just as you can find other people trained in all the syllogisms that are analyzed through the third rule or the fourth. And yet it is possible to analyze most of these differently and more concisely, as Antipater wrote; and besides, all the construction of such syllogisms is no small amount of effort wasted on a useless task, as Chrysippus himself testifies by his practice, for nowhere in his writings did he need these syllogisms in order to prove a doctrine.

(Galen, *On the Doctrines of Hippocrates and Plato* 2.3, 188.32–190.4)³²

There is an initial textual problem. Manuscript M and the Aldine edition have $\tau\rho\iota\omega\tilde{\nu}$ after $\delta\iota\alpha\ \delta\upsilon\omicron\ \tau\rho\omicron\pi\kappa\omega\tilde{\nu}$, and to make sense of the text Ricci³³ added η before $\tau\rho\iota\omega\tilde{\nu}$ so that the text reads: “the analysis of syllogisms $\delta\iota\alpha\ \delta\upsilon\omicron\ \tau\rho\omicron\pi\kappa\omega\tilde{\nu}\ \eta\ \tau\rho\iota\omega\tilde{\nu}$.” However, in his recent edition Philip De Lacy has removed $\tau\rho\iota\omega\tilde{\nu}$, and I see no reason to reintroduce it. But the problems raised by this passage do not change much even if the first reading is kept.³⁴ The syllogisms $\delta\iota\alpha\ \delta\upsilon\omicron\ \tau\rho\omicron\pi\kappa\omega\tilde{\nu}$ have been understood as inferences with two hypothetical (i.e., compound) premises.³⁵ But a passage in Origen (*Against Celsus* VII 15)³⁶ makes it clear that they are a very special kind of inference with two compound premises, namely inferences with the following form:

$$(9) \quad A \rightarrow B, A \rightarrow \neg B \vdash \neg A.^{37}$$

What Galen surely says in the text we are considering is that a syllogism such as (9) was reduced to indemonstrables by means of *themata*. *Prima facie*, he seems to add that the reduction is performed only by means of the first and second *themata*, but this is not the only possible interpretation of the passage, as we will see in a moment. Let us first try to accomplish the reduction of (9), which can be performed as follows:

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$$\begin{array}{ll} (a) & \frac{A \rightarrow B, A \vdash B}{} \\ (b) & \frac{\neg B, A \vdash \neg(A \rightarrow B) \quad A \rightarrow \neg B, A \vdash \neg B}{} \\ (c) & \frac{A \rightarrow \neg B, A, A \vdash \neg(A \rightarrow B)}{} \\ (d) & \frac{A \rightarrow \neg B, A \vdash \neg(A \rightarrow B)}{} \\ (e) & \frac{A \rightarrow B, A \rightarrow \neg B \vdash \neg A}{} \end{array}$$

In line (a) the first indemonstrable is stated. Line (b) is obtained by applying the first *thema* to (a) and by considering a version of the first indemonstrable. We get line (c) by applying the third *thema* and eliminating premise $\neg B$; line (d) only differs from line (c) insofar as the repetition of premise A is deleted; line (e) is again the result of an application of the first *thema* together with (PDN).

What is striking in this proof is that the third *thema* is used. It is because of this that Benson Mates and Oskar Becker put forward the hypothesis that Sextus’s dialectical theorem had to be taken as the second *thema*.³⁸ But we have seen that Sextus’s dialectical theorem does not differ from Alexander’s synthetic theorem, which has something to do with the Stoic third *thema*. On the other hand, it is impossible to perform the deduction of (9) without a cut of some sort. Therefore, the hypothesis that the second *thema* is a version of the cut rule, which differs from the third *thema* only in its generality, might be advanced. So one might offer

another interpretation of Alexander's text [G] according to which the second, third, and fourth *themata* are weakened versions of the synthetic theorem. As we have seen, a reasonable way of interpreting Alexander's claim is by imagining that the second, third, and fourth *themata* correspond to (T3_S), (T3_{A*}), and (T3*). Because the weakest form of the cut rule is needed in deduction (VI), to fit this interpretation of Galen's words, one might suppose that (T3_S) corresponds to the second *thema* and take (T3_{A*}) as the third, thereby following Alexander (at least to a certain extent), whereas (T3*) could be the counterpart of the fourth. But we have seen that the set of (T3_S), (T3_{A*}), and (T3*) does not fit Alexander's evidence. To the difficulties that have already been raised against this hypothesis we can add that a deduction such as (IV) cannot be justified by this set of rules, and this introduces a limitation on the Stoic deductive system that has no plausible reason.

To the standard interpretation of the second, third, and fourth *thema* a similar criticism can be made that is based on Alexander's remark about the relationship between the synthetic theorem and the last three *themata* and that tries to account for Galen's claim.³⁹ According to this interpretation, (T3_A) is called on to play the role of the third *thema*, and the second *thema* is identified with what we now call the "contraction rule":

$$(T2_F) \quad \frac{\Delta, A, A \vdash B}{\Delta, A \vdash B}$$

On the other hand, the fourth *thema* takes the following form:

$$(T4_F) \quad \frac{A_1, A_2 \vdash B \quad \Delta \vdash A_i \quad \Theta \vdash A_j}{\Delta, \Theta \vdash B}$$

where $i, j = 1, 2$ and $i \neq j$. It should be clear that (T4_F) is stronger than (T3_A), as we can readily see if we try to infer the former from the latter. A possible attempt might be

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$$\begin{array}{ll} \text{(a)} & \frac{A_1, A_2 \vdash B \quad \Delta \vdash A_i}{\Delta, A_j \vdash B} \\ \text{(b)} & \frac{\Delta, A_j \vdash B \quad \Theta \vdash A_j}{\Delta, \Theta \vdash B} \\ \text{(c)} & \Delta, \Theta \vdash B \end{array}$$

where $i, j = 1, 2$ and $i \neq j$. The first formula on line (b) can be reached by using (T3_A). But by adding the second premise of (T4_F), that is, $\Theta \vdash A_j$, we obtain (c) only if a full cut rule is applied, namely (T3), because (T3_A) is not sufficient. Therefore, to avoid the redundancy of the fourth *thema* on this interpretation, it is

crucial that the third *thema* is a restricted version of the cut rule. Because of this a deduction such as (IV) cannot be carried out in this system, because none of the versions of the cut rule that we have considered apply to this case, and (T3_A), (T2_F), and (T4_F) do not together cover the whole domain of application of (T3). Moreover, the interpretation does not square with the passage in Galen, because some of the reductions, as they are carried out by the supporters of the standard interpretation, require *themata* other than the first and the second.⁴⁰ Finally, I am not at all convinced that the Stoics used contraction as an explicit rule of inference. Sextus's analyses of arguments imply an implicit use of contraction,⁴¹ and this favors the idea that contraction was not an explicit rule of inference.⁴²

However, the problem of the Galenic text remains. If we look at its beginning once more, it is not difficult to see that the interpretation we have considered up to now is not at all compulsory and is complicated by a further textual problem. To begin with, let us keep to the text proposed by De Lacy. By stressing, then, the presence of the iterated $\pi\omega\varsigma$ at the beginning of the passage, one might think that the reference to the first and second *thema* does not concern every kind of syllogism mentioned in the previous line, which is to say those $\delta\iota\alpha\ \delta\upsilon\omicron\ \tau\rho\omicron\pi\iota\kappa\omega\tilde{\nu}$ and those $\acute{\alpha}\delta\iota\alpha\phi\acute{o}\rho\omega\varsigma\ \pi\epsilon\iota\rho\alpha\acute{\iota}\nu\omicron\nu\tau\epsilon\varsigma$, but only those unspecified items that immediately precede and are referred to by the vague formula: $\tau\iota\upsilon\epsilon\varsigma\ \acute{\alpha}\lambda\lambda\omicron\iota\ \tau\omicron\iota\omicron\upsilon\tau\omicron\iota$. I do not think that $\tau\omicron\iota\omicron\upsilon\tau\omicron\iota$, which marks a similarity between these inferences and the previous ones, is a difficulty for this interpretation, because it is not said that their likeness is a likeness in the way in which they are reduced. However, if we keep De Lacy's text, this interpretation is still not completely satisfactory because two groups of logicians appear to be contrasted here, and the second of these groups is characterized as using the third and fourth *thema* in the analysis of arguments. It is therefore natural to take the former group as qualified in a parallel way, namely by the use of the first and the second *thema* in reducing arguments. The arguments that the former group addressed were those $\delta\iota\alpha\ \delta\upsilon\omicron\ \tau\rho\omicron\pi\iota\kappa\omega\tilde{\nu}$, those $\acute{\alpha}\delta\iota\alpha\phi\acute{o}\rho\omega\varsigma\ \pi\epsilon\iota\rho\alpha\acute{\iota}\nu\omicron\nu\tau\epsilon\varsigma$, and some other unspecified items. If this is the sense of the passage, it is difficult to maintain that the use of the first and second *themata* need not be extended to all the syllogisms that the first group considered it their job to analyze.

But is this what Galen means? Does he mean that there were people specialized in deducing syllogisms that could only be deduced with the aid of the first and second *themata* and people who dedicated themselves to the deduction of syllogisms that can only be analyzed by means of the third and fourth *themata*? This is difficult to believe. Even in a sophisticated society like ours, such a degree of specialization would appear odd. What Galen wants to convey is the general idea that there are different ways of analyzing nonsimple syllogisms. He mentions Antipater for this reason, and from what he says one may even doubt that Antipater used *themata* to perform his analyses. His view is contrasted with Chrysippus's position, and one would find it difficult to put the leading logician of the Stoic school in one of the two groups apparently mentioned at the beginning of the passage. It seems more reasonable to think that the standard Stoic view about syllogistic analysis in which the four *themata* were used is being contrasted with the heterodox position of Antipater.

One might think that one way out of this difficulty is to interpret the text differently. If we side with the tradition and some of the manuscripts by reading at 114.4–5:

- (i) ὥσπερ ἀμέλει καὶ ἐπ’ ἄλλοι ὅσοι διὰ τοῦ τρίτου θέματος ἢ τετάρτου συλλογισμοὺς ἀναλύουσι

and reject De Lacy’s correction, that is, ὥσπερ ἀμέλει καὶ ἄλλοις ἐφ’ ὅσους, it may seem that we are able to find a way to avoid the unpleasant obligation to charge Galen with making a ridiculous statement. A possible translation of (i) might be:

- (ii) just as you can find other kinds of arguments with respect to which they analyze the syllogisms through the third *thema* or the fourth.

But I do not think that this translation is possible. It is based on a distinction between arguments and syllogisms: The syllogisms contained in arguments are those which are analyzed. But ἄλλοι, if it does not refer to people, refers to syllogisms. Therefore, the text would say that there are syllogisms whose syllogisms are analyzed by means of the third and fourth *themata* – a nonsense. Let us therefore accept De Lacy’s text.

I am inclined to interpret Galen’s point in a simpler and nontechnical way. I think that what he implies is that in analyzing pertinent syllogisms some people limited themselves to the application of the first and second *themata*, whereas others considered the application of the third and fourth *themata*. From a contemporary perspective, this attitude toward formal proofs looks strange. A modern logician is trained to perform a proof in such a way that any manipulation of a formula, however trivial it may be, must be the result of the application of an explicitly stated rule. From this point of view, the passage from (c) to (d) in deduction (VI) needs a rule, which in sequent logic is called the “contraction rule.”⁴³ The situation of ancient logicians was different. They did not have the formal concept of proof that is familiar to us, and they did not have at their disposal a formal language in which any change in a formula can be exactly checked. My impression is that Stoic logicians, or at least the logicians considered by Galen, were halfway between Aristotle and modern logicians. Aristotle, as we have seen, had no theory to explain the deduction of inferences from inferences, and he expressed everything in an informal metalanguage where no explicit rule was stated. Modern logicians possess a completely formalized theory perfectly controlled at each step. The logicians considered by Galen were probably in between. They had abandoned the informality of Aristotle by elaborating some deduction rules that they tried to apply to different inferences; but they were still far from modern rigor in the sense that they looked for some rules only and probably left other steps that they did not consider relevant or worth formal analysis to an informal metalanguage. In a proof such as (VI), they made explicit the application of the first *thema*, and they arrived at the conclusion by an implicit use of the third *thema*.

In order to evaluate this proposal we must consider the reductions of the other inferences that Galen attributes to the former group, namely the inferences that are called “ἀδιαφόρως πειραίνοντες.” From Alexander (*On Aristotle's Topics* 10.10–11) we know that a tautologous syllogism⁴⁴ is an inference in which the conclusion is the same as one of its premises. Two examples were normally given to illustrate this kind of inference. One has the form

$$(10) \quad A \vee B, A \vdash A^{45}$$

and the other

$$(11) \quad A \rightarrow B, A \vdash A.^{46}$$

The analysis of (10) and (11) can be performed together, and their deduction begins as follows:

VIII

$$\begin{array}{l} (a) \quad \neg(A \wedge A), A \vdash \neg A \\ \hline (b) \quad A, A \vdash A \wedge A \\ \hline (c) \quad A \vdash A \wedge A \end{array}$$

Up to this point, everything is all right. Line (a) is a version of the third indemonstrable; (b) is reached by applying the first *thema* and (PDN) to (a); (c) is the result of an implicit contraction. But now things get complicated. On the one hand, we need to simplify the consequent of (c) by reducing $A \wedge A$ to A ; on the other hand, we must enrich its antecedent by adding a further premise to premise A . To do that we must introduce two new rules. The first says that when a conjunction is derived from a set of premises, we can derive from the same set either of its conjuncts. Formally, we have

$$(AR_1) \quad \frac{\Delta \vdash A_1 \wedge A_2}{\Delta \vdash A_i}$$

where $i = 1, 2$. The second rule is based on the idea that if a conclusion follows from a set of premises, the same conclusion can be derived from the same set increased by any premise whatsoever. Thus we can state

$$(AR_2) \quad \frac{\Delta \vdash B}{\Delta, A \vdash B}$$

In modern terminology this rule is called “weakening.” It is easy to see how the deduction can be completed. As far as (10) is concerned, we have

$$\begin{array}{l} \text{(d)} \quad \frac{}{A \vdash A} \\ \text{(e1)} \quad \frac{}{A \vee B, A \vdash A} \end{array}$$

and with reference to (11) we obtain

$$\begin{array}{l} \text{(d)} \quad \frac{}{A \vdash A} \\ \text{(e2)} \quad \frac{}{A \rightarrow B, A \vdash A} \end{array}$$

Line (d) is obtained by applying (AR₁) to (c), whereas (e1) and (e2) are reached by means of (AR₂), namely by weakening. Of course, the deduction of (10) and (11) could be completed in a different way, but it cannot be performed without introducing rules beyond (T1) and (T3).⁴⁷

Shall we count (AR₁) and (AR₂) as the second and fourth *themata*? I see no reason either for affirming or for denying it. Surely, the system composed by the five indemonstrables plus (T1), (T3), (AR₁), and (AR₂) is not sufficient by itself to generate all the inferences the Stoics believed to be valid. If we try to reduce other inferences that the Stoics included as indemonstrables, we are compelled to introduce other rules.

One might object that (AR₂) cannot be taken as a *thema*, nor even as an implicit rule of inference for a Stoic system of inferences because the Stoics did not admit redundant arguments. For instance, Sextus Empiricus (*Outlines of Pyrrhonism* 2.146–147; *Against the Professors* 8.438–439) claims that an argument such as

$$\begin{array}{l} \text{(12*)} \quad \text{If it is day, it is light} \\ \quad \text{It is day} \\ \quad \text{Dion is walking} \\ \hline \quad \text{It is light} \end{array}$$

is inconclusive because of its redundancy. This position has been differently evaluated. Benson Mates thinks that the view referred to by Sextus is mistaken, and Jonathan Barnes takes it as an anticipation of the theory of relevance logic.⁴⁸ I do not want to discuss the problem of redundancy here. I must say that I find it difficult to believe that the same Stoics rejected arguments of the form of (12*), that is,

$$\text{(12)} \quad A \rightarrow B, A, C \vdash B$$

and accepted the ἀδιαφόρως πειραίνοντες, in particular, an inference such as (11). On any reasonable account of redundancy, if (12) is counted as redundant, (11) takes the same label. That (11) is sound can only be admitted if one thinks that its conclusion A follows from the fact that A is already asserted in the premises, that is, if one takes $A \vdash A$ as valid and considers $A \rightarrow B$ as a redundant premise. We do not know on what basis the Stoics acknowledged ἀδιαφόρως πειραίνοντες arguments. But if they did, it is difficult to believe that they at the same time rejected all redundant arguments. Therefore, if the Stoics, or at least some Stoics, not only accepted them but also reduced them to the indemonstrables, it is hard to think that they could do so without implicitly or explicitly relying on some sort of weakening.

However, with respect to the class of inferences to which the *themata* were supposed to apply, we do not even know what constituted the set of nonsimple inferences. Did it include all the valid propositional inferences other than the five indemonstrables, or was it limited to a subset of them, namely to those inferences that could be deduced by means to the *themata*? Remember that the only definition of nonsimple syllogisms that we have is syntactic and that it presupposes the list of *themata*.⁴⁹ Besides, Diogenes Laertius (7.78) reports that the Stoics admitted valid inferences different from the syllogistic ones.

Again, the Galenic passage strongly suggests the idea that the analysis of non-simple arguments into the indemonstrables was not understood by the logicians to whom he refers as a fully formalized deduction in the modern sense, but rather as a process in which some steps were accomplished by reference to general rules and other steps only accomplished informally. Shall we attribute this way of putting things to a particular group of Stoic logicians, or shall we think that it was common to the whole school? This question is hard to answer. What results from Galen's testimony, if we take it seriously, is that a rigidly formal notion of proof was not one of the tenets of the school on which its members had a unanimous and well-established view.

In this situation, it seems to me that all the efforts made in the last forty years to discover which kind of rules the second and fourth *themata* are can hardly yield a consensus among the interpreters, because they are based on too many speculative elements. Various attempts have assumed that the Stoic theory of inferences is complete. Starting from this hypothesis, and taking the five indemonstrables as axioms and the first and third *themata* as rules of inference, scholars have made different proposals in order to achieve completeness. But none of these efforts, however interesting from a logical point of view they may be, have any basis in the texts.⁵⁰ My conclusion is pessimistic but, at least in my view, reasonable: If we keep to the texts, two of the Stoic *themata* can be identified in a somewhat sure way, and we must simply confess that we have no sure way to detect the others.

Notes

1 Cf. Patzig 1969: 51 ff.

2 It is sufficient to refer to the reconstruction put forward by Łukasiewicz 1957²: 51 ff. and to the more recent proposal of Corcoran 1974: 85–131, where natural deduction is

used to formalize Aristotle's proof. The Łukasiewicz approach has been considerably improved by Patzig 1969: 137 ff.

- 3 I am thinking in particular of a passage from Alexander of Aphrodisias (*On Aristotle's Prior Analytics* 284.12 ff.; cf. text [G]). I will discuss it in detail later.
- 4 I have not translated the word "θέμα," which is difficult to render. See the discussion in Ierodiakonou 1990b: 58–60. In *LS*, "θέμα" is rendered by "ground-rule."
- 5 Cf., for instance, Galen, *The Doctrines of Hippocrates and Plato* 2.3, 188.32–190.1. I will come back to this text later (see text [K]).
- 6 Alexander of Aphrodisias, *On Aristotle's Prior Analytics* 274.7–11. There is something odd in the text. In the Wallies edition it reads as follows: καὶ οὕτως, φησί, τὸ αὐτὸ διὰ πλειόνων ὅρων δειχθήσεται, εἰ ἑκάτερα τῶν προτάσεων, ἢ τε A καὶ ἢ B, δι' ὧν συνήγετο τὸ E, εἰ εἷη διὰ προσυλλογισμοῦ εἰλημμένα, ὥς ἑκάτεραν αὐτῶν συμπέρασμα συλλογισμῶν εἶναι, τὴν μὲν A διὰ τῶν Δ, E προτάσεων τὴν δὲ B διὰ τῶν Z, Θ. Δεικνύοιτο γὰρ ἂν καὶ τὸ E διὰ τῶν Δ, E, Z, Θ προτάσεων. Here the conclusion of the main inference is marked by the same letter as one of the prosyllogistic premises. That this identity is not meant by Alexander is shown by the example that follows (274.14–19). The same example is found in the parallel passage of Themistius, *On Aristotle's Prior Analytics* 30.30–35.
- 7 I have kept the traditional translation, although perhaps something like "theorem of composition" would be better.
- 8 Themistius (*On Aristotle's Prior Analytics* 80.30–81.11) repeats Alexander's point verbatim. A parallel formulation of the synthetic theorem can be found in Alexander of Aphrodisias (*On Aristotle's Prior Analytics* 283.15–17), and it is once more copied by Themistius (*On Aristotle's Prior Analytics* 84.16–18).
- 9 Cf. Takeuti 1975. I deviate from his symbolism by using "⊢" instead of "⊢" and "→" instead of "⊃." Needless to say, "⊢" and "→" are left undefined.
- 10 Sextus's second example (*Against the Professors* 8.234–238) can be represented by means of

$$\frac{(A \wedge B) \rightarrow C, \neg C \vdash \neg(A \wedge B) \qquad \neg(A \wedge B), A \vdash B}{(A \wedge B) \rightarrow C, \neg C, A \vdash B}$$

where the dialectical theorem is used as before to eliminate proposition $\neg(A \wedge B)$ from the conclusion. Here the reconstruction of the proof is even smoother than in the previous example, because no deletion of a repeated premise is required.

- 11 That is, an inference such as (7).
- 12 Themistius, *On Aristotle's Prior Analytics*. 84.14–18 repeats the point.
- 13 Cf. Frede 1974a: 193; *LS* I 219.
- 14 In Alexander the formula "οἱ περὶ" combined with a proper name is sometimes a periphrasis for the proper name itself. Cf. Bochenski 1947: 18.
- 15 This was the view of the Neoplatonic commentators. See for instance Ammonius, *On Prophyry's Isagoge* 35.16 ff.; *On Aristotle's Prior Analytics* 7.6–11 and 8.9–12.
- 16 I owe this observation to Jonathan Barnes.
- 17 {Mignucci's intention was to say that A_i is omitted in the premises of the conclusion}
- 18 To show this, it is sufficient to consider Alexander's interpretation of Aristotle's definition of syllogism (*Prior Analytics* 1.1, 24b18–22; *Topics* 1.1, 100a25–27). In a genuine inference the conclusion must be different from its premises; otherwise, it could not perform its cognitive task (*On Aristotle's Topics*. 9.22–10.7). For this reason the Stoic tautologous arguments are judged useless inferences (Alexander of Aphrodisias, *On Aristotle's Prior Analytics* 18.18–19.3). On this question see also Mignucci 1990: 389–397.
- 19 Unfortunately, Alexander's commentary on the second book of the *Prior Analytics* is lost. His interpretation is probably reflected in Philoponus's commentary (*On Aristotle's Prior Analytics* 423.4 ff.).

- 20 As Kneale and Kneale 1978⁷: 169 have pointed out, “*constitutio vel expositum* [. . .] sounds like a phrase of the Roman imperial administration; perhaps ‘guiding principle’ or ‘directive’ is the nearest English equivalent.” *LS* I 218 renders these expressions by “first principle” or “ground-rule.”
- 21 Cf. *LS* I 221.
- 22 This point has been made by Becker 1957: 30n5, who claims that the genuine Stoic formulation of the first Stoic *thema* is more in the sense of (T1_{A1}) than (T1_{A2}). The same view has been adopted by Mueller 1979: 204–205.
- 23 The sense requires that we interpret the triple “every/not-every/some” as “every/not-every/some not.”
- 24 See also Philoponus, *On Aristotle’s Prior Analytics* 423.4–9.
- 25 See also Alexander of Aphrodisias, *On Aristotle’s Topics* 582.23–26, where the definition of syllogistic conversion is repeated. The text has a lacuna, but the word ἀντικείμενον is preserved in the definition.
- 26 In the standard Aristotelian doctrine, opposition covers contradiction, contrariety, relation, and privation. See *Categories* 10, 11b17–19; *Topics* 2.3, 109b17–20; *Metaphysics* 5.10, 10.18a20 ff.; 10.4, 1055a38 ff. Of course, the oppositions that are relevant for the conversion rule are those of contradiction and contrariety.
- 27 {Nothing is implied here regarding the validity of (T1_{A3}).}
- 28 It must be pointed out that (PDN) should not be taken as a disguised rule of inference, but as a linguistic way of simplifying statements. Therefore, the use of (PDN) does not allow us to state, for instance, $A \vdash \neg\neg A$ and $\neg\neg A \vdash A$, but rather to justify the substitution of $\neg\neg A$ for A and *vice versa*.
- 29 There is no clear evidence that the Stoics recognized inferences such as (d). They surely accepted: $A \rightarrow A$, $\neg A \rightarrow A$, $A \vee \neg A \vdash A$ (cf., for instance, Sextus Empiricus, *Outlines of Pyrrhonism* 2.186; *Against the Professors* 8.281–284), where at any rate premises $A \rightarrow A$ and $A \vee \neg A$ are redundant.
- 30 Cf. Sextus Empiricus, *Against the Professors* 8.443; Sextus Empiricus, *Outlines of Pyrrhonism* 2.167; Alexander of Aphrodisias, *On Aristotle’s Topics* 8.16–19; Ps-Apuleius, *On Interpretation* 184.16–23.
- 31 Cf. Mueller 1979: 203–204.
- 32 I follow the text and translation offered by De Lacy.
- 33 One of the two editors together with V. Trincavelli of the Latin Galen (Venice 1541–1545); cf. De Lacy in his introduction to Galen, *On the Doctrines of Hippocrates and Plato* (1981–1984).
- 34 The traditional reading is maintained by *LS* II 36H, whereas *FDS* 1160 sides with De Lacy.
- 35 Cf. Becker 1957: 37n13. For the meaning of “τροπικόν” as “compound” or “molecular premise,” see for example Alexander of Aphrodisias, *On Aristotle’s Prior Analytics* 262.29–32.
- 36 Cf. Sextus Empiricus *Outlines of Pyrrhonism* 2.3.
- 37 Cf. Mates 1961²: 136; Frede 1974a: 182.
- 38 Cf. Becker 1957; Mates 1961²: 78. They are followed by Kneale and Kneale 1978⁷: 169; Gould 1970: 163.
- 39 Cf. Frede 1974a: 172 ff.
- 40 Cf. Frede 1974a: 186–190. This becomes particularly clear if we keep the traditional reading διὰ δύο τροπικῶν ἢ τριῶν, as Frede does. Syllogisms are not just inferences with three compound premises, but a special kind of them, although it is not completely clear what their structure is. In their more general form they might amount to this:

$$(*) \quad A \rightarrow B, C \rightarrow B, A \vee C \vdash B$$

Cf. Frede 1974a: 182–183, and also Ierodiakonou 1990a: 139. The reduction of (*) can be performed as follows:

- (a)
$$\frac{A \vee C, \neg C \vdash A}{}$$
- (b)
$$\frac{\neg A, \neg C \vdash \neg(A \vee C) \quad C \rightarrow B, \neg B \vdash \neg C}{}$$
- (c)
$$\frac{\neg A, \neg B, C \rightarrow B \vdash \neg(A \vee C) \quad A \rightarrow B, \neg B \vdash \neg A}{}$$
- (d)
$$\frac{A \rightarrow B, C \rightarrow B, \neg B, \neg B \vdash \neg(A \vee C)}{}$$
- (e)
$$\frac{A \rightarrow B, C \rightarrow B, \neg B \vdash \neg(A \vee C)}{}$$
- (f)
$$A \rightarrow B, C \rightarrow B, A \vee C \vdash B$$

It is easy to see that in order to get (c) from (b) the third *thema* must be used. Alternatively, one might apply (T4_F) to get (d) immediately from (b) and the independent premise of (c) (this is what Frede 1974a: 187 actually does). At any rate, we must introduce a *thema* different from the first and the second.

41 See deduction (II).

42 An interesting variant of Frede's standard interpretation has recently been proposed by Katerina Ierodiakonou in Ierodiakonou 1990b: 60–75. According to her, whereas the Stoic third *thema* has to be identified with Simplicius's version of the cut rule, namely (T3_S), and the fourth *thema* corresponds to Frede's (T4_F), the second *thema* has the following form

$$(T2_K) \quad \frac{A_1, \dots, A_n \vdash B \quad A_i, B \vdash C}{A_1, \dots, A_n \vdash C}$$

where $i = 1, \dots, n$. It is clear that (T2_K) is a sort of restricted cut rule to which contraction is added. I suppose that Ierodiakonou must admit contraction as an implicit rule of inference; otherwise, deductions such as those referred to by Sextus would be impossible. But if contraction is admitted as an implicit rule, (T2_K) does not add anything to (T3_A). Therefore, (T2_K) is not redundant only because the cut rule is given in an implausibly weak form. Besides, (T2_K), (T3_S), and (T4_F) do not account for all analyses of the ἀδιαφόρως πειραίνοντες arguments mentioned in the Galen passage.

43 To be precise, a further rule should be added to the deductions we have performed, which allows us to change the order of the formulas to the left of “ \vdash ”, the inference sign. In sequent logic this rule is called “exchange.”

44 I follow Mates 1961: 66 in calling tautologous inferences the ἀδιαφόρως πειραίνοντες syllogisms.

45 Alexander's example (*On Aristotle's Topics* 10.11) is:

Either it is day or there is light
It is day

It is day.

See also Ps-Apuleius, *On Interpretation* 183.23–31.

46 The standard example is:

If it is day, there is light
It is day

It is day

Cf. Alexander of Aphrodisias, *On Aristotle's Prior Analytics* 18.14–17. It is clear that at 18.16 we must read ἡμέρα ἄρα ἐστὶ and not, as Wallies does, φῶς. Wallies does,

however, accept this reading in his *addenda et corrigenda* (711) printed at the end of his edition of Alexander's commentary on the *Topics*. Cf. Ammonius, *On Aristotle's Prior Analytics* 28.5–6.

- 47 Mueller 1979: 204 has shown that an inference such as $A \vdash A$ cannot be obtained from the indemonstrables by means of (T1) and (T3). Frede 1974a: 188–189 reconstructs the deduction of (10) differently, because he takes its disjunctive premise as expressing complete exclusiveness and formulates the fourth and the fifth indemonstrables in the same way. If strong disjunction is formalized by $A \downarrow B$ and defined as $A \leftrightarrow \neg B$, then

$$(10^*) \quad A \downarrow B, A \vdash A$$

can be proven as follows:

$$\begin{array}{ll}
 (a) & \begin{array}{c} A \downarrow B, A \vdash \neg B \qquad A \downarrow B, \neg B \vdash A \\ \hline \end{array} \\
 (b) & \begin{array}{c} A \downarrow B, A \downarrow B, A \vdash A \\ \hline \end{array} \\
 (c) & A \downarrow B, A \vdash A
 \end{array}$$

Line (a) is formed by two instances of the fourth and fifth indemonstrables, the third *thema* is used to get (b) from (a), and contraction is used to obtain conclusion (c). However, it is not clear to me that the same kind of disjunction was used by the Stoics in the fourth and fifth indemonstrables, and for this reason I prefer the weaker version (10) to (10*). Frede 1974a: 189–190 does not offer any reduction for (11). If he had tried to do so, he would have recognized that further rules must be introduced in order to perform the analysis. The same holds for Ierodiakonou's interpretation. She is able to derive (c) immediately from (a) by means of her (T2_K), and in this way she remains faithful to the letter of Galen's text, but she has no way of reducing (11).

- 48 Mates 1961²: 83; Barnes 1980: 172.

- 49 Cf. Diogenes Laertius 7.78 and Sextus Empiricus, *Against the Professors* 8.229.

- 50 Cf. the discussion of the whole question of completeness with reference to the proposals of Becker 1957; Kneale and Kneale 1978⁷: 165 ff.; Mueller 1979.



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Part II

Identity, predication, and quantification



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4 Remarks on Aristotle's theory of predication

I

Consider the following two sentences:

(1) Richard is handsome

and

(2) every man is rational.

In traditional terms, we can say that these two sentences have a common grammatical structure. In (1), two terms, “Richard” and “handsome,” are linked by the copula “is”; the terms “every man” and “rational” are connected by the copula in exactly the same way. However, the logical analyses of these propositions are different. The formal counterpart of (1) is

(1*) $\phi(a)$

whereas a more complex structure corresponds to (2), namely

(2*) $\forall x(\phi(x) \rightarrow \psi(x)).$

In accordance with the different syntactical structures assigned to (1) and (2) we have different semantic interpretations for (1*) and (2*). Once a domain **D** of individuals is fixed, we say that (1) is true if and only if the particular of **D** assigned to a is an element of the subset of **D** that corresponds to ϕ . However, (2*) is true if the set corresponding to ϕ is included in the set corresponding to ψ . Thus, the set-theoretical membership relation, usually expressed by “ \in ,” is the semantic counterpart of the copula in (1), whereas inclusion, “ \subseteq ,” corresponds to the copula in (2). Modern logicians are well aware that these two relations should not be confused simply because they are expressed using the same word, “is.”

This interpretation gives us a reassuring symmetry. On the one hand, it is easy to be convinced that the “is” of (1) is much more fundamental than the “is” of (2).

It expresses the basic predicative relation out of which the meaning of the “is” of (2) is built up with the help of some logical notions such as universal quantifier and material implication. On the other hand, the membership relation, which is the semantic counterpart of the “is” of (1), is the primary relation by means of which the semantic counterpart of the “is” of (2) is characterized, because inclusion can easily be defined in terms of membership belonging.

II

This approach to the syntax and semantics of propositions is so widespread and usual that we are inclined to think that it is a distinctive feature of any logical theory of propositions. We start from simple singular propositions, and from them we build up more complex predicative relations that involve logical operators and quantifiers. In a parallel way, we interpret simple singular propositions through the set-theoretical membership relation, and from this basic operation we proceed to define truth conditions for more complicated propositions by introducing new set-theoretical relations defined in terms of the membership relation.

Historians of logic have not been able to escape the appealing simplicity of this model. To be convinced of this, it is sufficient to think of the way in which the following passage from Aristotle’s *Prior Analytics* is interpreted:

(A) We say that one term is predicated of all of another whenever nothing of the subject can be taken (μηδὲν ἧ λαβεῖν [τοῦ ὑποκειμένου]) of which the other is not said; equally for being predicated of none.

(Aristotle, *Prior Analytics* 1.1, 24b28–30)

The Greek is not certain because Ross has deleted τοῦ ὑποκειμένου at 24b29 on the basis of Alexander of Aphrodisias,¹ although the expression is present in most manuscripts.² However, even if we decide to follow Alexander in omitting this expression, the sense clearly requires that “τοῦ ὑποκειμένου” be tacitly understood.

The text contains a well-known definition of universal predication and is normally taken to mean that asserting that a term *A* is predicated of every *B* amounts to stating that there is no particular of which *B* is true and *A* is not. According to this interpretation, “rational” is predicated of every man if there is no particular man who is not rational, and in general “*A* is predicated of every *B*” means that there is no particular that is *B* and not *A*. A familiar pattern comes to mind, and we are immediately tempted to formalize the Aristotelian definition as

$$(3) \quad \neg \exists x (B(x) \wedge \neg A(x)).$$

Thus, because (3) is equivalent to (2), it is reasonable to claim that (2*) is the adequate formal representation of (2) and in general of universally quantified affirmative propositions. Therefore, we are inclined to think that (2), even from

the Aristotelian point of view, is dependent for its form on the form of (1), just as (2*) is based on (1*).

We can find other apparent confirmations of this view. Let us consider, for instance, the definition of κατὰ παντός in the *Posterior Analytics*, where Aristotle may be taken to be even more explicit than in text (A). What he says is as follows:

(B) Now I say that something holds of every case if it does not hold in some cases and not others (ὁ ἅν ἢ μὴ ἐπὶ τινὸς μὲν τινὸς δὲ μὴ), nor at some times and not at others; e.g., if animal holds of every man, then if it is true to call this a man (εἰ ἀληθὲς τόνδ' εἰπεῖν ἄνθρωπον), it is true to call him an animal too; and if he is now the one, he is the other too; and the same goes if there is a point in every line. Evidence: when asked if something holds of every case, we bring our objections in this way – either if in some cases it does not hold or if at some time it does not.

(Aristotle, *Posterior Analytics* 1.4, 73a28–34; J. Barnes translation)

The example of the proposition that every man is an animal, the truth of which implies that if this particular is a man (εἰ ἀληθὲς τόνδ' εἰπεῖν ἄνθρωπον) then he is an animal, seems to guarantee that in the sentence “it does not hold in some cases and not others” (ὁ ἅν ἢ μὴ ἐπὶ τινὸς μὲν τινὸς δὲ μὴ) “in some cases” (ἐπὶ τινὸς) means “for at least a particular under the general term to which the quantifier κατὰ παντός is applied.”

Because of this way of putting things, one is led to attribute the same sort of semantic interpretation to Aristotelian propositions as one finds attached to formulas such as (1*) and (2*). Therefore, particulars are associated with singular terms and classes with predicates, and a more or less explicit set-theoretical interpretation is imposed on Aristotelian propositions by introducing something similar to the membership and inclusion relations in order to express the copula in its different possible situations.

III

I do not claim that this interpretation is locally false. If we keep solely to the texts from the *Posterior Analytics* we have quoted, there is no reason to reject it. However, doubts can be cast on it if we consider the logic in which Aristotle's definitions are embedded and the way in which universal propositions are used by him.

It is well known that this interpretation of Aristotelian quantified propositions has revealed some embarrassing discrepancies between standard predicate logic and the Aristotelian construction. The often-mentioned problem of the logical square and the absence of singular inferences in the theory of syllogisms are only two of the many difficulties that the modern rendering of Aristotelian propositions entails. It is probably for this very reason that Łukasiewicz, in his reconstruction of Aristotelian logic, preferred to treat the Aristotelian quantified propositions as primitive relations not to be interpreted via quantifiers and logical operators.³

I think that Łukasiewicz's approach to the problem is wise and that his prudence must be commended. However, I believe that we can find clues to understanding Aristotle's ideas about predication by considering the way in which he employed predication. To analyze the inadequate and unsystematic evidence at our disposal, I will depart from traditional modern logic and instead use mereological ideas. In making my point, I will first propose an alternative reading of texts (A) and (B), which in my view is as plausible as the standard interpretation. I will then try to justify my position by referring to some general features of Aristotelian logic that do not fit the traditional understanding of (A) and (B) and lead to an alternative interpretation.

IV

Let us consider again text (A). In characterizing universal predication, Aristotle does not say that there is no *particular* of which the subject is said such that the predicate is not true of it or, equivalently, that every *particular* in the extension of the subject is in the extension of the predicate. In a much more elliptical and vague way, he claims that, in a universal predication, there is *nothing* of the subject of which the predicate is not said. It is our biased view regarding the nature of predication that persuades us to interpret this "nothing" (μηδέν) in the sense of "no particular." If we steer clear of the influence of modern semantics, we could make a different suggestion and suppose that "nothing" (μηδέν) means simply "no part." Taken from this standpoint the characterization of universal predication could be understood in the following way: *A* is predicated of every *B* if there is no part of *B* of which *A* is not predicated. By a natural expansion of Aristotle's words, we will say that *A* is predicated of every *B* if and only if there is no part of *B* that is not part of *A*.

This view is consistent with text (B) as well. If parts of *B* are *A*, even those parts that correspond to the particulars to which *B* applies are *A*, and to reject the claim that every *B* is *A*, it is sufficient to show that a minimal part of *B*, namely a particular, is *B* without being *A*.

Before presenting some evidence for this move, let us try to clarify the meaning and the consequences of this proposal by comparing it with the traditional interpretation. Consider again proposition (2) and imagine that with its subject, "man," we associate what I call an *extension*. In a parallel way, let us suppose that an extension is attached to the predicate "mortal" as well. Both extensions – one could claim – have parts. The extension associated with "man" has the sum, or better, the fusion, of all extensions of the instances of humans as its extension, and the same holds for the predicate "mortal."⁴ To claim that (2) is true amounts to claiming that every part of the extension associated with "man" is mortal, or, which is the same, every part of the extension associated with "man" is also part of the extension of "mortal." A similar situation applies to (1). We can associate an extension with "Richard," namely the extension that has itself as its only part, and the fusion of the extensions of all handsome things with the predicate "handsome." Therefore, we can claim that "Richard is handsome" is true, as it is,

because every part of the extension associated with “Richard” is handsome, or it is part of the extension of “handsome.” If we take “ $|\xi|$ ” as the extension associated with a term (whether universal or singular) and “ $x \leq |\xi|$ ” to mean “ x is part of the extension of $|\xi|$,”⁵ we may state

$$(4) \quad \forall x(x \leq |\phi| \rightarrow x \leq |\psi|)$$

as the semantic interpretation of (2). What semantically corresponds to (1) has exactly the same structure, namely

$$(5) \quad \forall x(x \leq |\alpha| \rightarrow x \leq |\phi|).$$

We can easily express the truth conditions for a particular proposition, which in usual logic has the form:

$$(6) \quad \exists x(\phi(x) \wedge \psi(x))$$

stating that (6) is true if at least one part of the extension of ϕ is ψ :

$$(7) \quad \exists x(x \leq |\phi| \wedge x \leq |\psi|).$$

One might be surprised at this result and wonder whether there is any rationale in this approach. Extensions look mysterious and clumsy. Why should we introduce them? The reason is that we cannot interpret quantification in the mereological terms we have adopted if we give up extensions. Consider sentence (1) and suppose that we interpret its subject “Richard” not as the extension of “Richard” but as Richard himself. This means that we claim that (1) is true if and only if every part of Richard is handsome. One might agree with that, but if Richard is a professor we should conclude in the same way that every part of Richard is a professor, and students would surely be confused by such a scholarly body.⁶

This view may look strange, but we can avoid the difficulty if we associate “Richard” not with the fully fleshed Richard, but, let us say, with a representative of him whose special characteristic is that it has no proper part or, to use the jargon of mereology, is *atomic*.⁷ To insist on this point, particulars normally have parts; “Richard” has an extension, and his hand has an extension as well. But in the extension of “Richard” there is not the extension of his “hand,” and the extension of his “finger” is not a part of the extension of his “hand.”

An atomic extension is associated with a singular term. What kind of extension should we associate with a general term? A possible answer to this question is to take a mereological fusion of individual extensions as the extension of a general term. In general, we say that γ is a fusion of α and β if it has α and β as parts and no other part distinct from them. In relation to our extensions we can write:

$$(8) \quad |\xi| = |\mu| + |\nu| = |\mu, \nu|.$$

It is important to point out that in the case of $|\mu|$, μ is not a proper part of $|\mu|$; and in the case of $|\mu, \nu|$, μ and ν are not proper parts of $|\mu, \nu|$, the only proper parts of it being $|\mu|$, $|\nu|$, and $|\mu, \nu|$. In general, we can say that an extension with $n > 0$ elements has $2^n - 1$ parts because, by definition, there is no empty extension.

If one is not yet reassured about extensions, there is another way to present them by means of the more familiar notion of class. We take atomic extensions as singletons (classes that have only one element), we take fusions of extensions as unions of singletons, and we interpret the part–whole relation as the inclusion relation. By making some quite natural assumptions, it can be demonstrated that a class is part of another class if and only if it is one of its subclasses and further that no class has any part that is not itself a class. From these two claims it follows that the parts of a class are exactly its subclasses. In this way, extensions can be treated as classes.⁸

But it should be observed that recourse to the familiar notions of class, union, singleton, and inclusion does not bring the theory to the reassuring patterns of standard set theory. To keep pace with the mereological theory of extensions we must either take the notion of inclusion as primitive (and not defined by means of the membership relation) or interpret it in terms of the part–whole relation. In both ways, we are outside the usual set theory, because we have no longer anything to do with collections logically determined by the fundamental membership relation, but we are dealing with collections whose basic relation is the part–whole relation. And, of course, these relations are logically different because they have different logical properties. It is sufficient to think that the part–whole relation is normally taken as transitive, whereas the membership relation is not, or that for collections governed by the part–whole relation the following axiom holds:

$$(*) \quad \forall x(x < a \rightarrow x < b) \rightarrow a \leq b$$

which is not the case for the corresponding membership-collections.⁹ If every proper part of the Heeramanek collection is a proper part of the oriental collection of the Los Angeles County Museum, then the Heeramanek collection is a part of the oriental collection of the Los Angeles County Museum. The situation is different with classes. If all the elements of the class that constitutes the Heeramanek collection are elements of the class of the oriental collection of the Los Angeles County Museum, we should not infer that the Heeramanek collection is an element of the class that corresponds to the collection of the County Museum.

In general, there is no need to follow the prejudiced view largely endorsed by logicians according to which the only logically safe way of dealing with collections is by taking them as sets. Collections are not necessarily sets, nor is there any reason to claim that taking a collection as a set is the only way to make our confused intuitions about a multiplicity of objects precise. To limit collections to being sets, we must abandon some of our natural intuitions about collections, and more than one philosopher has stressed this point.¹⁰ The mereological approach we have proposed is as precise as the set-theoretical one; it is philosophically interesting, and it enables us to capture some of Aristotle's intuitions about predication.

The conclusion is that interpreting extensions in terms of classes takes us outside the domain of the standard set theory. Therefore, let us stay with the mereological interpretation of extensions, which, as we shall see, better suits Aristotle's views.

V

We cannot develop this theoretical approach any further here and discuss its merits and difficulties because we must return to Aristotle. A difficulty awaits us. Aristotle has no word for extension, and nowhere, as far as I know, are these dubious logical entities mentioned by him. Moreover, even if we were allowed to introduce extensions to explain Aristotle's views, why should we think of them in mereological terms and not in terms of collections or sets? Let us introduce some evidence that points in the direction of the introduction of extensions conceived in mereological terms. While distinguishing the different senses of "μέρος" ("part"), Aristotle states:

(C) We call a part in a sense that into which a quantity can in any way be divided (for that which is taken from a quantity *qua* quantity is always called a part of it, i.e., two is called in a sense part of three). In another sense it means, of the parts in the first sense, only those which measure the whole; this is why two, though in one sense it is, in another is not, a part of three. Moreover, apart from the quantity, the things into which the species might be divided are also called parts of it; for this reason we say the species are parts of the genus (τὰ εἶδη τοῦ γένους φασὶν εἶναι μέρη).

(Aristotle, *Metaphysics* 5.25, 1023b12–17; ROT, slightly modified)

Two points are clearly made here. First, we can think of both species as parts of their genera and particulars as parts of their species. Second, species are said to be parts of their genera in a sense that is different from the sense that allows us to say, for instance, that segment *AB* is a part of line *AC*, or my head is a part of me. As we shall see in a moment, the first claim can easily be extended: Not only are species parts of their genera and particulars of their species, but in general what is an instance of something is a part of it. From this point of view a universal can be conceived of as a whole, the parts of which are its instances.

Text (C) does not explain where the difference between the parts which constitute a genus and the parts which constitute a material whole lies. To begin to understand Aristotle's point, we must consider another text where the question is examined in a symmetrical way, that is, from the point of view of the whole. With reference to the various meanings of "ὅλον" ("whole"), Aristotle states:

(D) We call a whole that from which is absent none of the parts of which it is said to be naturally a whole, and that which so contains the things it contains that they form a unity; and this in two senses – either as each and all one or as making up the unity between them. For what is universal and what is said

as a whole, implying that it is a certain whole, is universal in the sense that it contains many things by being predicated of each and by being all those and each of them one, as for instance man, horse, and god are one because they are all living things. On the other hand, the continuous and limited is a whole, when there is a unity out of several things that are constituents of it especially if they are potentially present or, failing this, even if they are present actual.

(Aristotle, *Metaphysics* 5.26, 1023b26–34; ROT, slightly modified)

The translation is difficult, and the Greek is probably clearer. What I understand of this passage is, first, that the idea of being a whole is extended here not only to species with respect to individuals and to genera with respect to species but also to universals with respect to that of which they are predicated. Second, the point is repeated that a universal is a whole in a different sense, for instance, the sense in which a living thing is a whole made up of its parts. What is added here to the previous text is a reason for the difference, and this difference is said to consist in the fact that they form a unity in a different way. A universal unifies its parts differently than, for instance, a living body unifies its parts. A universal unifies its parts in a way that allows predication of the whole with respect to its parts, and we must understand that this is not required in the case of a natural whole.

One might think that an obvious way to explain the manner in which a universal is one with respect to its instances is to think of the universal as a common feature of its instances. Animal is predicated of horse and man because it is something that horse and man have in common, a feature shared by them. However simple, this cannot be the right explanation of Aristotle's point. If it were, a genus, or a universal, would be a part of its species, or its instances. Animal is a part of horse and of man such that man and horse are both indistinguishably animals. This is how Aristotle considers genera with respect to species when he claims that the genera are parts of their species.¹¹ What he says here is the opposite: Species are parts of their genera and in general are instances of the corresponding universals.

In what sense, then, are instances parts of their universals? It is obvious to point out that instances, even if they are species, are not parts of the notion of their universals. Horse is not a part of the notion of animal that I understand when I heard the word "animal." Aristotle is very clear about this point: From the fact that we know that a species is ϕ we cannot infer anything about the instances of that species unless we further know that *a* is an instance of that species.¹² Therefore, of what is horse a part if it is not a part of the notion, the conceptual content, of animal? My answer is: It is a part of the *extension* of animal.

This interpretation is reinforced if we think of logical division. As is well known, Aristotle took the method of division from Plato and developed it in an original way by rejecting the dichotomic interpretation of the method elaborated in the Academy.¹³ Although a considerable amount of work has been dedicated to the analysis of this process,¹⁴ I have never found an answer to the following question: When a genus is divided, what exactly is divided? We cannot say that the notion or the conceptual content is expressed by a genus that is divided because its species are not parts of such a notion. It is something else. In some places,

Aristotle compares a genus to a subject or even to matter out of which the species are generated by applying to it the differentiae.¹⁵ Would it be too daring to think of a genus *qua* matter or *qua* subject of the differentiae as an *extension* of the genus? And why are we not entitled to expand the point to all universals with respect to their instances?

Moreover, particulars are at the bottom of a division whose top is constituted by the most universal genus. Particulars in this situation are called “ἄτομα,” and this means that they cannot be further divided. Why? Because if particulars had parts, these parts would be parts of their species and genera, because the part–whole relation is transitive. But full-blooded particulars such as Richard, Robert, and Mario *do* have parts. Therefore, we must conclude that full-blooded particulars are not at the bottom of a process of division. The particulars that close a division, being atomic, are logical particulars, or particulars logically taken, or, which is the same, *extensions* of particulars.

One might claim that it is not the case that the ἄτομα at the bottom of a division are indivisible in the sense that they do not have parts, but rather in the sense that their parts are not homogenous. Richard is an ἄτομον, one might say, because Richard’s parts are not that what Richard is, namely a man. I am not sure that this view can easily be applied to Aristotle. There are at least two ways to interpret this claim. One is stronger and consists in saying that a is an ἄτομον if, and only if, whatever is true of a is true of no proper part of a :

$$(P1) \quad \forall \phi \forall x (x < a \wedge \phi(a) \rightarrow \neg \phi(x)).$$

The other way amounts to saying that a is an ἄτομον if and only if at least one proper part of a is such that there is something true of a that does not hold of that part:

$$(P2) \quad \exists \phi \exists x (x < a \wedge \phi(a) \wedge \neg \phi(x)).$$

The advantage of (P1) and (P2) is that they do not require the notion of extension to explain what an ἄτομον is. But they hardly fit Aristotle’s views. In Aristotle’s terminology, “ἄτομον” normally equated to “καθ’ ἑκάστων” or what is “ἐν ἀριθμῳ,” and in most cases denotes a particular.¹⁶ Now if “ἄτομον” were characterized by (P1) there would be entities that cannot be qualified as particulars. Take, for instance, a segment AB . One would be prepared to say, I think, that AB is an individual segment according to our obvious intuition of particularity. But in fact AB is not particular according to (P1) because it is not true that whatever holds of AB holds of no part of it. AB is a segment, and the same is true of segment AC , which is a part of AB . This can be repeated *ad infinitum*, and therefore no segment is an individual segment.

On the other hand, if we consider (P2) a better candidate for explaining what an ἄτομον is, we surely solve the problem of the segment. AC may be taken to be a part of segment AB , and AC definitely has characteristics that are not shared by AB , as, for instance, the property of being lesser than AB . But we face a new

difficulty. Suppose that the part–whole relation is transitive and that ἄτομα, as Aristotle says, are parts of their species. It follows that parts of the ἄτομα are parts of the species. Now, take the head of Richard, which is part of the species man, Richard being part of man. It is not true to say that the head of Richard is a species. Therefore, according to (P2), “man” must be classified as a particular – a nonsense in Aristotelian terms. The conclusion is that we must stick with our original explanation of ἄτομα and think of them as logical particulars, namely as atomic extensions.

By introducing mereological extensions we are able to explain the difference Aristotle claims to exist between a universal whole and a natural whole. Consider, for instance, a natural whole such as a human body. Its parts are its material parts, for instance, its head or its arms, out of which the body is made. In the case of a genus and its species or of a universal and its instances, the species or instances are said to be parts in a different sense because, properly speaking, the species are not that out of which the genus is made but what determine that out of which the extension of the genus is made. And, in general, the instances of a universal are not that out of which the universal is made but what determine the extensions out of which the extension of the universal is constituted.

VI

In light of this interpretation, some features of Aristotle’s terminology that *prima facie* look odd can easily be explained. As he frequently calls the “εἶδη” (“species”) “μέρη” (“parts”) of their genera,¹⁷ Aristotle sometimes refers to a universal as that which contains (περιέχειν) that of which it is true.¹⁸ In a parallel way, what is contained by something (τὸ περιεχόμενον ὑπό τινος) is also what is under something else (ὑπὸ τί ἐστι).¹⁹ Again, it is perfectly natural to take the species as partaking (μετέχοντα) of their genera.²⁰ Although the relation of partaking is normally used to qualify the way in which species are related to their genera, sometimes “partaking” is also used to express the relation between particulars and species (or even genera). Of course, one can take all these as metaphorical expressions. But the interpretation we are proposing can explain why Aristotle has preferred this metaphor to other possible ones. And indeed, if we take our interpretation seriously, we can claim that his terminology is not at all metaphorical. We can make sense of this way of speaking by postulating extensions that are parts of other extensions in the manner we have described.

This terminology should be understood carefully in the light of a passage that introduces text (A). Before giving his famous definition of κατὰ παντὸς κατεγορεῖσθαι, Aristotle states an equation between “be in another as in a whole” and “being predicated of all” by saying:

(E) That one term should be in another as in a whole (ἐν ὅλῳ εἶναι ἕτερον ἑτέρῳ) is the same as for the other to be predicated of all of the first (τὸ κατὰ παντὸς κατεγορεῖσθαι θατέρου θάτερον).

(Aristotle, *Prior Analytics* 1.1, 24b26–28; ROT translation)

This means that saying that ϕ is in ψ as in a whole is the same as saying that ψ is predicated of every ϕ . A similar point is made when Aristotle considers the universal modes of the first figure, namely Barbara and Celarent. He says:

(F) Whenever three terms are so related to one another that the last is in the middle as in a whole ($\epsilon\tilde{\nu} \ \omicron\lambda\omega \ \epsilon\tilde{\iota}\nu\alpha\iota \ \tau\tilde{\omega} \ \mu\acute{\epsilon}\sigma\omega$) and the middle is either in, or not in, the first as in a whole ($\epsilon\tilde{\nu} \ \omicron\lambda\omega \ \tau\tilde{\omega} \ \pi\rho\acute{\omega}\tau\omega \ \eta \ \epsilon\tilde{\iota}\nu\alpha\iota \ \eta \ \mu\eta \ \epsilon\tilde{\iota}\nu\alpha\iota$), the extremes must be related by a perfect syllogism.

(Aristotle, *Prior Analytics* 1.4, 25b32–35; ROT translation)

If we take the relation of being in a whole as synonymous with the relation of being a part of a whole, both texts become immediately intelligible. Suppose that ϕ is a general term and that its extension is part of the extension of another general term ψ . If the extension of ϕ is part of the extension of ψ , then every part of the extension of ϕ is part of the extension of ψ , because the part–whole relation is transitive. Therefore, ψ is predicated of every ϕ , and Aristotle’s terminological point becomes perfectly clear.

However, one might take text (E) as evidence against the interpretation we are trying to defend here. If Aristotle really had in mind a reduction of universal quantification to a relation between mereological extensions, why should he have privileged the relation of “being predicated of all” over the relation of “being contained in” and reduced the latter to the first? By focusing on the relation of “being predicated of all” and by defining only this, one might claim that Aristotle reduced the mereological to the predicative relation, and not vice versa, as we are trying to establish.

To answer this objection we have to make a point that cannot be fully justified here, namely that the crude mereological language such as “being in another as a whole” used by Aristotle to denote universal predication probably antedates his theory of quantification, in which particular and universal propositions are systematically related and opposed. In other words, the hypothesis I am considering here is that Aristotle first thought of predicative relations in terms of connections between species and genera, and between particulars and species, and he used crude mereological language to express them. By reflecting on the logical aspects of propositions, he developed a theory of quantification in which universal is distinguished from particular predication. At that point, it was crucial to find an expression to denote particular, and oppose universal, predication. Because “ἐπὶ τινὸς κατηγορεῖσθαι” is a good candidate to do the job, the obvious choice for universal predication became “κατὰ παντὸς κατηγορεῖσθαι,” and expressions such as “ἐν ὅλῳ εἶναι” were left behind because they had no *pendant* for particular predication. If this is the story – and I am aware that more should be said in order to make it plausible from an historical point of view – I do not think that we can infer an objection to our interpretation from the reduction of mereological to quantificational language. The point is simply that crude mereological language is not able to stress the relation between the two kinds of quantified predications. But abandoning pure mereological language did not necessarily imply Aristotle’s setting aside the ideas that are behind its introduction.

VII

There are deeper reasons that support our interpretation. A crux of the analysis of Aristotle's syllogistic is that no mention is made of inferences containing singular propositions.

As is commonly stated after Łukasiewicz's book on Aristotle's syllogistic,²¹ an inference such as

- (9) every man is mortal
 Socrates is a man

 Socrates is mortal

is by no means an example of an Aristotelian syllogism because it contains a premise that is singular and Aristotle does not consider such premises in his theory.²² The type of inference closest to (9) that Aristotle examines is an inference such as

- (10) every man is mortal
 every Athenian is a man

 every Athenian is mortal.

If we consider the logical form of (9) and (10) according to the usual patterns, we are compelled to conclude that their formal structure is different. Whereas the form of (9) is

- (9*) $\forall x(B(x) \rightarrow A(x)), B(a) \vdash A(a)$

the logical structure of (10) is given by

- (10*) $\forall x(B(x) \rightarrow A(x)), \forall x(C(x) \rightarrow B(x)) \vdash \forall x(C(x) \rightarrow A(x)).$

Formulas (9*) and (10*) are different schemes, and they are proven in different ways. Therefore, taking (10*) as valid is not, by the same token, a justification for (9*), even if the deduction of (9*) from (10*) may be considered to be trivial. Although (10*) corresponds to Barbara, which Aristotle mentions and takes as a perfect, that is, evident, syllogism,²³ there is no account of (10*) in his theory of deduction. On the other hand, Aristotle seems to admit inferences that are instances of (9*),²⁴ and tradition has solved the problem by taking inferences such as (9) as special cases of inferences like (10). But if we interpret the logical structure of (9) by means of (9*) and take (10*) as representing the form of (10), it is difficult to see how (9) can be a special case of (10). Having denied that inferences such as (9) and (10) are instances of the same logical schema, modern scholars have tried to find more or less complicated logical motivations to explain why Aristotle, in his official presentation of syllogisms, does not even mention

inferences with singular premises.²⁵ The obvious point is that there is no logical reason to reject or avoid inferences such as (9*), and in fact I believe that Aristotle had no logic-based argument to disregard singular inferences.

We cannot tackle this question here. What matters to us is that the problem becomes logically less acute if we take another approach to the question: one based on our interpretation of quantification. This approach consists in proposing a different formalization of (9) and (10) that eliminates any formally relevant difference between the two logical structures involved in the inferences. As we have seen, universal propositions, as all the premises of (10), may be represented as relations holding between the mereological extensions of their terms, and the same is true for the singular proposition that constitutes the minor premise of (9). Therefore, if we take, “ ζ ,” “ η ,” and “ θ ” to stand for three different terms, either singular or universal, we can represent the logical structure of both (9) and (10) by

$$(11) \quad \forall x(x \leq |\eta| \rightarrow x \leq |\theta|), \forall x(x \leq |\zeta| \rightarrow x \leq |\eta|) \vdash \forall x(x \leq |\zeta| \rightarrow x \leq |\theta|).$$

Needless to say, both (9) and (10) can be considered instances of (11). Therefore, the validity of (11) implies the validity of both (9) and (10).

Of course, Aristotle did not formulate Barbara with the generality of (11) because the terms he considers in his deductions are general terms, and the problem still remains as to why he omits any mention of singular propositions in his logical classification of premises. But from this perspective it is easy to see how syllogistic patterns can trivially be extended to cover singular inferences, and we can explain why Aristotle uses inferences of the same form as (9) without having explicitly considered them. This view reinforces the impression that in order to explain Aristotle’s silence concerning singular inferences we should look not for a logical but for an epistemological explanation.

VIII

The fact that the predicative relation between a general and a singular term can be explained in the same way as that between two general terms accounts for another feature of the Aristotelian system, which might appear disconcerting to people considering it from the point of view of standard logic.

Aristotle’s usual words for “particular” and “individual” are “τὸ καθ’ ἑκάστων” and “τὸ ἄτομον.” For instance, he calls that which is opposed to a universal a καθ’ ἑκάστων.²⁶ Along the same lines, he calls what is not said of a subject or is in a subject an ἄτομον.²⁷ These terms, “καθ’ ἑκάστων” and “ἄτομον,” are sometimes used to indicate general terms, namely the so-called “*infimae species*,” which is to say the species under which there are only particulars.²⁸ If we keep to the traditional interpretation, we should judge this way of speaking to be misleading. By contrast, it has a natural explanation in our perspective. The tendency to distinguish sharply between singular and general terms depends mainly on the fact that they are thought to be bearers of different logical relations. The way in which an individual is under a concept is supposed to be different from the way in which

a concept is subordinated to another concept. In the first case we are dealing with the membership relation; in the second with the inclusion relation. Because of the difference between membership belonging and inclusion, one is led to carefully separate singular from general terms. But the main reason for such a sharp distinction vanishes if we suppose that singular terms are related to concepts in the same way as general terms are related to them, namely through the part–whole relation.

If we think of the relation between a singular and a general term as something that can be expressed in the same way as the relation between two general terms, we may conceive the singular as the borderline case of the less general. Both less general and singular terms are particular cases of the corresponding universals, and their difference lies only in the fact that in the case of the singular there is nothing less general below, its extension being constituted by only one atom. This way of putting things explains what we would otherwise be inclined to call an unexpected shift from singular to universal, which is sometimes found in Aristotle's logical writings.

To give an idea of what such a shift is like, it is sufficient to recall the famous definition of “induction” (ἐπαγωγή) at the beginning of the *Topics*. Aristotle characteristically defines induction as a progress from particulars to the universal (ἡ ἀπὸ τῶν καθ'ἕκαστα ἐπὶ καθόλου ἔφοδος).²⁹ But it can hardly be maintained that τὰ καθ'ἕκαστα mentioned in this definition are the particulars we would be inclined to put at the beginning of any induction. To understand the point, it is sufficient to consider the example given by Aristotle as an illustration of his definition. To show that the skilled man is the best man in any particular field, Aristotle uses the examples of the skilled pilot who is the best pilot and of the skilled charioteer who is the best charioteer.³⁰ None of them represents an individual case, but rather a case less general than the one that is aimed at as a conclusion. Once more, the distinction between what is really particular and what is less general than a proposed case is not something on which any stress is imposed. Thus, the nonstandard interpretation seems to be more appropriate than the traditional one to account for Aristotle's view.

Aristotle's inclination to avoid any precise distinction between singular and general terms matches an epistemological problem that Aristotle shares with his teacher Plato. Particulars in the extension of a general term may be infinite, or at least so many that they cannot easily be considered and dealt with. Suppose that we have to establish whether all animals have a property ϕ . We cannot list all individual animals and see whether each of them has ϕ , because animals are too many. Aristotle's solution to the problem is as follows:

(G) Another rule is to examine the subjects to which a predicate is said to belong universally or is taken as belonging to none. Look at them species by species, and not in their infinite multitude (σκοπεῖν δὲ κατ'εἶδη καὶ μὴ ἐν τοῖς ἀπειροῖς); for the inquiry will proceed more directly and in fewer steps. You should look and begin with the first divisions, and then proceed down to the indivisibles (ἕως τῶν ἀτόμων).

(Aristotle, *Topics* 2.2, 109b13–16; ROT, slightly modified)

The general idea is clear. One must stick to universals in order to make the inquiry as systematic and complete as possible, because particulars are indefinitely many and there is no means to survey them all. In order to prove that ϕ holds all the members of a genus α , it is sufficient consider all its species β , γ , and δ . If β , γ , and δ are all ϕ and they exhaust α , we are entitled to conclude that α , the genus, is ϕ . In this way we do not need to consider the many particulars under α , and we have to deal with a limited number of cases. This is a reasonable part of an epistemological strategy, which matches our interpretation of universal predication very well. Species β , γ , and δ of genus α are parts of its extension, and they cover the whole of it. If we are able to say that (the extensions of) β , γ , and δ are all in (the extension of) ϕ , we can infer that also (the extension of) α is included in (the extension of) ϕ , because there is no part of α other than β , γ , and δ .³¹

IX

I would like to point out that the interpretation proposed here should be taken *cum grano salis*. I do not claim that the mereological approach to Aristotle's conception of predication we have proposed here corresponds in detail to what he had distinctly in mind when he wrote the *Analytics* or the treatise *On Interpretation*. The simple fact that there is no trace of an analysis of the notion of predication in his remaining works would suggest that he did not have a fully developed theory of predication. From this point of view, our interpretation goes far beyond the texts, and I understand that a cautious historian might be upset by my exegetical arrogance. However, I feel entitled to defend my view not only because the standard position violates Aristotle's self-restraint (as does the interpretation proposed here) but also because I think that the mereological approach has the advantage of explaining some features of Aristotle's logical theory, which would otherwise be considered either wrong or nonsensical. It is for this better reason, in particular, that I ventured to present my proposal.

Notes

- 1 Alexander of Aphrodisias, *On Aristotle's Prior Analytics* 24.27–30.
- 2 See Ross 1949: 292.
- 3 Łukasiewicz 1957²: 14 and 77–78.
- 4 {Here Mignucci is assuming that every human being is identified with an extension that has no proper parts. Later on Mignucci also speaks of extension in connection with the denotation of a name.}
- 5 For reasons of simplicity, I depart from the terminology used by Simons 1987. I represent the strict part–whole relation by “<,” leaving “≤” to express the simple part–whole relation. Peter Simons uses “<” to indicate the part–whole relation and introduces “≪” for the strict part–whole relation.
- 6 With a similar argument, Bertrand Russell correctly denied that the membership relation of an individual to a class can be interpreted as a part–whole relation (Russell 1979¹⁰: § 21: 19).
- 7 An object is atomic or an atom if it has no proper part. Formally, we state

$$At(a) =_{df} \neg \exists x (x < a).$$

Moreover, a is a proper part of b if a is a part of b without being the same as b :

$$a < b =_{df} a \leq b \wedge a \neq b.$$

- 8 This has been proven in Lewis 1991: 1–10.
- 9 For the meaning of this axiom, see Simons 1987: 28 ff. and 114 ff.
- 10 See, for instance, Bealer 1982: 102–111.
- 11 Aristotle, *Metaphysics* 5.6, 1015b27; 5.25, 1023b24; cf. *Physics* 4.3, 210a19.
- 12 Aristotle, *Posterior Analytics* 1.1, 71a17 ff.
- 13 Aristotle never abandoned division. Balme 1961: 195–212; Pellegrin 1982: 38 ff. are surely right in pointing out that in *Parts of Animals* 1.2–3 Aristotle criticizes not the method of division but its dichotomic interpretation favored in the Academy. On the relation between Aristotelian and Academic division see Kullmann 1974: 342 ff.
- 14 In addition to the works quoted in the previous footnote, interesting comments on the method of division can be found in Fragstein 1967: especially 86–87; Solmsen 1929: 189–190. For the evolution of this notion in Aristotle's thought, see Düring 1943: 109–114; Lloyd 1961: 59–81, *sed contra* Pellegrin 1982: 25 ff.
- 15 See, for instance, Aristotle, *Metaphysics* 5.6, 1016a24 ff.
- 16 Sometimes the *species infimae* are also called “ἄτομα” (see endnote 28).
- 17 See, for instance, Aristotle, *Physics* 4.1, 208b11–12; *Metaphysics* 5.14, 1020b16. For more instances, see Bonitz 1955²: 455b46 ff.
- 18 See, for instance, Aristotle, *Generation and Corruption* 1.3, 317b7. For more instances, Bonitz 1955²: 581a37 ff.
- 19 Aristotle, *Topics* 7.1, 152a16 and 29–30.
- 20 See, for instance, Aristotle, *Topics* 4.1, 121a12 ff.
- 21 Łukasiewicz 1957²: 1–7.
- 22 I will not discuss here the claim made by Łukasiewicz (Łukasiewicz 1957²: 1 ff.) that the only appropriate way to express an Aristotelian syllogism is by representing it as a conditional.
- 23 See Aristotle, *Prior Analytics* 1.4, 26b29–30 for equating syllogistic perfection with evidence cf. Patzig 1969: 51 ff.
- 24 See, for instance, Aristotle, *Prior Analytics* 1.33, 47b18 ff.; 2.21, 67a33 ff.
- 25 Łukasiewicz 1957²: 4–8.
- 26 See, for instance, Aristotle, *On Interpretation* 7, 17a37 ff.
- 27 See Aristotle, *Categories* 2, 1b3–9.
- 28 Aristotle, *Posterior Analytics* 2.13, 97b28 and *Metaphysics* 7.15, 1040a5–7 for καθ' ἑκάστων; *Posterior Analytics* 2.13, 96b17 and Bonitz 1955²: 120a58 ff. for ἄτομον.
- 29 Aristotle, *Topics* 1.12, 105a13–14.
- 30 Aristotle, *Topics* 1.12, 105a14–16.
- 31 I note, in passing, that this is the epagogical syllogism examined in Aristotle, *Prior Analytics* 2.23, 68b15 ff.

5 Puzzles about identity

Aristotle and his Greek commentators

I

Aristotle's conception of identity is too large a subject to be analyzed in a single chapter.¹ I will try to discuss here just one of the many problems raised by his views on the topic of identity. My subject will be Aristotle's references to what is today called "Leibniz's Law" (*LL*): If x and y are the same, they both share all the same properties. A formal version of it could be

$$(1) \quad x = y \vdash \forall F(F(x) \leftrightarrow F(y)).^2$$

It is perhaps worth remembering that (*LL*) must be distinguished from what is normally called the "principle of substitutivity" (*SP*) according to which substitution of co-referring expressions is truth preserving. As has been shown, (*LL*) does not entail (*SP*), because there are counterexamples to (*SP*) that do not falsify (*LL*).³ Both (*SP*) and (*LL*) have been doubted by some modern logicians. The question is far from being settled, and it is thus perhaps still of interest to examine how ancient logicians tried to manage this problem.

First, I will consider Aristotle's statements about (*LL*) and the analysis he gives of some supposed counterexamples to this principle. Second, I will consider the interpretations of his view among his Greek commentators, and I will evaluate their distance from Aristotle's position.

A preliminary remark is in order here. Aristotle distinguishes several senses of the word "same" (τὸ αὐτόν). He says that one is allowed to speak of numerical, specific, generic, and analogical identities.⁴ It is numerical sameness that corresponds to what we understand today by identity. The identity involved in (*LL*) is precisely this kind of identity. Therefore, the whole of our discussion will be limited to this kind of sameness.

Scholars have called attention to some passages where they believe that Aristotle introduces a somewhat restricted version of (*LL*). The most impressive evidence is given at *Topics* 7.1, 152b25–29. There, Aristotle considers the τόποι

concerning identity that are useful for disproving definitions and not for corroborating them (*Topics* 7.2, 152b36–153a5). He says:

(A) To speak generally, you should look at every possible predicate of each of the two terms and at the subjects of which they are predicated and see whether there is a discrepancy anywhere; for (i) anything which is predicated of the one ought also to be predicated of the other, and (ii) of anything of which the one is a predicate the other also ought to be a predicate.

(*Topics* 7.1, 152b25–29; E. S. Forster translation, slightly modified)

That a reference to (*LL*) is supposed to be implied in this text is a position held by many scholars.⁵ But things are not as clear as one would like. The problem is to which entities the alleged version of (*LL*) is thought to apply. Are individuals meant here? Aristotle says in (ii) that we have to look at the subjects of the terms that are asserted to be the same, but individuals are not predicated of anything (*Prior Analytics* 1.27, 43a25–29).⁶ General terms could be expected to be the appropriate candidates for the role of objects that (i) and (ii) refer to. Let us use capital Greek letters A, B, Γ, \dots as variables for classes; a, b, c, \dots as variables for individuals; and x, y, z, \dots as variables ranging over both classes and individuals. According to the hypothesis that we are discussing, (i) says that, given two classes A and B , if they are the same, then every predicate of the one is also a predicate of the other. Now the problem is: What does “ Γ is a predicate of A ” mean? If the terminology of set theory can be borrowed, we can say that it means either “ $A \subseteq \Gamma$ ” or “ $A \in \Gamma$.”⁷ If we choose the first interpretation, (i) becomes:

$$(2) \quad A = B \vdash \forall \Gamma (A \subseteq \Gamma \leftrightarrow B \subseteq \Gamma).$$

If it is the second interpretation that is to be preferred, we have for (i):

$$(3) \quad A = B \vdash \forall \Gamma (A \in \Gamma \leftrightarrow B \in \Gamma).$$

Both (2) and (3) are versions of (*LL*) expressed in the language of set theory. Correspondingly, two different translations of (ii) can be proposed. If predication is expressed by means of the inclusion sign, something like

$$(4) \quad A = B \vdash \forall \Gamma (\Gamma \subseteq A \leftrightarrow \Gamma \subseteq B)$$

has to be stated. On the other hand, if the \in -relation is the formal correspondent for predication, (ii) becomes:

$$(5) \quad A = B \vdash \forall x (x \in A \leftrightarrow x \in B).$$

Formulas (4) and (5) are trivially equivalent, whereas the difference between (2) and (3) is sharp. If (3) is admitted, classes are to be considered as individuals. Taken in this sense, let us call them “abstracts.” Consider, for instance, the proposition “man

is white.” If this proposition is to be interpreted by means of the \in -relation, it means that among white things there is the object called “man.” Of course, the more natural way of interpreting our example is by means of the \subseteq -relation, so that the proposition means that everything that is a man is white. In this case, classes play the role not of abstracts, but just of general terms or predicates.⁸

Even if our first reaction would in all likelihood be in favor of the interpretation that takes (2) as the correct translation of (i) (hereafter called “the \subseteq -interpretation”), the other possibility, namely the “ \in -interpretation,” cannot be dismissed without further examination. Some facts support the \in -interpretation. The examples Aristotle chooses in the discussion of identity are apparently of classes taken not as abstracts, but as general terms. Let me consider just one case. In *Topics* 7.1, 152a5–30, Aristotle discusses Xenocrates’s identification of the happy life with the good life. Xenocrates’s argument for his thesis is that the happy life and the good life are each the most desirable life. But only a single thing can be the greatest in a given area. Therefore, the happy life and the good life are the same.⁹ In his answer, Aristotle takes for granted that what is greatest in a given area is a single thing in that area, if this thing is an individual thing. But the same does not hold if “to be the greatest” is a general term like “Peloponnesians” or “Spartans.” We can state that the Peloponnesians are the bravest of the Greeks and add that the Spartans (a subclass of the class of Peloponnesians) are likewise so, without implying that the Spartans are the same as the Peloponnesians (*Top.* 152a12–26). Therefore, Aristotle concludes that Xenocrates is wrong because the happy life and the good life are not individual, but general terms, one of which is a subclass of the other (*Top.* 152a26–30). A possible formalization of Xenocrates’s argument would be the following. Let “ A_1 ” stand for “the happy life,” “ A_2 ” for “the good life,” and “ $x \in B_1$ ” for “ x is the greatest.” Then the principle stating that what is the greatest is a single thing becomes

$$(*) \quad \exists x(x \in B_1) \vdash \exists! x(x \in B_1).^{10}$$

Because $A_1 \in B_1$ and $A_2 \in B_1$, it follows from (*) that $A_1 = A_2$. Aristotle does not deny (*). He only points out that when a class is said to be the greatest, as for instance, that of the Peloponnesians (“ Γ_1 ”), the uniqueness expressed by (*) becomes an equivalence, so that what we have to state is $\Gamma_1 = B_1$, that is, $\forall x(x \in \Gamma_1 \leftrightarrow x \in B_1)$. From such an assumption it does not follow that anything else is B_1 . Take, for instance, a subclass Γ_2 of Γ_1 , the Spartans. In this case $\Gamma_2 \subseteq B_1$ holds, but $\Gamma_2 \equiv \Gamma_1$ does not. Now the happy life and the good life have to be interpreted as playing the same role as the Peloponnesians and Spartans. Therefore, the happy life and the good life are not abstracts, but general terms. One feels justified in thinking that Aristotle takes the classes of all his examples of identity statements as general terms and not as abstracts.

A second point in favor of the \subseteq -interpretation comes from the *Topics*. Discussing *propria* Aristotle says:

(B) Next, for destructive criticism, you must see whether the same thing fails to be a *proprium* of things that are the same, in so far as they are the same;¹¹

for what is stated to be a *proprium* will not be a *proprium*. For example, since appearing good to people is not a *proprium* of the object of pursuit, neither could appearing good to people be a *proprium* of object of choice, because object of pursuit and object of choice are the same thing. For constructive argument, on the other hand, you must see whether the same thing is a *proprium* of the same subject, in so far as it is the same; for what is stated not to be a *proprium* will be a *proprium*. For example, since it is said to be a *proprium* of man *qua* man to possess a tripartite soul, it would also be a *proprium* of mortal *qua* mortal to possess a tripartite soul.

(*Topics* 5.4, 133a24–32; E. S. Forster translation, slightly modified)

This text is normally taken to support the view that Aristotle states a version of (LL) restricted to *propria*. But the difficulty is even greater here than before. A *proprium* is a predicate that (1) does not enter the definition of the term to which it is attributed and (2) is predicated reciprocally of it.¹² The more natural way of expressing this last condition is by means of an equivalence. If y is a *proprium* of x , then $x \equiv y$, namely $\forall z (z \in x \leftrightarrow z \in y)$.¹³ It would be odd to formalize condition (2) through the \in -relation, so that $x \in y$ and $y \in x$. But this kind of reciprocity would be required in order to see a formulation of (LL) restricted to *propria* in this passage. (LL) restricted to *propria* takes the form:

$$(6) \quad x = y \vdash \forall \Delta (x \in \Delta \leftrightarrow y \in \Delta)$$

where “ Δ ” ranges over the *propria* of x and y . Suppose then that Δ' is a *proprium* of x . Thus, $x \in \Delta'$ implies by definition $\Delta' \in x$. Therefore, what is stated by Aristotle is not (6), but rather a restricted version of (2).

Against this conclusion, one could urge that Aristotle sometimes takes a *proprium* to mean what is predicated of only one subject. Of course, such a condition cannot be fulfilled by an equivalence between two general terms and it has to be expressed by means of the \in -relation. We could say that if x is a *proprium* of y (in this sense of *proprium*), then $y \in x \wedge \exists! z (z \in x)$.¹⁴ It is precisely this requirement that forms the basis of the argument attributed to a sophist against the existence of *propria* and of Aristotle’s reply in *Topics* 5.4, 133b15–134a4.¹⁵ But this way of conceiving *propria* is not the standard one and it cannot, therefore, be applied every time Aristotle speaks of *propria*. It is more likely that the *propria* that are considered here are the *propria* covered by the definition of *Topics* 1.5, 102a18–19, as we have presupposed in our interpretation.

Shall we conclude that the passages we have analyzed have nothing to do with (LL) and that something like (2) and (4) – or restricted versions of (2) – are the logical laws expressed by them? Before answering this question let us consider another text. Among the *τόποι* concerning identity the following is considered by Aristotle:

(C) Furthermore, you must examine them <i.e. two identicals> from the point of view of their accidents or of the things of which they are accidents; for any accident of the one must also be an accident of the other, and, if the one of

them is an accident of something else, so must the other be also. For if there is any discrepancy on these points, obviously they are not the same.

(*Topics* 7.1, 152a33–37; E. S. Forster translation)

In this passage Aristotle makes the same point he defends in text (A). The only difference is that the range of the predicates considered is limited to accidental predicates. Because accidental predications are in question, it seems more natural to understand them by means of the \in -relation rather than by means of the inclusion relation.¹⁶ The reason is that accidental predicates are for the most part predicates of individuals. If F is a predicate of x , where x is an individual, it is correct to express this relation as $x \in A$, where $A = \{x: F(x)\}$, whereas $x \subseteq A$ would be inappropriate. But accidental predication can also take place between general terms. Nothing prevents us from asserting that for an appropriate A and B , $A \subseteq B$ is an accidental inclusion.¹⁷ Besides, Aristotle mentions here “the things of which they (= the identicals) are accidents.” So, as in text (A), he cannot be thinking of individuals. Once more, restricted versions of (2) and (4) seem to be meant rather than particular cases of (3) and (5).

What about (*LL*) then? Shall we think that other interpreters have too hastily admitted its presence in Aristotle’s texts and that they have nothing to do with it? Such a statement appears to me hardly defensible, because in *Sophistical Refutations* 24, where some alleged counterexamples to (*LL*) are considered, (*LL*) is recognized as a legitimate assertion, as we will see in due course. Perhaps one way of getting free of the impression of uneasiness inspired by the result of our previous inquiry could start from reconsidering the distinction between the \in -relation and the \subseteq -relation, which we have assumed to be so sharp. We do not need to insist that $x \in y$ and $x \subseteq y$ are different, nor must we suppose that Aristotle systematically confuses them. We can think of the \in -relation and the \subseteq -relation as two different interpretations of what is expressed in natural language by expressions such as “to be said of” (“λέγεσθαι”), “to be predicated of” (“κατηγορεῖσθαι”), “to belong to” (“ὑπάρχειν”), and so on.¹⁸ White *is predicated of* Socrates as much as mortal *is predicated of* man, but in the first case we have an \in -relation, whereas in the second an \subseteq -relation. If the subject of a proposition is an individual, the \in -relation has to be chosen in order to express the logical form of that proposition. However, if the subject is a class, we have to decide whether it is a general term or an abstract. If it is a general term, we use the \subseteq -relation; otherwise, the \in -relation has to be applied. Aristotle states (i) and (ii) of text (A) with reference to classes taken as general terms, as we have seen. Therefore, (2) and (4) are the correct interpretations of his statements. But we would force Aristotle’s words if we inferred that (i) and (ii) have nothing to do with (*LL*), or better, that any reference to (*LL*) is ruled out by them. Incidentally, a version of (*LL*) for individuals, namely

$$(7) \quad a = b \vdash \forall \Gamma (a \in \Gamma \leftrightarrow b \in \Gamma)$$

can easily be deduced from (2) by using some elementary theorems and an axiom of set theory.¹⁹ Such a proof cannot be attributed to Aristotle. However, the

formalization of what he says in (i) of text (A) must consider the ambiguity, or at least the variety of meaning, of the expression “being predicated of” that is at work in natural languages. With this fact in mind, one could translate (i) as

$$(8) \quad x = y \vdash \forall \Gamma (\Gamma(x) \leftrightarrow \Gamma(y))$$

where the interpretation of “ $\Gamma(x)$ ” depends on the way in which x is interpreted. If “ x ” and “ y ” stand for individuals or abstracts, $\Gamma(x)$ becomes $x \in \Gamma$; if they stand for general terms, $\Gamma(x)$ becomes $x \subseteq \Gamma$. In this way, we can say that *(LL)* is implied in (8) and therefore in (1). Naturally, the fact that “being predicated of” has a double meaning is not by itself a sufficient reason for asserting that Aristotle, when he states (i), is prepared to extend it to (3) and (7). But we will see in the following that Aristotle is committed to defending *(LL)* against some alleged counterexamples. That makes it reasonable to suppose that he had nothing against the idea that (i) could be said to cover both (2) and (3).²⁰ Our next task, therefore, will be to consider this part of Aristotle’s doctrine of identity.

II

In *Physics* 3.3, Aristotle discusses some consequences of his definition of change. If change is actualization both of the mover and of the moved, the actualization of the mover has to be located in the moved (*Physics* 3.3, 202a13–21). This view raises some difficulties. Suppose that the actualization of the mover is the same as the actualization of the moved. As a particular case, we may state that teaching ($\delta\acute{\iota}\delta\alpha\chi\iota\varsigma$) and learning ($\mu\acute{\alpha}\theta\eta\sigma\iota\varsigma$) are the same. Thus to teach ($\delta\iota\delta\acute{\alpha}\sigma\kappa\epsilon\iota\nu$) is the same as to learn ($\mu\alpha\nu\theta\acute{\alpha}\nu\epsilon\iota\nu$). But if so, a person who is teaching will necessarily be learning what she is teaching (*Physics* 3.3, 202b2–5).

Aristotle’s answer to this objection is developed in several steps, and the first of them interests us because it has something to do with *(LL)*. Aristotle says:

(D) (i) Nor is it necessary that one who is teaching should learn even if to act and to be acted on are the same, provided they are not the same in the sense that the formula that states the essence is one, as in the case of cloak ($\lambda\acute{\omega}\pi\iota\omicron\nu$) and mantle ($\acute{\iota}\mu\acute{\alpha}\tau\iota\omicron\nu$), but are the same in the sense in which the road from Thebes to Athens and the road from Athens to Thebes are the same, as was stated before <at 202a18–20>. (ii) For it is not things that are in a way the same that have all their predicates the same, but only such as have the same being.

(*Physics* 3.3, 202b10–16; R. P. Hardie and
R. K. Gaye translation, modified)

This passage deserves a few words of comment. Let us ask which logical thesis (ii) refers to. The answer to this question depends on the logical status we are disposed to attribute to the terms “teaching” and “learning.” One might think that teaching and learning refer to individual events, because they are put on the same level as the road from Thebes to Athens and from Athens to Thebes, which points to an

individual. But Aristotle says that teaching and learning do not share the same λόγος or definition (*Physics* 3.3, 202b20–22), and it is precisely for this reason that they differ from cloak and mantle, which, on the contrary, have one and the same definition. Because individuals cannot be defined according to Aristotle, teaching and learning are either general terms or abstracts that admit definition, although they are in some sense individuals. Correspondingly, (2) or (3) is the formal version of the logical law expressed by (ii). After what we have said on the distinction between the \in - and the \subseteq -relation, solving this problem is not immediately relevant to our analysis. What is probably meant by Aristotle in (ii) is (8), which takes the form of (2) or (3) according to what is substituted for its variables x and y . Because, in a parallel passage (*Sophistical Refutations* 24, 179a26–b4) we will consider later, Aristotle discusses two alleged counterexamples to (LL) in a way similar to the present one, I will restate his argument in the form of (3). Therefore, I take “teaching” and “learning” as names of abstracts. This perhaps arbitrary choice should not invalidate my interpretation because I am confident that the reader will easily be persuaded at the end of this section that the interpretation I am going to propose works even if (2) were considered. My proposal is in fact less arbitrary than it might *prima facie* sound. If (8) is the logical thesis considered by Aristotle and (3) is a particular case of it, to construct an interpretation with reference to (3) does not compromise the result if it is reached without exploiting the peculiarity of (3).

This conclusion is not warranted by the traditional interpretation of text (D) because the version of (LL) that is normally supposed to be asserted here by Aristotle is weaker than (3): Given that A is the same as B , for them to share all the same predicates, they must be not only numerically identical but also have the same definition. In other words, they must be in the same situation as cloak and mantle are – namely, they must be synonymous. As a consequence, we find that (LL) cannot be applied to a statement of identity in which the two terms involved cannot be defined in the same way. A and B must be not only the same but also synonymous.

This standard interpretation seems doubtful to me in many respects. First of all, it is not clear what role the informal version of (LL), which is supposed to be hinted at by our text, has in the argument. A possible way of representing the thesis to be refuted could be:

$$(9) \quad \text{teaching } P = \text{learning } P \vdash x \text{ teaches that } P \text{ iff } x \text{ learns that } P$$

(what one teaches and what one learns is taken to be a proposition). Now, (9) seems to be justified by something like a substitution principle and not by a version of (LL). What then is the sense of the reference to the properties of identicals contained in Aristotle’s text? If this point is to be taken seriously, (9) needs to be formulated differently, namely as a (wrong) instantiation of (LL). As a preliminary step let us simplify (9), eliminating its free variables. We may write

$$(10) \quad \text{teaching } p_k = \text{learning } p_k \vdash m \text{ teaches that } p_k \text{ iff } m \text{ learns that } p_k$$

where “ m ” and “ p_k ” stand, respectively, for an individual name and a definite proposition. We now have to express “ m teaches that p_k ” and “ m learns that p_k ” as properties of “teaching p_k ” and “learning p_k .” A possible way of obtaining this result is to transform “ m teaches that p_k ” into “teaching p_k is (an activity) of m ” and “ m learns that p_k ” into “learning p_k is (an activity) of m .” Even if this translation sounds artificial, the resulting formula looks like an instantiation of (LL), that is,

- (11) teaching p_k = learning p_k \vdash
 teaching p_k is (an activity) of m iff learning p_k is (an activity) of m .

According to the traditional interpretation, (11) is an instantiation of (LL), which is therefore a principle that does not hold generally, its validity being limited to cases in which x and y are identical in essence or definition. Of course, “teaching p_k ” and “learning p_k ” are not so. Therefore, (11) is not a correct inference, and consequently (9), because (11) is not a sound instantiation of a valid logical principle.

We are well acquainted with the problems connected with (LL).²¹ The modern standard view is that in order to get free of any trouble in applying (LL) we have to avoid intensional contexts and limit ourselves to extensional properties of the objects that are asserted to be identical. Aristotle was aware of some of the difficulties connected with (LL). A version of the so-called paradox of the veiled man, which is discussed in *Sophistical Refutations* 24, can be interpreted as a counterexample to (LL). According to the traditional interpretation, Aristotle’s answer to an uncritical application of (LL) would be that we have to restrict not the range of the properties considered by the principle, but rather the range of the subjects involved. (LL) holds only in the cases in which the terms said to be identical are essentially the same, namely they have the same definition. At least three unwanted consequences derive from this interpretation. First, the allegedly true version of (LL) does not apply to the identity of individuals. If individual things cannot be defined, as Aristotle maintains, it is hard to see how two different names (or descriptions) of an individual could have the same definition. “Cloak” and “mantle,” the standard Aristotelian examples of terms sharing the same definition, are not names of an individual cloak or of an individual mantle, but of general terms or, at most, of abstracts, the definition of which is said to be the same. Second, it is difficult to see how the restriction to essential identities without any further proviso saves us from incorrect applications of (LL). Even if x and y are the same and share the same definition, a predicate of x that is not true of y can be found, provided that no limitation on the range of the predicates considered is given. Two synonyms, one of which is known by someone and the other is not, could be taken as a counterexample to this restricted version of (LL).²² Third, an inconsistency has to be attributed to Aristotle. If it is agreed that (8) is asserted by him in the *Topics* and that (8) implies (3) as a special case, then the passage that we are now discussing, which shows that (3) – and therefore (8) – is not always true, contradicts *Topics* 7.1. One must think that Aristotle changed his mind in the time between the composition of *Topics* 7.1 and *Physics* 3.3.

The mere fact that the standard interpretation attributes an incorrect doctrine to Aristotle is not in itself sufficient grounds for dismissing it, nor is charging him for having changed his mind. But if one is able to present a different analysis of the relevant texts that does not lead to such undesirable results, charity forces us to prefer the second interpretation. In order to attain this goal we first have to consider the sense in which teaching and learning are asserted by Aristotle to be the same. Their identity is said not to be sameness in being (*Physics* 3.3, 202b8–9; 15–16), but it is considered of the same kind as the identity that holds between the road from Thebes to Athens and the road from Athens to Thebes (*Physics* 3.3, 202b13–14), and between the intervals between 1 and 2 and 2 and 1, and again between the downward and upward slopes of a hill (*Physics* 3.3, 202a18–20). Just as we are entitled to speak of the same road, the same interval, the same slope, so are we entitled to speak of teaching and learning as the same event, for instance, the same flux or movement of information. Teaching and learning can be said to be the same with reference to the event of transferring information, which may be considered to be the same if no mention of the acts of giving and receiving it is made. But, as Aristotle says, their identity does not imply sameness in being. Sameness in being is surely identity in essence or definition, as can be seen by comparing 202b9 to 202a20 and 202b21–22, and as the parallel passage of *On the Soul* 3.2, 425b26–27 confirms. That teaching and learning do not exhibit the same being or essence has been taken as an assertion that they are not synonymous, as, on the contrary, is the case of cloak and mantle. Hence, the standard interpretation: (LL) in (ii) is asserted to hold only with reference to synonymous terms. But this conclusion cannot be inferred so easily.

Let us consider again what “same in being” means. This expression has at least two meanings. First, it can be taken as a condition on the synonymy of the terms that are said to be the same. Using a modern terminology, one could say that identity in being implies sameness of sense: x and y are the same in being if, and only if, they denote the same object and share the same sense. “Sameness in being” is sometimes used by Aristotle with this meaning as, apparently, in *Topics* 5.4, 133b34–35: The man and the man who is pale are different in being (ἕτερον εἶναι αὐτοῖς τὸ εἶναι), although they refer to the same person. The second meaning can be drawn out of the *Metaphysics*; Aristotle says:

(E) For things which either are non-continuous or do not have one form or do not have one formula (ὧν ὁ λόγος μὴ εἷς) we in fact reckon as more than one thing.

(*Metaphysics* 1.6, 1016b9–11; Ch. Kirwan translation)

Things that do not have the same formula are things that are not the same in being: “ὧν ὁ λόγος μὴ εἷς” and “οἷς τὸ εἶναι τὸ αὐτό” are equivalent expressions in *Physics* 3.3 (202a18–20 and 202b13–16). Suppose now that, in the passage just quoted, identity in definition means sameness in sense, as before. Then, if x and y have different senses, they cannot be considered numerically identical. The morning star and the evening star would be reckoned as two objects. This idea is contradicted

by Aristotle when he states that numerical identity is implied by descriptions of the same object that are different in sense (*Topics* 1.7, 103a29–31) or when he defines numerical identity as sameness in matter, that is, substrate, as in *Metaphysics* 1.6, 1016b32–33. What is meant here by sameness in definition or sameness in being is something quite different. I take it to mean sameness in the definition of what is denoted by the terms said to be identical. In other words, x and y are the same in being if what is denoted by x has the same definition as what is denoted by y . It is easy to convince ourselves that sameness in being taken in this sense simply amounts to numerical identity. If x and y denote one and the same object, the definition (if there is one) of what is denoted by x cannot be different from the definition of what is denoted by y , because one and the same object cannot have two definitions. For the same reason, if what is denoted by x and what is denoted by y share the same definition, x and y denote the same object.

With the distinction between the two meanings of the expression “sameness in being” in mind, let us come back to our passage from *Physics* 3.3. My claim is that teaching and learning are said by Aristotle to be different in being in the sense that what is denoted by teaching and what is denoted by learning do not have the same definition, namely they are not numerically identical. This may seem awkward, because the kind of sameness normally attributed to learning and teaching is sameness in substrate, which is a kind of numerical identity.²³ According to the standard interpretation, teaching and learning are said to describe the same event in two different ways, just as the same planet, namely Venus, is described in two different ways as the morning star and the evening star. But is this really the case? Upon reflection, it is far from true that one could take “learning” to denote an oriented process, namely a process in which some information is received by someone. Correspondingly, “teaching” could denote a differently oriented process of giving something to someone. Of course, these differently oriented processes have something in common, precisely an *un-oriented* event, the same shifting or flux of information. For one and the same flux of information becomes an act of learning when it is considered as received by a pupil and becomes an act of teaching when it is taken as given by a teacher. The other examples mentioned by Aristotle can be analyzed in a similar way. If the interval between 1 and 2 is taken as an oriented interval, namely an interval in which order is relevant, what is denoted by the “interval between 1 and 2” is different from what is denoted by the “interval between 2 and 1.” As in the preceding case, the two intervals share the fact of being constituted by the same un-oriented interval, that is, by the same distance. The examples of the road and of the hill can be treated accordingly. In what sense, then, are teaching and learning said to be identical if they denote different objects? I propose to interpret this identity as that which Aristotle in *Metaphysics* 1.9, 1018a5–6 calls identity in matter as opposed to identity in substance (οὐσία). Teaching and learning are said to be the same because the processes they denote share a common element, the same flux of information that in the one case is given by the teacher and in the other case received by the pupil. This use of “being the same as” is similar to the use of “is” in statements such as: “That heap of fragments there is the porcelain jug you saw the last time you came to this house.”

The heap of fragments and the jug cannot be the same jug, but they might be the same collection of material bits, because they are constituted by the same bits of porcelain.²⁴ The two processes of teaching and learning have then a flux of some bits of information as a common constitutive part.

This interpretation seems to me to be confirmed by what Aristotle says in what follows text (D). He states that even if one were led to assert that teaching and learning are the same, it does not follow that to teach is the same as to learn:

(F) But indeed it by no means follows from the fact that teaching is the same as learning that to learn is the same as to teach, any more than it follows from the fact that there is one distance between two things that are at a distance from each other, that the two vectors AB and BA are one and the same. To generalize, teaching is not the same as learning or agency as patiency in the full sense ($\kappa\rho\acute{\iota}\omega\varsigma$), though they belong to the same subject, the motion; for the “actualization of X in Y ” and the “actualization of Y through the action of X ” differ in definition.

(*Physics* 3.3, 202b16–22; R. P. Hardie and R. K. Gaye translation, modified)

The argument runs as follows. Say teaching and learning are the same in the sense that they denote the same object: an un-oriented flux of information. Even if that is admitted, one cannot conclude that to teach and to learn are the same, because two different processes are denoted by them; in one of them the flux of information is produced by a teacher, whereas in the other the same flux of information is captured by a pupil. Aristotle is also ready to dismiss the premise of the alleged argument, that is, that teaching is the same as learning. Teaching and learning cannot be reckoned as one and the same thing, because their definitions are different.

Against this interpretation one could argue that in text (D) Aristotle contrasts the case of teaching and learning with the case of cloak and mantle, a standard example of synonymy (*Topics* 6.11, 149a3–4; *Physics* 1.2, 185b19–20). Therefore, one could say, cloak and mantle are two synonymous terms, but teaching and learning are not, although both pairs are cases of numerical identity. This conclusion does not follow from what Aristotle says. Cloak and mantle are not only an example of synonymous terms but also an example of terms that denote one and the same object (*Topics* 1.7, 103a9–10). Thus, the case of teaching and learning contrast with that of cloak and mantle because the latter case is an example of numerical identity, whereas the former is not.

Should we adopt this interpretation, it becomes clear why (11) cannot be taken as a sound instance of (3). Teaching p_k = learning p_k is not a special case of $A = B$, because it does not express a proper identity. Therefore, (11) does not constitute a counterexample to (3). The latter formula is precisely what is stated in clause (ii) of text (D), where it is emphasized that in order to apply (3) we have to make sure that the terms taken to be the same really are the same. The doctrine of the *Topics* is then confirmed in a plain way, and there is no need for thinking that Aristotle changed his mind about (*LL*).

III

I hope to make my interpretation more convincing by considering the main passage where Aristotle deals with some alleged counterexamples to (LL). In *Sophistical Refutations* 24, Aristotle considers several kinds of fallacious arguments that he groups under the label of “κατὰ συμβεβηκός παραλογισμοί,” “fallacious arguments depending on the accident.” It has been frequently observed that the arguments considered by him are quite different and that the fallacies involved in them have different logical status. Nevertheless, Aristotle insists on their common ground. This common basis could perhaps be understood along the following line of interpretation. First of all, I propose that “συμβεβηκός” in the formula “κατὰ συμβεβηκός παραλογισμοί,” does not refer to accident in its technical opposition to essence or substance, but simply means “attribute” or “property” without any explicit hint at the distinction between essential and nonessential properties of a substance.²⁵ Without this hypothesis, it would be difficult to understand how Aristotle could class an argument such as the following among the fallacies depending on the accident:

I	Coriscus (<i>m</i>) is a man (<i>M</i>)
	Man is different from Coriscus
	<hr/>
	Coriscus is different from Coriscus

that is

(12) $M(m), m \neq M \vdash m \neq m.$ ²⁶

No accidental predication is involved here.

Aristotle characterizes a fallacy depending on the accident as follows:

(G) Fallacies, then, that depend on accident occur whenever any attribute is claimed to belong in a like manner to a thing and to its accident. For, since the same thing has many accidents, there is no necessity that all the same attributes belong to all of a thing’s predicates and to their subject as well.

(*Sophistical Refutations* 5, 166b28–32;
W. A. Pickard-Cambridge translation)

If “συμβεβηκός” is taken here to mean simply “attribute,” a difficulty arises with respect to this way of defining fallacies depending on the accident. It seems that Aristotle is here denying transitivity of the $\acute{\upsilon}\pi\acute{\alpha}\rho\chi\epsilon\iota\nu$ -relation – namely, the relation of being affirmed of. But the $\acute{\upsilon}\pi\acute{\alpha}\rho\chi\epsilon\iota\nu$ -relation, when applied to terms belonging to the same category, is said to be a transitive relation (*Categories* 3, 1b10–12; 5, 3b4–5). In order to save Aristotle from inconsistency, one might be led to conclude that the nontransitivity of the $\acute{\upsilon}\pi\acute{\alpha}\rho\chi\epsilon\iota\nu$ -relation holds among accidental predications. Therefore, the strict sense of “accident” should be restored to “συμβεβηκός” in this

context. But Aristotle's claim seems to me to be different. He does not point out that the ὑπάρχειν -relation is not transitive in some cases, but simply that ὑπάρχειν covers different logical relations, some of which are not transitive.

In general, Aristotle considers the case in which an attribute F is predicated of a subject – let us assume for the sake of simplicity that the subject is an individual thing m , as in the case of Coriscus in (I). We have $F(m)$. Now, Aristotle continues, suppose that something, for instance G , is predicated of F . Let us express this fact by means of the somewhat barbarous formula $G(F)$. Aristotle's claim is that from these premises one is not allowed to conclude $G(m)$. In other words, the inference

$$(13) \quad G(F), F(x) \vdash G(x)$$

is not valid. This general scheme can already be seen in (12), if “ m ” is substituted for “ x ,” “ M ” for “ F ,” and “ $\neq m$ ” for “ G .” I do not say that every example of fallacies depending on the accident that Aristotle considers matches (13) as directly as (12) does. The example Aristotle gives immediately after (I), for instance, does not fit (13) as well as (I). His example is the following:

(H) If Coriscus is said to be different from Socrates and Socrates is said to be a man, then, they say, it has been admitted that Coriscus is different from man, because it happens that the person from whom Coriscus is said to be different is man.

(*Sophistical Refutations* 5, 166b33–36;
W. A. Pickard-Cambridge translation)²⁷

If “ m ” stands for “Coriscus,” “ n ” for “Socrates,” and “ $M(x)$ ” for “ x is a man,” the wrong inference may be formalized by

$$(14) \quad m \neq n, M(n) \vdash m \neq M.$$

If something like our schema (13) is applied here, we must take “ n ” as “ F ,” a predicate of something that is different from m , and “ M ” for “ G .” A similar difficulty arises with the example of *Sophistical Refutations* 24, 179a34–35. What (13) is intended to make evident is that the common origin of all these fallacies is, according to Aristotle, that “ ὑπάρχειν ” covers different logical relations, some of which are not transitive. If the terminology of set theory is used, an example of what I am trying to say can easily be put forward. If “ $G(F)$ ” is interpreted as “ $F \subseteq G$,” (13) is perfectly valid. On the contrary, if “ $G(F)$ ” is taken to mean “ $F \in G$,” (13) is not always true. Incidentally, the distinction between the \subseteq -relation and the \in -relation explains a traditional example of fallacy depending on the accident, such as:

Socrates is white
White is a color

Socrates is a color.

As is well known, the \in -relation is not transitive. The relevant case for our discussion is the case in which “ $G(F)$ ” is taken to mean $F = G$, where “ F ” and “ G ” stand for abstracts and identity is not supposed to mean anything different from identity between individuals. Two very similar fallacies are considered by Aristotle, where a version of (LL) plays a crucial role. He says:

(J) All arguments such as the following depend upon accident. “Do you know what I am going to ask you?”, “Do you know the man who is approaching” or “the veiled man?”.

(*Sophistical Refutations* 24, 179a32–34;
W. A. Pickard-Cambridge translation)

The first example is normally reconstructed as an argument containing the following premises:

- (P 1) You do not know what I am going to ask you
- (P2) What I am going to ask you is P
- (P3) You know P .

From these premises, it is inferred:

- (C 1) You know and at the same time you do not know what I am going to ask you.

Let us call this argument the “argument of the imminent question.” It is not clear what P amounts to in this example. From what is said at *Sophistical Refutations* 24, 179a39–b1, it is something that concerns goodness. What cannot be decided is whether “ P ” stands for a propositional content, for instance, “to be virtuous is good,” or for a concept, for example, “the nature of good.” However, our interpretation of the text is not affected by choosing one or the other alternative. “ P ” can stand either for propositional content, a *Gedanke* in Frege’s terminology, or for a concept. Of course, the copula involved in premise (P2) stands for identity. Aristotle connects this argument in some way to a version of (LL).²⁸ In order to clarify this relationship, the premise (P3) can be restated in the form “ P is something known by you,” so that the whole argument takes a form that implies a use of (LL) in one of its parts, namely in the inference

- II What I am going to ask you = P
 P is something known by you

What I am going to ask you is something known by you.

The second example given by Aristotle can be analyzed along the same lines. Its expanded premises are:

- (P4) You do not know the man who is approaching (the veiled man);

- (P 5) The man who is approaching (the veiled man) is Coriscus;
 (P6) You know Coriscus.

The corresponding conclusion is:

- (C2) You know and at the same time you do not know the man who is approaching (the veiled man).

Like the preceding one, this argument – let us call it “the argument of the approaching man”²⁹ – can be restated in such a way that it is shown to imply an instance of (*LL*), namely

- III The man who is approaching (the veiled man) = Coriscus
 Coriscus is someone known by you

The man who is approaching (the veiled man) is someone known by you.

Because the conclusions of (II) and (III) are supposed to be inconsistent with (P1) and (P4), respectively, the contradictions expressed by (C1) and (C2) arise. According to Aristotle the wrong steps are constituted by (II) and (III). These inferences, which are instances of (*LL*), are also instances of (13) where *G(F)* is taken as an identity between two individual descriptions or between a description and a name of an individual. In this sense (II) and (III) are cases of fallacious arguments depending on the accident.

After giving some other examples of invalid arguments depending on the accident that we will not consider here, Aristotle puts forward what he believes to be the reason why (II) and (III) are fallacious:

- (K) (i) For it is evident, in all these cases, that there is no necessity for the attribute that is true of the thing’s accident to be true of the thing as well. (ii) For only to things that are indistinguishable in substance and that are one is it generally agreed that all the same attributes belong. (iii) But in the case of a good thing, to be good is not the same as to be about to be the subject of a question; (iv) nor in the case of a man approaching or veiled is to be approaching the same as to be Coriscus, (v) Therefore, if I know Coriscus but do not know the man who is approaching, it does not follow that I both know and do not know the same man.

(*Sophistical Refutations* 24, 179a35–b4;
 W. A. Pickard-Cambridge translation, modified)

Whereas clause (i) is a general statement concerning all kinds of fallacies depending on the accident, which is intended to underline their common root, (ii) seems to be designed to solve the cases in which we are interested. The traditional interpretation of this passage is very similar to the one I criticized when commenting on the text of *Physics* 3.3. Many authors attribute a distinction between essential

(or substantial) and nonessential (or nonsubstantial) identity to Aristotle, and (*LL*) is said to hold when the first but not the second kind of sameness is in question.³⁰ Because Coriscus is not essentially the same as the man who is approaching, what is predicated of the man who is approaching cannot be truly asserted of Coriscus himself.³¹ According to this interpretation, the phrase

(a) κατὰ τὴν οὐσίαν ἀδιαφόροις καὶ ἓν οὖσιν (*Sophistical Refutations* 24, 179a38)

is normally translated: “things that are indistinguishable and one in essence (or in substance).” But *Metaphysics* 1.15, 1021a10–11 and *Metaphysics* 3.4, 999b21–22 show that (a) does not characterize anything more than the usual identity in which the expressions taken to be identical denote the same substrate. Besides, the traditional solution is unsatisfactory in cases where propositions containing verbs such as “to know” are involved. Even if a strong notion of identity is implied, where sameness of sense is meant, nothing prevents us from imagining a case in which the first member of the identity relation is said to be known by someone but the second is not.

The interpretation that I would like to defend is in the same vein as before. Only for things identical in their substrate (κατ’οὐσίαν) is it true that the predicates of one of them are also the predicates of the other. Where no such identity is given, namely where no identity at all is given, such an inference is not allowed. Clauses (iii) and (iv) confirm this interpretation. Aristotle distinguishes there, on the one hand, being good from being the subject of a coming question and, on the other, being a man who is approaching (or a veiled man) from being Coriscus. Given these assumptions, he concludes in (v) (ὥστε: 179b2) that from the facts of knowing Coriscus and of not knowing the man who is approaching, it cannot be inferred that I know and I do not know the same person. It seems to me that this conclusion is warranted only if it is assumed that what is known by me when I claim that I know Coriscus is not the object Coriscus, but an aspect of him, his being Coriscus. In other words, when I say that I know Coriscus, I am not committed to asserting that I know the object that “Coriscus” refers to, but rather that I know the object Coriscus *qua* conceived. To generalize, one could maintain that according to Aristotle here “to know *x*” means “to know of something its being *x*” and not “to know what is denoted by ‘*x*.’” If I knew what “Coriscus” referred to, my knowledge would be inconsistent with the assertion that I do not know the man who is approaching, given that what “Coriscus” refers to is the same as the man who is approaching. This point would perhaps be clearer if we were allowed to use a Fregean terminology. My knowledge of Coriscus is the knowledge of a certain conceptual content: the sense attached to the word “Coriscus.” What this word refers, namely its *Bedeutung*, lies outside the reach of my knowledge. If this is so, it is easy to see why (III) fails. Coriscus, of whom it is said that he is one of the things known by you, is not the same man as Coriscus who is identical with the man who is approaching (or the veiled man). In the first case, we are speaking of the sense of “Coriscus,” whereas in the second case we are speaking of what “Coriscus” refers to. Therefore, if something is true of the sense of “Coriscus” or,

if you prefer a less committed terminology, of “Coriscus” *qua* conceived, it does not follow that it will be true of that which is the same as the thing referred to by “Coriscus.” Coriscus *qua* conceived and the object named “Coriscus” are not necessarily the same. Consequently, (III) is not a real instance of (LL) and cannot be counted as a counterexample to this principle. The same analysis can very easily be extended to (II).³²

This interpretation finds support in the criticism Aristotle makes of another attempt at solving the puzzle of the approaching man. The alleged solution consists in distinguishing the conclusion of the argument such that no inconsistency remains. As Aristotle puts the matter, people who adopt this view

(L) Say that it is possible to know and not to know the same thing, only not in the same respect: accordingly, when they do not know the man who is coming toward them, but do know Coriscus, they assert that they do know and do not know the same object, but not in the same respect.

(*Sophistical Refutations* 24, 179b7–11;
W. A. Pickard-Cambridge translation)

It is not clear whether this view has something to do with what now is called a transparent interpretation of expressions like “to know that . . .” But we do not have to worry about that. What interests us is to consider one of the objections that Aristotle brings up against this position. He states:

(M) Possibly, indeed, the present suggestion may very well apply in some cases: but in these cases, at any rate, not even this would be generally agreed. For he knows both of Coriscus that he is Coriscus and of the approaching man that he is approaching. To know and not to know the same thing is generally thought to be possible, when e.g. one knows that someone is white, but does not realize that he is musical. For in that way he does know and does not know the same thing, though not in the same respect. But as far as the approaching man and Coriscus are concerned, he knows both that he is approaching and that he is Coriscus.

(*Sophistical Refutations* 24, 179b26–33;
W. A. Pickard-Cambridge translation, modified)³³

I take this passage to be a refutation of the view that it is possible to affirm that we do not know in some respect the person who is coming, given that we know that the same person is Coriscus and the approaching man. If this premise is admitted, one cannot say that there is a respect in which the coming person is not known because we are in a position to assert of him both that he is Coriscus and that he is the approaching man. If this interpretation is right, Aristotle’s objection concerns the possibility of stating that we know that the same man is both the approaching man and Coriscus. We actually know the approaching and the being Coriscus, which happen to be properties of him who is really one and the same, yet we do not know that they are properties of the same man. But such a statement is possible

only if it is implied that to know Coriscus means to know his being Coriscus, which is the sense of Coriscus. Of course, Aristotle's answer makes sense only if the possibility of construing "to know" transparently is dismissed. Inside an opaque interpretation of knowing Coriscus, one can deny that there is a respect in which one can assert that Coriscus is not known, even if it is affirmed of him that he is Coriscus and the approaching man.

In the light of this interpretation, it cannot be maintained that Aristotle admitted exceptions to (LL). Neither (11) nor (II) and (III) are counterexamples to some version of this principle, because they are apparent but not sound or real instantiations of it. The puzzles arise because we are sometimes not able "to distinguish what is the same and what is different" as Aristotle says, listing the possible causes of the fallacies depending on accident (*Sophistical Refutations* 7, 169b3–4). Therefore, (LL) retains its full force of necessary condition for identity, at least in the sense that if x has a property that y does not share, x is not the same as y .³⁴

IV

If the interpretation of Aristotle's doctrine I have proposed is correct, one must conclude that his Greek commentators, insofar as their position can be reconstructed, did not follow him. Some of them, notably Simplicius and Philoponus, took a position that is very similar to the view that I have called in the previous section the "traditional interpretation." Others developed a completely different and independent view. Before starting a detailed analysis of these positions, a preliminary remark has to be made. The main reason for the difficulty of sketching out an account of the different solutions proposed is that our sources are disappointingly meager and sometimes far from clear. Simplicius and Philoponus commented on *Physics* 3.3 but not on *Sophistical Refutations* 24. In addition, we possess an unfortunately very late commentary on the *Sophistical Refutations*, an even later paraphrasis of the same work, and some relevant *scholia* coming from different sources, which have been collected and edited by Sten Ebbesen.

With reference to *Physics* 3.3, Simplicius says that an inference like (11) is not warranted by the law according to which things, if they are identical, share all the same properties, because this law holds only for a particular kind of identicals among which teaching and learning are not included (*On Aristotle's Physics* 447.20 ff.). Actually, it is difficult to decide whether Simplicius was thinking of (LL) or of something like (2) as the logical thesis that has to be restricted in order to avoid paradoxes. Because of the ambiguity of "being predicated of" in natural language, he was perhaps in the same situation as Aristotle. The thesis considered by him was probably a statement that would have (8) as its formal correspondent and that would be intended to cover both (2) and (3). In order to make his position comparable with the doctrine expounded in the commentary and in the paraphrasis of the *Sophistical Refutations* and for the sake of simplicity, let us suppose that Simplicius refers to (LL). As before, this restriction is a harmless loss of generality, because my interpretation of his statements can very easily be extended to the case of (2).

How, then, has (LL) to be restricted according to Simplicius? He says:

(N) Things that are identical only in substrate (κατὰ τὸ ὑποκείμενον) do not have the same set of properties; that happens only to things that are identical in *logos* (κατὰ λόγον) and that have the same being and definition.

(Simplicius, *On Aristotle's Physics* 447.33–448.1)

Teaching and learning are not the same in *logos*, but only according to their substrate. Therefore, they do not necessarily share all the same properties. What matters is to understand what identity in substrate and identity in *logos* are according to Simplicius. One could be led to think that what we called “sameness in matter” corresponds to identity in substrate and that saying that *x* and *y* are the same in *logos* means saying that the definition of what is denoted by *x* is the same as the definition of what is denoted by *y*. If it were so, sameness in *logos* would coincide with numerical identity and Simplicius would have interpreted Aristotle in what I think to be the right way. But things are otherwise. Discussing Parmenides’s view about unity, Simplicius clarifies what he means by identity in *logos* and identity in substrate. He says:

(O) Because of their ignorance of this matter also the so-called philosophers of Megara, taking as evident the premise that things whose *logoi* (λόγοι) are different are different, and that things that are different are separated, believed to prove that each thing is separated from itself. For since the *logos* (λόγος) of Socrates who is musical is different from the *logos* of Socrates who is pale, Socrates would be separated from himself. But it is clear that Socrates is the same in the substrate according to which he is Socrates, while he is different in respect to the accidents, being one and many in different respects.

(Simplicius, *On Aristotle's Physics* 120.12–20)

“Socrates who is pale” and “Socrates who is musical” denote the same object, and for this reason they are said to be the same in substrate. Their identity is the same identity that holds between the morning star and the evening star, namely numerical identity. But they do not share the same *logos*. That could only mean that they do not have the same sense. The morning star and the evening star are not the same in *logos* because they do not have the same sense, and it is the same with teaching and learning.

If this is the interpretation that should be given to identity in substrate and identity in *logos*, Simplicius’s view becomes clear. (LL) is a valid principle provided that it is limited to identicals that not only denote the same object but also share the same sense. Therefore, (LL) holds only for synonymous terms. Simplicius’s word for “synonymy” is “polyonymy.” If *x* and *y* are polyonymous, they share the same sense (λόγος), being different only in name (*On Aristotle's Categories* 22.24–26).³⁵ Let us call the identity that holds between two synonymous or

polyonymous terms “strong identity” and let us denote it by “ \equiv .” Then the version of (LL) stated by Simplicius may be expressed in the following way:

$$(15) \quad x \equiv y \vdash \forall F(F(x) \leftrightarrow F(y)).$$

Of course, (15) can be generalized to correspond to (8), if “ $F(x)$ ” is taken to represent ambiguously “ $x \in A$ ” or “ $x \subseteq A$,” where $A =_{df} \{x: F(x)\}$.³⁶

Simplicius’s view, if it is taken as a general point about the validity of (LL), is very restrictive for the range of applications of (LL). Is this restriction sufficient for saving (LL) from counterexamples constructed in the same vein as the arguments of the imminent question and of the approaching man? The problem could be stated as follows. Let us suppose that one asserts that

(P7) cloak is the same as mantle

(in fact a case of strong identity), and that

(P8) cloak is known by me.

Is one allowed to infer that

(C3) mantle is known by me?

If this conclusion is admitted as a consequence of the application of (15), then (LL) even in the restricted version adopted by Simplicius has a counterexample, because it seems that I can know what cloak is without knowing what mantle is. Nevertheless, one could try to defend the position of the commentator by emphasizing more than he does that strong identity, holding between *polyonyma*, is a relation not between objects but between names.³⁷ It is a well-known Fregean view that a (true) identity statement in which identity is considered a relation between signs is not informative and does not differ from a statement of self-identity.³⁸ Because no increment of our knowledge is produced by an identity statement, when one knows the referent of “cloak,” one also knows the referent of “mantle.” On this interpretation, (C3) *does* follow from (P7) and (P8) by means of (15), but nothing paradoxical happens because the knowledge of cloak actually implies the knowledge of mantle. In this way, (LL) is sheltered from being falsified.

Upon reflection, such a defense does not reach its goal, even if one is prepared to swallow that according to Simplicius identity is just a relation between names or signs. When we say that “cloak” is the same as “mantle,” this identity statement, being a statement about words, increases our knowledge not of things but of words. Knowing that “cloak” is a name of an object does not entail knowing that “mantle” is another name for the same object. Because identity statements about words can be informative, (C3) is unjustified. If it is admitted to follow from (P7) and (P8) by means of (15), a counterexample to the restricted version of (LL) adopted by Simplicius is provided.³⁹

Let us make a little digression on the position of Alexander of Aphrodisias before considering the extant works on the *Sophistical Refutations*. An interpretation of Alexander's view about (LL) is complicated by the fact that both his commentary on the *Physics* and his commentary on the *Sophistical Refutations* (supposing that he wrote one) are lost.⁴⁰ Besides, when he speaks of predicates of identicals, it is not clear if he means (2) or (3). I think that what we have said about Aristotle on this subject can be applied also to Alexander. Because of the ambiguity of "being predicated of," we may suppose that he has in mind a generalized version of (LL) that can be represented by (8). Actually, the passages giving some clues of Alexander's attitude toward the laws of identity are concerned especially with that restriction of (8) expressed by (2). Even if no direct reference to (3) or (7) is made, it is perhaps worth noticing that Alexander imposes on (2) restrictions that are analogous to those that Simplicius puts on (LL).

The clearest evidence comes from what Alexander says with reference to *Topics* 5.4, 133a32–34. As we have seen, in this text Aristotle states a thesis that looks more like a restricted version of (2) than a particular case of (3), even if accidents are primarily concerned with individuals. Alexander misunderstands Aristotle's point by taking it to mean that if x and y are two accidents, then the *propria* of x are the same as the *propria* of y (*On Aristotle's Topics* 395.20–26). If its triviality did not raise suspicion about the authenticity of this passage, we would not need to be concerned with this error. What matters, however, is the way in which Alexander makes his point. He says:

(P) The same holds for accidents, if some of them are polyonymous. A *proprium* of what is named by one of the two names must also be a *proprium* of what is named by the other name.

(Alexander of Aphrodisias, *On Aristotle's Topics* 395.20–23)

Given two accidents x and y (but accidents of what?), the *propria* of x are also *propria* of y on the condition that x and y are polyonymous. We have here stated in full what in Simplicius is implicit. Polyonymy is asserted as a necessary requirement for the truth of the restricted version of (2) that is considered here.

Further evidence can be added. Shortly before the passage we have just discussed, Alexander comments in the following way on Aristotle's statement that if x and y are the same, then the *propria* of the one are also the *propria* of the other:

(Q) Aristotle uses an example taken from things that are numerically identical, that is from things which differ only in name, while their meaning, i.e. their substrate ($\tau\tilde{\omega}$ ὑποκειμένῳ), is the same. For man and mortal ($\beta\rho\omicron\tau\acute{o}\varsigma$) are the same in substrate, that is numerically, while they differ in name.

(Alexander of Aphrodisias, *On Aristotle's Topics* 395.17–20)⁴¹

Alexander does not maintain elsewhere that numerical identity is sameness of things that differ only in their name.⁴² The simplification he adopts here might depend on the need he feels for imposing a condition on the logical thesis asserted

here, namely a restricted version of (2). This thesis holds only if a strong identity is assumed – namely, identity of synonyms. Again Alexander’s view does not seem very far from Simplicius’s position, even though he speaks of (2) and Simplicius is concerned with (*LL*) or with a generalized version of this principle.⁴³

I am not sure that the evidence I have gathered is sufficient to warrant the hypothesis that Alexander had views about the laws of identity similar to the views held by Simplicius. If so, Simplicius is just echoing an old tradition, the development of which deserves to be studied more carefully.

It is now time to turn to the only extant commentary on the *Sophistical Refutations*. It is attributed to Michael of Ephesus, a scholar of the twelfth century AD belonging to the circle gathered around Princess Anna Comnena in Constantinople. It has been edited by Maximilian Wallies in the *Commentaria in Aristotelem Graeca* (*CAG*) as a pseudo-Alexander work.⁴⁴ Following Ebbesen, I will refer to it by “Ps-Alex. 1.” With respect to the argument of the imminent question, Michael thinks that what is going to be asked is whether virtue is a good thing. His point is as follows:

(R) The solution is that “virtue is a good thing” and “being about to be asked whether virtue is a good thing” are not the same thing, but it happens (*συμβέβηκεν*) to virtue, which is a good thing, that it is going to be asked whether it is a good thing.

(Ps-Alex. 1, 161.10–12)

Michael’s words are not *prima facie* clear. In the first half of his statement he denies that “virtue is a good thing” is the same as “being about to be asked whether virtue is a good thing.” Is this a claim that premise (P2) must not be interpreted as an identity statement? This view could be taken to be a development of Simplicius’s position. Nonessential identity is no identity at all. One might suspect that behind this thesis there is the idea that (*LL*) is a criterion of identity: Where (*LL*) fails, sameness fails. But I do not think that Michael’s interpretation moves along this line. His point about nonidentity is probably directed to emphasizing that (P2) is not an essential predication. To be the subject of an imminent question happens (*συμβέβηκεν*) to virtue, which is by itself a good thing, and for this reason, according to Michael, the argument is not sound. What I am thinking is that Michael had in mind, as a counterpart of (II), an argument like the following:

IV What I am about to ask you is *P*
 P is known by you

What I am about to ask you is known by you

where “*P*” stands for “whether virtue is a good thing.” The first premise of (IV) does not need to be conceived as an identity statement. It is just an accidental predication. Now Michael’s point seems to be that (IV) is not a correct inference because its first premise is an accidental predication. In order to understand his view we have to look at his doctrine about the fallacies depending on accident. One of the explanations

he gives for this kind of sophism is based on the idea that if accidental predications are involved in the premises of an argument, we cannot appeal to the transitivity of the relation of “being said of” or “being predicated of” in order to justify that inference (Ps-Alex. 1, 38.18–21; 39.6–9). Because (IV) involves at least one accidental predication, its conclusion does not follow from its premises. This interpretation is confirmed by what Michael says about the related fallacy of the veiled man. According to him the premise “Coriscus is the veiled man” is a proposition that expresses an accidental predication. But what is predicated of an accident of Coriscus is not necessarily a predicate of Coriscus himself. Therefore, the conclusion stated in the argument is not proven (Ps-Alex. 1, 125, 26–36).⁴⁵

Michael’s position about fallacies depending on accident is in fact a little more complicated than it appears from what I have said up to now. In his commentary, we find two interwoven interpretations of this kind of sophism. The first one is the explanation bearing on the presence of accidental predications, the “AP-explanation,” as it could be christened. But there is also another interpretation that refers to arguments such as

V	Socrates is white
	White is a color
	<hr style="width: 20%; margin-left: 0;"/>
	Socrates is a color.

In this case, the solution is that white does not refer to the same object in the two premises. In “Socrates is white” it refers to what is colored, whereas in “white is a color” it refers to a particular kind of color.⁴⁶ Therefore, because a *quaternio terminorum* takes place, (V) cannot be labeled a valid inference (Ps-Alex. 1, 37.27–38.3). Let us call this explanation the “QT-explanation.” Michael does not seem to be concerned with the difference between this explanation and the AP-explanation. He uses one or the other according to convenience. For instance, in Ps-Alex. 1, 59.17–25 the QT-explanation is preferred, whereas in Ps-Alex. 1, 38.35–39.9 it is the AP-explanation that is considered. This probably depends on the way in which his commentary has been put together, in addition to his not having a very high level of logical skill. As Sten Ebbesen has demonstrated, Michael’s commentary printed in the *CAG* is the final product of a very complicated editorial process. Its starting point is a collection of *scholia* dating from 900 AD called by Ebbesen “*Commentarium II*,” which were inserted in a larger collection gathered or invented by Michael himself. The result was a first edition of his commentary that Ebbesen calls “Ps-Alex. 2.” When copies of Ps-Alex. 2 began to circulate, Michael started a revision of it, the final product being Ps-Alex. 1, the commentary edited by Wallies.⁴⁷ Now only the QT-explanation is found in the *Commentarium II* (1–60, Ebbesen II, 42–44) in terms very similar to the version adopted by Ps-Alex. 1, 37.23 ff. In Ps-Alex. 2, the standard QT-explanation is given (fr. 7, 6–26, Ebbesen 1981: II, 166–167), but the AP-explanation is added (fr. 7, 26–32, Ebbesen 1981: III, 167), as in Ps-Alex. 1, 38.18–21; 39.6–9. It is worth noticing that in what is called by Ebbesen “*Commentarium III*,” a late

Byzantine commentary of the twelfth to thirteenth century,⁴⁸ the two explanations are presented as possible alternative interpretations of Aristotle's view (fr. 5, 1–16, Ebbesen 1981: II, 205). On the other hand, in the anonymous paraphrasis of the *Sophistical Refutations*, almost contemporary with the *Commentarium* III, a work probably to be attributed to Sophonias,⁴⁹ only the AP-solution is put forward (10.24–27; 36–37), according to a tradition represented by the earlier (1007 AD) so-called Anonymous Heiberg (§ 57, Heiberg; Ebbesen 1981: III, 92–93).⁵⁰

It is easy to see that the AP-explanation derives from the accepted interpretation of *Categories* 3. There, Aristotle says that if a predicate *A* is said of *B* as of a subject and *B* is said of *Γ* as of a subject, *A* is said of *Γ* as of a subject (*Categories* 3, 1b10–12). This statement was interpreted as an assertion that not every predication is transitive, but only the predication of the relation of being said of another as of a subject (ὅταν ἕτερον καθ' ἑτέρου κατηγορεῖται ὡς καθ' ὑποκειμένου: 1b10–11), in which essential or substantial predications are in question.⁵¹ That one cannot dispense with limiting the transitivity of the relation of “being said of” to substantial or essential predications was proven by recourse to examples such as the following:

VI	Animal is a genus Man is an animal
	Man is a genus

or

VII	Animal is trisyllabic Man is an animal
	Man is trisyllabic.

Because “animal is a genus” and “animal is trisyllabic” in (VI) and (VII), respectively, were considered accidental predications, by restricting transitivity of predication to essential predications, commentators were able to dismiss (VI) and (VII) as invalid inferences.⁵² Inferences such as (VI) and (VII) were normally regarded as examples of fallacies depending on accident. Therefore, the AP-explanation comes from the interpretation of Aristotle's doctrine of predication. It is more difficult to find the origin of the QT-explanation. Perhaps Simplicius gives us a clue (*On Aristotle's Categories* 54.8–21). He says that Andronicus and some other Peripatetics held the view that it is not only essential predications that are transitive but also others, as in the case of when we say that Aristoxenus is musical or that Socrates is an Athenian. Simplicius characterizes this kind of predication, opposing it to predications such as Socrates is walking.⁵³ Simplicius immediately raises an objection: If such predications are to be counted as transitive, how are we to avoid wrong inferences such as the following one?

VIII A body is white
 White is a color

 A body is a color.

His answer is that “white” refers to different objects, namely the color and the being colored of a body, and that the word is used in different senses in (VIII). May we then conclude that the QT-explanation was proposed by Andronicus and his followers? Even if one does not want to commit himself to an affirmative answer to this question, one has to admit that the QT-explanation has something to do with a view about the transitivity of predication that is different from the accepted one. Andronicus might have been the starting point of the process that led to the unorthodox position.

This very long detour through the history of the interpretations of the fallacies depending on accident was necessary for a better understanding of the differing views of (LL) held by Simplicius, on the one side, and Michael of Ephesus, on the other. Simplicius seems to interpret fallacies depending on accident as a consequence of incorrectly taking as transitive predications that in fact are not.⁵⁴ On the other hand, he does not hesitate to propose a different explanation for the sophism concerning the activity of teaching and learning. He accepts (LL) as a valid principle, provided that it is limited to strong identities. Nothing is said about the range of the predicates involved in the principle. The predicates can be essential or accidental properties of *x*. If his solution of the sophism of *Physics* 3.3 has to be taken as a general point about all arguments of the same kind, one can conclude that the same solution was applied by him to the fallacies of the approaching man and of the imminent question. Although we lack direct evidence, suppose that this point is adopted. If so, Simplicius’s general attitude regarding fallacies is much more liberal than the attitude we find in Michael of Ephesus. No uniform solution for all fallacies of the same kind is prescribed. Although cases like (V)–(VIII) are met by restricting transitivity to essential predications, the arguments of the approaching man and of the imminent question are solved by means of a weaker version of (LL), namely (15), where accidental predications are admitted. Therefore, the fact that in (15) a particular kind of predication, namely identity predication, is under consideration is exploited in order to obtain valid consequences. We do not know how Simplicius interpreted Aristotle’s claim that all fallacies depending on accident have the same origin (*Sophistical Refutations* 24, 179a26–27; b11–12). As we have seen, this passage may be interpreted such that it is consistent with the possibility of analyzing different instances of fallacies depending on accident in different ways. The variety of solutions Simplicius presents therefore is not incompatible with Aristotle’s view.

Michael of Ephesus is much less tolerant of logical differences. Although he hesitates between different traditions, his claim seems to be that one single solution has to be given for every fallacy depending on the accident. His treatment of the sophisms of the approaching man and of the imminent question is paradigmatic. No place is left for (LL) even in the form (15). Propositions such

as “what I am going to ask you is *P*” are not considered identity propositions but just normal predications. Correspondingly, a restriction is made on the range of predicates that can safely be used in inferences if fallacies depending on accident are to be avoided. They must be essential predicates. In this way, fallacies like the approaching man and the imminent question can be treated along the same pattern as (V)–(VIII).

Even if some distinctions should be introduced or stressed in order to make the statement historically acceptable, Aristotle’s view reminds us of Frege’s position. Full validity has to be given to (*LL*), and paradoxical counterexamples can be avoided by exploiting a distinction that has something to do with the Fregean distinction of sense and reference. His Greek followers took other lines. Simplicius and, perhaps, Alexander preferred to restrict the range of applicability of (*LL*) to particular kinds of identicals, precisely to identicals that have the same reference and the same sense. It is not my concern to advance judgements in such a matter. Let me just say that it is difficult to conceive a characterization of identity that does not respect (*LL*) and, at the same time, that fits our intuition of sameness.⁵⁵ If *x* and *y* are the same in the ordinary sense of the word “same,” they have to obey (*LL*). I am inclined to think that Simplicius’s view comes down to a restriction in the notion of sameness. Even though we can speak of “identity” in different ways, real sameness holds just between synonyms. If *x* and *y* are the same in the way in which two nonsynonymous terms are, they are not properly the same because one of them can have a property that the other has not. A weaker meaning of sameness is working here – relative identity perhaps. But another story begins here.

Notes

- 1 I am indebted to many friends and colleagues for helpful discussions of the problems dealt with in this chapter. I benefited particularly from a seminar given at the University of Padua by David Wiggins in December 1982. Jonathan Barnes, Philip Drew, Pierdaniele Giaretta, Paolo Leonardi, Anthony Lloyd, Carlo Natali, and Richard Sorabji kindly read a first draft of this chapter and contributed substantially with their criticism to its improvement and refinement.
- 2 Despite its name, (*LL*) is not unanimously attributed to Leibniz. Cf. Feldman 1970: 510–522; *sed contra* Curley 1971: 497–504. See Mates 1968: 516–519; Angelelli 1967: 94–100; Angelelli 1976: 154–155. (*LL*) has to be distinguished from the so-called “Principle of Identity of Indiscernibles” (*PII*), which is supposed to be the most characteristic Leibnizian thesis about identity: If *x* and *y* share all the same properties, they are the same. There is an uninteresting version of (*PII*) that applies to a trivial consequence of the reflexivity of identity: *x* has among its properties also the property of being identical with itself. By hypothesis, *x* shares this property with *x*. Therefore, *y* is the same as *x*. The most controversial version of (*PII*) rules out predicates involving identity or place–time–thing individuation from the range of the properties to be considered. For (*PII*), see Black 1976: 250–262; Ayer 1976: 263–270; O’Connor 1976: 271–280; Wiggins 1980: 55–57.
- 3 See Cartwright 1971: 121 ff.
- 4 Aristotle, *Topics* 1.7, 103a8–9; 7.1, 152b30–33; *Nicomachean Ethics* 1.4, 1096b27–29; *Metaphysics* 1.4, 1070b16–19.
- 5 White 1971: 171; Barnes 1979: 48–49; Angelelli 1976: 144–145; Lewis 1982: 15–16.

- 6 The traditional interpretation could be defended by taking (i) and (ii) to refer to different criteria for different kinds of identicals: (i) would apply to individuals, (ii) to general terms. But the most natural reading of the text does not exclude the case of two general terms that are supposed to be the same and whose identity is checked by means of (i) and (ii), namely by looking at their predicates and subjects. If (i) would be interpreted as expressing (*LL*), this case would have to be left out.
- 7 The fact that the terminology of set theory is used here does not imply that we attribute to Aristotle such commitments as extensionality of classes.
- 8 The distinction I am trying to make corresponds to some extent to the distinction between nonvirtual and virtual classes that Quine develops in his set theory (Quine 1970²: 15–21). Sorabji 1980: 189–191 and 201–203 offers evidence that Aristotle, when discussing the necessary truths of science, sometimes takes them to be referred to abstracts and sometimes to general terms.
- 9 Xenocrates fr. 82 Heinze = fr. 240 Isnardi Parente. The meaning of Xenocrates's thesis has been clarified by Isnardi Parente 1982: 422. But her judgement of Aristotle's criticism – “*obiezioni alquanto capziose*” – is unfair.
- 10 “ $\exists!x (\dots x \dots)$ ” means “There is only one x such that \dots ”.
- 11 τῶν αὐτῶν ἢ ταυτά: 133a24. This expression is difficult, but I can omit an interpretation of it for the purpose of my present discussion.
- 12 Aristotle, *Topics* 1.5, 102a18–19. I follow Barnes 1970: 137 in taking ἀντικατηγορεῖται τοῦ πράγματος as explanatory of μόνω δ' ὑπάρχει.
- 13 Cf. Brunschwig 1967: 122n1.
- 14 Other places where *propria* have such a meaning have been singled out by Barnes 1970: 148–151. He shows that Aristotle sometimes relies on this characterization of *propria* where the standard one is expected (see, for instance, *Topics* 5.4, 132b19–34).
- 15 It does not seem to me that (*LL*) is presupposed in this text (*pace* White 1971: 179). Nor can I share Lewis's position (Lewis 1982). I am inclined to think that Lewis confuses the relation of being compounded, which holds between a substance and an accident and is denoted by “+”, and the relation of being predicated of.
- 16 Bochenski 1968⁴: 67 (11.42) takes the passage as referring to individuals.
- 17 The question will be where to put the modal operator meaning accidentalness, where $\alpha \subseteq \beta$ is expressed in first-order logic. The problems of *de re* and *de dicto* interpretations of Aristotle's modal propositions are well known to students of his logic.
- 18 On the terminology used by Aristotle in the *Topics* for expressing a predicative relation, see De Strycker 1968: 148–149.
- 19 Take

$$(!) \quad \{a\} = \{b\} \vdash \forall \Gamma (\{a\} \subseteq \Gamma \leftrightarrow \{b\} \subseteq \Gamma)$$

as a particular case of (2). By means of the following theorem of set theory

$$(!!) \quad a \in \Gamma \leftrightarrow \{a\} \subseteq \Gamma$$

we obtain

$$(!!!) \quad \{a\} = \{b\} \vdash \forall \Gamma (a \in \Gamma \leftrightarrow b \in \Gamma)$$

from (!). Therefore, if

$$(!!!!) \quad a = b \vdash \{a\} = \{b\}$$

could be proven, (7) would be derived by transitivity from (!!!!) and (!!!). Proof of (!!!!). Let us assume $\{a\} =_{df} \{x: x = a\}$ and $\{b\} =_{df} \{x: x = b\}$. Take $x \in \{a\}$. By definition of $\{a\}$, $x = a$. But $a = b$ by hypothesis. Thus, $x = b$ by transitivity of identity, and consequently $x \in \{b\}$ by definition of $\{b\}$. Therefore, $\{a\}$ and $\{b\}$ have the same elements. By the axiom of extensionality, we get $\{a\} = \{b\}$.

- 20 Someone could think that resorting to set-theoretical relations in order to explain Aristotle's view about predication is not appropriate. For instance, the largely accepted opinion that "being predicated of" covers different logical relations is challenged by Fred Sommers (Sommers 1967: 47–81). According to him, "Socrates is mortal" and "Men are mortal" have the same logical form, their difference being only that in the first proposition there is a predication with a singular subject and in the second one a predication with a plural subject (*ibid.* 48). If Sommers's view fits Aristotle's doctrine better than the traditional one, then our problem can be restated as follows: Singular and plural predications do not have all the same logical properties (*ibid.* 58), although they have the same logical form, nor can the criterion for distinguishing them be merely grammatical (*ibid.* 58–59). We can think of (2) as (*LL*) applied to terms that give rise to plural predications, and of (3) and (7) as (*LL*) applied to terms that give rise to singular predications. Then our problem becomes: Does Aristotle maintain that (*LL*) holds not only in the form of (2) but also in the form of (3) and (7)? The text of the *Topics*, which we have already considered, and *Sophistical Refutations* 24, which we will discuss later, assure us that an affirmative answer to this question is legitimate at least as far as (2) and (7) are concerned. Therefore, the main result of my inquiry will not change very much if a perspective *à la* Sommers is adopted. Although his view may look attractive for analyzing Aristotle's theory of predication, I prefer to follow the less exciting path of the traditional interpretation.
- 21 See, for instance, Angelelli 1976. For the difficulties connected with the theory of relative identity, see Wiggins 1980: 18 ff.
- 22 I am not claiming that such a theory, if interpreted, for instance, as being what David Wiggins calls a "same reference same sense theory," cannot get rid of such difficulties. My point is that mere restriction to synonymy, however favorably interpreted, is not by itself sufficient to avoid paradoxical applications of (*LL*). See Wiggins 1976: 237–241.
- 23 See, for instance, Ross 1966⁴: 362; Wagner 1967: 497.
- 24 For an analysis of such statements and the distinction between the "is" of identity and the "is" of constitution, I refer the reader to Wiggins 1980: 27–28 and 30–35.
- 25 This view is confirmed in *Sophistical Refutations* 7, 169b4–6 where, among the grounds of the fallacies depending on the accident, Aristotle mentions the difficulty of establishing which predicates of an object's predicates are predicates of the object itself: μηδὲ τοῖς ποῖος τῶν κατηγορημάτων πάντα ταῦτά καὶ τῷ πράγματι συμβέβηκεν. Here συμβέβηκεν looks like a mere terminological variation for κατηγορεῖται.
- 26 Aristotle, *Sophistical Refutations* 5, 166b32–33.
- 27 Other similar sophisms based on the relation of being-different-from can be found in Plato, *Euthydemus* 298 A–B.
- 28 That (*LL*) is in question here can easily be proven by the fact that singular terms such as "Coriscus" are considered and said to be the same.
- 29 This is probably the Aristotelian version of the argument of the veiled man (ἐγκεκαλλυμένος) known to Eubulides (Diogenes Laertius 2.108 = *DM* 64) and Diodorus Cronus (Diogenes Laertius 2.111 = *DM* 109) and invented in the Megarian circles (for its origin see *DM* 107–108). Besides being considered by Aristotle, it was discussed by Epicurus (Περὶ φύσεως, fr. 13, col. IX sup., Sedley = fr. 18 IV, Arrighetti), but there is no agreement among scholars about his view (cf. Arrighetti 1973²: 622; *contra* Sedley 1973: 72–73; Giannantoni 1983: 17 on the side of Arrighetti's interpretation). Chrysippus is said to have written a work in two books on it (Diogenes Laertius 7.198 = *SVF* II 15), but we do not know how he solved this puzzle. A version of the riddle is reported by Lucianus (*SVF* II 287).
- 30 Bochenski 1968⁴: 68 (11.46) and Kneale and Kneale 1978⁷: 42 take (ii) as referring to (*PII*). But the context shows rather clearly that (*LL*) is in question here.
- 31 Cf. Angelelli 1976: 146–149; Barnes 1979: 52 and 62; Pelletier 1979: 301–302. Barnes's interpretation is made obscure by a strange statement. He says: "Aristotle

[. . .] takes ‘the approaching man’ to denote an accident, and one that he suggests, freakishly, belongs to Coriscus. This leads to the paradoxical conclusion that a substance and an accident are identical. It also explains Aristotle’s denial that Coriscus and the approaching man are identical in substance. But of course this conclusion not only sounds absurd but is absurd. A substance cannot be identical with anything but a substance.” It looks odd to say that “the approaching man” denotes an accident. What this description denotes is an individual, precisely the same person as the person denoted by “Coriscus.” The difference between “Coriscus” and “the approaching man” is that in the first case an individual is picked out by means of his name, whereas in the second case he is picked out by a description containing one of his accidental properties. A similar, more elaborated, point is proposed in Matthews 1982: 223–240. According to Matthews, “Socrates seated” picks out a “kooky object,” which is different from what is denoted by “Socrates” (224–228). I cannot discuss here this position as fully as it deserves. I limit myself to point out that (i) *Topics* 1.7, 103a32–39, as Matthews himself admits, is against his thesis. His counterargument (*ibid.* 226) is unconvincing, because it is not difficult to explain why Aristotle distinguishes the identity between the man who is sitting and Socrates and the identity between the man and the animal that walks on two feet. In the first case what is said to be the same is a sensible thing, whereas in the second case it is not a sensible thing – let us take it to be, for instance, an abstract. Moreover, (ii) if “Coriscus” and “the approaching man” are not co-referential expressions at all, as Matthews maintains (*ibid.* 227), in what sense can they be said to be the same, and numerically the same? In *Topics* 5.4, 133b31–134a4 Aristotle states that “the man” and “the white man,” although they do not share the same being, are nevertheless the same, at least as far as they cannot be supposed to denote two different things. If “Coriscus” picks out a substance and “the approaching man” a kooky object, they pick out two different objects. What does it mean that they are numerically the same? Accidental sameness must preserve numerical identity. Matthews says that “Socrates and Socrates seated [. . .] are only in a sense the same [people]. Yet they are not two people nor, indeed, two of anything else” (*ibid.* 226). But if the terms that express them are not co-referential, they can be counted as two in an obvious sense. Finally, (iii) Matthews’s evidence for the existence of kooky objects comes from *Physics* 1.7, 190a17–21. But his interpretation, the consequences of which are “staggering” (*ibid.* 225), is neither the only possible one, nor is it the most natural. From the fact that Aristotle says that when a man becomes musical the unmusical man does not survive, it does not follow that the kooky object denoted by “the unmusical man” ceases to be. What simply ceases to be is being unmusical of the man who becomes musical. Suppose that the object *m* (our Socrates) is appropriately described at *t*₁ as “the unmusical man.” If at *t*₂ *m* ceases to be unmusical and becomes musical, he ceases also to be an unmusical man. Therefore, at *t*₂ “the unmusical man” is no longer an appropriate description of *m*, its description now being “the musical man.” But the fact that a description becomes inappropriate and is supplanted by another does not entail that “the unmusical man” and “the musical man” do not refer to *m*. Similar and other objections can be raised against Lewis’s position (Lewis 1982), especially with reference to his interpretation of Aristotle, *On Generation and Corruption* 1.4, 319b25 ff.

- 32 Another approach to the solution of these paradoxes might consist of pointing out that the verb “to know” has different meanings in (P1) and (P3) on one side and in (P4) and (P6) on the other side. But Aristotle does not consider this further possibility.
- 33 I do not see the necessity of adding ὅτι in 179b32, as Ross does.
- 34 I do not agree with Angelelli 1976: 144–145, who maintains that (II) and (III) are conceived by Aristotle as inferences falsifying (*LL*). Because Angelelli thinks that (*LL*) is referred to in *Topics* 7.1, 152b25–29, he is compelled to charge Aristotle with an inconsistency. The same point can be brought against White 1971: 179; Lewis 1982: 13 ff.

- 35 The distinction between *polyonyma* and *heteronyma* goes back to Speusippus. See fr. 45 Isnardi Parente (= fr. 68 Tarán). In addition to the commentaries of the last two editors of Speusippus (Isnardi Parente 1980: 262–267; Tarán 1984: 406–414), see Barnes 1971: 65–80; Tarán 1978: 73–99.
- 36 Like Simplicius, Philoponus (*On Aristotle's Physics* 373.17–20; 374.1–5, 19–21; 382.25–383.2) introduces the distinction between identity in substrate and identity in *logos* or definition in order to explain *Physics* 3.3. He probably gives the same meaning as Simplicius to this distinction.
- 37 Simplicius's statements on this point are less clear than one might hope. On the one hand, he puts the *polyonyma* on the same level as the Aristotelian *synonyma* and *homonyma*, which denote relations between things (*On Aristotle's Categories* 22.22–30). The way in which *polyonyma* are defined points in the same direction. Even if Simplicius uses a pronoun in the neuter gender saying that they are polyonymous ὅσα have the same definition and differ only in name, it seems legitimate to refer the neuter to something like “things” or “objects” (*On Aristotle's Categories* 22.24–25). On the other hand, he is ready to follow Syrianus in maintaining that Aristotle abstained from considering *polyonyma* and *heteronyma* in the *Categories*, because this pair regards names and not things and therefore has to be considered by rhetoricians (*On Aristotle's Categories* 23.12–19; cf. also 19.7–8). We must, of course, look to a relation between names in order to understand what the relation of polyonymy means.
- 38 Frege 1967: 143–144 (26–27).
- 39 The same objection has been made against Frege, when he criticizes in *Über Sinn und Bedeutung* his own earlier view about identity. See Caton 1976: 167 ff.; Suter 1976: 258–261.
- 40 The evidence we possess does not allow us to conclude that Alexander composed a commentary on the *Sophistical Refutations*, even if it is likely. See Ebbesen 1981: I, 242–244.
- 41 It is clear that in this passage ὑποκείμενον has a different meaning than in Simplicius's texts (N) and (O). In other places, Alexander is closer to Simplicius's use. For instance, in *On Aristotle's Metaphysics* 377.22–27 he recognizes that although *x* and *y* have different senses, they can be numerically the same if they share the same ὑποκείμενον.
- 42 See the previous note and, for example, *On Aristotle's Topics* 58.8–14; 60.2–61.13.
- 43 It is perhaps in the same vein that Alexander, in explaining *Topics* 7.1, 152a33–37, uses as examples βροτός and μέρος, two *polyonyma* (*On Aristotle's Topics* 499.28–500.2). Read in this light, the statement found in *On Aristotle's Topics* 502.3–6 claiming that the κυρίως τὰντά cannot have different properties might have an interesting nuance. As κυρίως τὰντά we have to count either things numerically identical versus things specifically or generically identical (e.g., *On Aristotle's Topics* 496.14–497.2), or synonyms (*On Aristotle's Topics* 60.2–5). If the latter is meant by κυρίως τὰντά here, the restriction to strict identity might be understood to be hinted at in this passage.
- 44 S. Ebbesen 1981: I, 268 ff.
- 45 It is worth noticing that this solution is equated by Michael to the solution of the following fallacy:

White (λευκός) is disyllabic
Alcibiades is white

Alcibiades is disyllabic.

(Ps-Alexander 1, 125.32–35)

- 46 For a careful analysis of this doctrine and an interpretation of its meaning I refer the reader to Lloyd 1971b: 357–364.
- 47 Ebbesen 1981: I, 268–285; III, 12–63.
- 48 Ebbesen 1981: I, 290 ff.

- 49 Ebbesen 1981: I, 333 ff. This commentary has been edited by M. Hayduck in *CAG* XXIII 4.
- 50 On the Anonymous Heiberg, see Ebbesen 1981: I, 262–265.
- 51 An analysis of this doctrine can be found in Rohr 1978: 379–385. See also Dancy 1975: 351 ff.
- 52 Cf. Porphyry, *On Aristotle's Categories* 80.32–81.22; Ammonius, *On Aristotle's Categories* 30.25–31.12; Simplicius, *On Aristotle's Categories* 52.9–18; Philoponus, *On Aristotle's Categories* 38.19–39, 15; Olympiodorus, *On Aristotle's Categories* 50.12–22; Elias, *On Aristotle's Categories* 153.11–26; Anonymous, *On Aristotle's Categories* 7.20–28.
- 53 On the meaning of this view for Andronicus's doctrine of categories, see Moraux 1973: 104–105.
- 54 Philoponus's position is less clear, because in *On Aristotle's Prior Analytics* 154.25–28 he considers an inference like (VI) invalid because of the ambiguity of its middle term.
- 55 Cf. Wiggins 1980: 18–44.

6 Aristotle's *Topics* and contingent identity

I

A chapter in the first book of the *Topics* is dedicated to exploring the various senses or uses of the word “same” (ταὐτόν), and its style is, to a certain extent, reminiscent of the famous lexicon of the philosophical terms in *Metaphysics* 5 (*Delta*). The reason for its insertion at the beginning of the *Topics*, after the definition of predicables, probably lies in the fact that a reference to identity is made not only in relation to definition, where it is said that checking a definition in most cases consists in testing identity and difference,¹ but also in relation to genus, where the question of whether two things fall under the same genus is relevant in many ways.² If we stick to the received view that the *Topics* is one of Aristotle's earliest works,³ the analysis of identity we find in *Topics* 1.7 is probably the first attempt made by the philosopher to make a map of the senses of sameness.⁴ He starts by introducing a tripartite division of the use of “ταὐτόν.” Let us recall the passage:

(A) We may regard the same as being divided, in outline, in three parts, for we are accustomed to describe what is the same as in number or in species or in genus. Those are the same in number which have several names but the thing is one, for example cloak and mantle. Those are the same in species that, though many, are indistinguishable (ἀδιάφορα) with respect to species, for instance a man and another man and a horse and another horse (for those things are said to be the same in species which fall under the same species). Similarly, those are the same in genus which fall under the same genus, as horse and man.

(Aristotle, *Topics* 1.7. 103a1–4; R. Smith's translation, modified)

This is not the only place where an examination of the word “ταὐτόν” is offered⁵ or where this tripartition of the uses of sameness is mentioned.⁶

Numerical identity is probably the only kind of identity that one would expect to find under the heading of sameness, so much so that, later, when referring to numerical identity, I will simply call it identity, without any further qualification, unless this is required by the obvious necessity of following Aristotle's terminology.

Aristotle claims that μ and ξ are (numerically) the same if they have different names denoting one thing. To formalize his characterization of identity and the ways in which, according to him, “ταὐτόν” is used, we must make use of a language slightly richer than the usual one adopted in a standard first-order calculus with identity. We have to introduce not only “ a ,” “ b ,” “ c ” . . . as arbitrary names for individuals and “ x ,” “ y ,” “ z ” . . . as variables ranging over a domain of individuals but also “ μ ,” “ ν ,” “ ξ ” . . . as arbitrary names for general or individual entities and “ ρ ,” “ σ ,” “ τ ” . . . as variables ranging over a domain of general or individual entities. Then, if we take Aristotle’s words as a sort of definition of identity, we can rephrase them in the following way:

- (1) $\mu = \xi$ if
- (i) “ μ ” is a name for μ
 - (ii) “ ξ ” is a name for ξ
 - (iii) “ μ ” \neq “ ξ ”
 - (iv*) “ μ ” and “ ξ ” denote one thing.⁷

We can express condition (iv*) more elegantly by stating

- (iv) “ μ ” and “ ξ ” are co-referential.

This characterization of identity deserves some comment. The term “name” (ὄνομα) cannot be taken in the strict sense it has, for example, in the treatise *On Interpretation*, where it is contrasted with “discourse” (λόγος), being considered an expression whose parts are not significant.⁸ Evidence for that is Aristotle’s claim that the relation between a *definiens* and a *definiendum* is a case of (numerical) identity and a *definiens* is not a proper ὄνομα.⁹ Moreover, he allows identity statements in which one term of the relation is what looks like a definite description. Even if in his examples he uses definite descriptions in which only one significant term appears (τὸ καθήμενον, τὸ μουσικόν), there is no reason to deny that they can be expanded in more complex expressions.¹⁰ But the most important fact to underline in (1) is that, in Aristotle’s view, identity is a relation between things and not between names, despite the linguistic approach chosen here. It is not that the names “cloak” and “mantle” are the same. In fact, they are not the same: “Cloak” is a different name from “mantle.” Rather, it is what is denoted by “cloak” that is the same as what is denoted by “mantle.” Therefore, identity holds between things (πράγματα), not between linguistic entities. As we shall see, this observation is crucial for the development of our analysis.¹¹

There are, however, at least three disturbing aspects of this way of putting things. First of all, one could consider condition (iii) in the definition of identity odd. This condition is required by Aristotle’s words in text (A) (“which have several names but the thing is one”: 103 a 10–11). However, its inclusion in our definition seems to rule out obvious cases of identity such as “Callias is Callias.” I do not think that Aristotle would have denied that “Callias is Callias” is an identity statement. Probably, he did not consider this case an interesting one with respect

to the use of identity that he was going to make in the dialectical examination of definitions and relations between genera in the *Topics*.

More importantly, one should consider the example of numerical identity offered in the text. We would expect Aristotle to quote the case of an individual with two names, as, for instance, “Caravaggio” and “Michelangelo Merisi” for the famous painter. Instead, he mentions “cloak” and “mantle,” which are general terms. For those of us raised in the protective shadow of Frege, it may be shocking to accept an identity relation between the *denotata* of nonindividual terms. The impression that Aristotle is not interested in distinguishing the case of sameness among individuals and sameness among general or abstract entities is confirmed by the fact that elsewhere he states a proposition expressing sameness of a particular with itself as an example of (numerical) identity.¹² I will not discuss this apparent anomaly of Aristotle’s approach to identity. Let me only observe that this particular attitude toward sameness does not seem to be conditioned by the context in which the analysis of identity is carried out in the *Topics*. In the *Metaphysics*, where he seems very concerned with avoiding the assignment of an ontological import to the *denotata* of general terms, we find the same free attitude toward the bearers of the identity relation: They cannot be only individuals but must also be what is referred to by universal terms.¹³

Let us concentrate on the third awkward feature of Aristotle’s way of characterizing identity in text (A). The linguistic aspect of it should not go unnoticed. Identity is said to occur when the names (or descriptions) of the entities involved by the relation refer to one and the same object. This approach to identity implies that things to which identity applies must have a name, and this is not at all obvious. What is worse is that identity seems to apply only to things that have at least two names, as we have seen. Although it is not at all clear whether we can assign names to the most imperceptible objects in the world as a grain of sand in the sea, and although we can claim that such objects are self-identical, Aristotle is well aware that names cannot match things, because the former are finite and the latter infinite in number.¹⁴ Even if we include definite descriptions in the notion of an Aristotelian name, we do not solve the problem. All possible combinations of a finite number of names are themselves finite, and they cannot equate the number of infinite things to which sameness applies.¹⁵ Of course, identity has a linguistic aspect. We cannot express identity without referring to a language and a way of picking out things because, as we have seen, identity holds between things. But this obvious remark does not entail that we are compelled to assign a name (or a definite description) to any object of which we say, for instance, that it is identical to itself. In a natural language we have quantifiers and expressions – such as “thing,” “object,” and “entity” – to help cope with the limited number of names at our disposal, and some refined versions of these devices are used in formal languages.¹⁶

Needless to say, (1) cannot be counted as a definition of sameness. Condition (iv) – or (iv*) – contains a clear reference to identity, and the same holds for condition (iii). Therefore, conditions (i)–(iv) cannot be taken as proper *definiens* of sameness, because they include what must be defined. As one should expect, (1) can be considered only an elucidation of the notion of identity, and this

elucidation naturally depends on the context in which sameness is supposed to operate. The context of Aristotle's analysis is dialectic, namely the technique of discussion between two opponents. It is from this point of view that definitions are examined and, as we have seen, discussion about definitions and genera is one of the main reasons for appealing to identity. For instance, a definition can be disproved if one is able to show that its alleged *definiens* is not the same as the *definiendum*. Similarly, we can reject the claim that μ is the genus of ν and ξ if it can be maintained that π is the genus of ξ , μ is the genus of ν , and μ is different from π .

If this is the context of Aristotle's use of identity, it should not surprise us that he has recourse to a linguistic approach in his elucidation of this notion. In some sense *definientia* and *definienda* can be taken as names of entities, and genera can be considered in the same way. This view is confirmed by the fact that when Aristotle treats identity in the context of his ontology, his approach changes. Consider, for instance, the following passage:

(B) The same has several meanings: we sometimes mean the same numerically; again we call a thing the same if it is one both in definition and in number, e.g., you are one with yourself both in form and in matter; and again if the definition of its primary substance is one, e. g., equal straight lines are the same, and so are equal and equal-angled quadrilaterals – there are many such, but in these equality constitutes unity.

(*Metaphysics* 10.3, 1054a32–b3; ROT translation, modified)

For our purposes, it is sufficient to underline the different perspective according to which identity is characterized in this passage. It is no longer explained in terms of the sameness of names' *denotata*, but in terms of the sameness of the ontological constituents of the bearers of the relation, matter and form. The details of the passage are far from clear. It is not obvious in what sense numerical identity differs from unity of definition and number. Maybe, as pseudo-Alexander thinks,¹⁷ Aristotle is here hinting at the distinction between accidental and essential propositions in which statements of identity are included.¹⁸ Nor is the example of the equal straight lines as a special case of sameness in definition, namely specific identity, perspicuous. However, what really matters for us is the ontological shift in the characterization of sameness with respect to the approach in the *Topics*. The same ontological commitment in characterizing identity we find in other passages of the *Metaphysics* in which no mention of the co-referentiality of names is made.¹⁹

There is, however, an aspect in which the ontological characterization of identity and the linguistic one coincide, and this is the attempt that Aristotle makes to reduce sameness to oneness. In the *Topics* it is said that μ and ξ are the same if their names denote one thing, and in text (B) it is oneness in number that establishes sameness. More explicitly, in another passage of the *Metaphysics*, Aristotle clearly says that identity is a kind of oneness.²⁰ It is difficult to evaluate the meaning of this reduction. One might be tempted to take it seriously and spell it out in terms of indistinguishability: μ and ξ are the same if they are one, that is, if they

cannot be distinguished, and, of course, μ and ξ cannot be distinguished if they share all properties and attributes.²¹ We could express this by positing

$$(1^*) \quad \mu = \xi \quad \text{if} \quad \forall F(F(\mu) \leftrightarrow F(\xi)).$$

But the textual evidence for such a move is not very strong,²² and one might wonder what is to be gained by such a characterization of identity. Surely (1*) is not a real definition of identity because the notion of sharing the same attributes involves the notion of identity.

II

In text (A), (numerical) sameness is contrasted with specific and generic identity. We can say that two individuals a and b are specifically identical, namely $Spec(a, b)$, if the species to which a belongs is the same as the species to which b belongs:

$$(2) \quad Spec(a, b) \quad \text{if} \quad S(a) = S(b).$$

It should be clear that “ $S(x)$ ” in (2) stands for “the species of x .” Here we can safely use, as Aristotle does, identity to define specific identity because we can draw a clear-cut line between the two notions. To speak in the Aristotelian way, if a and b are specifically but not numerically identical, they still count as two, which does not happen with numerical identity. However, specific identity is based upon numerical identity, because specific identity occurs only when the species of a is numerically the same as the species of b . Moreover, numerical identity implies specific identity, in the sense that

$$(3) \quad a = b \rightarrow Spec(a, b)$$

holds, as is easy to see. The converse of (3) is not valid, because it may be that $Spec(a, b)$ is satisfied also in the case in which $a \neq b$, as the example of Coriscus and Socrates shows, being different individuals in the same species.

Generic identity does not add much to specific identity, in the sense that a similar pattern works in this case, with the only difference being that here individuals and species can enjoy generic identity. Therefore, we can write

$$(4) \quad Gen(\mu, \xi) \quad \text{if} \quad G(\mu) = G(\xi)$$

where “ $G(\mu)$ ” stands for “the genus of μ ” and “ $Gen(\mu, \xi)$ ” for “ μ and ξ are generically the same.” As before, generic identity is implied by numerical identity and is based upon the latter. We can also add that things which enjoy specific identity also enjoy generic identity, because individuals that fall under the same species fall also under the same genus.²³

The problem that these definitions raise concerns the role that we are expected to assign them. Are they introducing distinctions in the notion of identity in such

a way that, for instance, we are allowed to equate specific and generic sameness to something like the idea of partial identity endorsed by Peter Geach and criticized by David Wiggins?²⁴ Or are they specifying a generic notion of identity in three different kinds of sameness? I believe that we must strongly resist these views. The notion of identity that is implied in the tripartition of the meanings of “ταὐτόν” is always the same, and it is simple identity. In the case of the so-called numerical sameness, it is identity of what is denoted by two names (or descriptions) for objects, whereas in the case of specific and generic identity, it is identity concerning the species or the genus of two objects. I am inclined to think that Aristotle is not classifying kinds of sameness but is considering linguistic uses of same in connection with propositions such as “Coriscus and Callias are the same in species,” or “man and horse are the same in genus.” Consider, for instance, the proposition “Coriscus and Callias are the same in species.” Aristotle’s analysis is probably intended to warn anyone who is dealing with this proposition that it must be taken as asserting not that Coriscus and Callias are the same thing, but that they share the same species. No new notion of identity is introduced, and Aristotle’s distinctions are simply meant to make it clear that to which identity refers.

III

In the chapter in which we find text (A) we find also a puzzling passage concerning (numerical) identity that is worth considering. Aristotle says:

(C) What is one in number (τὸ ἓν ἀριθμῶ) is most uncontroversially called the same in everyone’s judgement. But even this is customarily indicated in several ways. The strictest and primary case is when that which is the same is indicated by means of a word or a definition, e.g., mantle for cloak or two-footed terrestrial animal for man. The second case is when it is indicated by means of a *proprium*, e.g., capable of knowledge for man or carried upward by nature for fire. The third case is when it is indicated with an accident (ἀπὸ τοῦ συμβεβηκότος), e.g. the one sitting or the musical one for Socrates. All these cases are intended to signify what is one in number.

(*Topics* 1.7, 103a23–31; R. Smith translation, modified)

This text is meant not only to present cases in which (numerical) identity or oneness is expressed but also to offer a sort of hierarchy of them. To have a better understanding of what is behind this strange passage, let us reverse the order of cases and start from the last one, namely the case of identity concerning individuals. If we put Aristotle’s examples in a proper sentential form, we get propositions asserting a relation of identity in which one and the same particular is picked out by a proper name, “Socrates,” and by what looks like a definite description, “the one sitting” or “the musical one.” If this view is correct, we can formulate one of these examples in a semi-formal way by stating:

(5) Socrates = the one sitting

Aristotle says that in such a case the definite description is taken ἀπὸ τοῦ συμβεβηκότος, “from an accident,” surely because being sitting is an accident of Socrates.

Because in (5) identity is expressed by means of an accident, one can confidently infer that (5) and all propositions of this kind are contingent statements.²⁵ This can easily be seen if we generalize (5) by introducing the “ι” operator used by Russell to represent definite descriptions, so that we can express the form of a definite description by “ι x $F(x)$,” which means “the unique x which F -s.”²⁶ In this way, the logical form of Aristotle’s example is

$$(6) \quad a = \iota x F(x).$$

Moreover, the accident that plays a role in (5) is “being sitting,” and it is obviously an accident of Socrates. Therefore, the condition is added that $F(a)$ is an accidental predication. What is characteristic of accidental predicates is that they do hold, but they do not necessarily do so, of the subjects to which they are attributed. Therefore, if $F(a)$ is constituted by an accidental predication, we can assume that $F(a)$ is contingent. Accordingly, the following conditional holds:

$$(7) \quad F(a) \rightarrow \Diamond \neg F(a).$$

On the other hand, it is obvious that the following implication

$$(8) \quad a = \iota x F(x) \rightarrow F(a)$$

is always the case. We can easily take (8) as a necessary proposition and therefore state

$$(9) \quad \Box(a = \iota x F(x) \rightarrow F(a)).^{27}$$

By distributing the necessity operator as per implication (9), we get

$$(10) \quad \Box a \quad \iota x F(x) \rightarrow \Box F(a).^{28}$$

And by contraposition we obtain

$$(11) \quad \Diamond \neg F(a) \rightarrow \Diamond a \neq \iota x F(x).$$

By transitivity from (8), (7), and (11) we get

$$(12) \quad a = \iota x F(x) \rightarrow \Diamond a \neq \iota x F(x).$$

Thus, in every case in which a definite description is made out of an accidental attribute, we are allowed to conclude that $a = \iota x F(x)$ is a contingent proposition.

If we put this result within the framework of Aristotle's ontology and his theory of definition, we might reinforce it by saying that in every case in which identity is about an individual and is expressed by means of a definite description, we are dealing with a contingent statement. To prove this assertion, it is sufficient to show that every definite description of a particular entails a reference to accidental properties of that particular so that the argument we used for (5) can apply to every case of identity. Of course, to put this claim in proper logical form, we should accommodate our language to include in a definite description not only a simple accidental attribute but also a complex condition that is uniquely satisfied by an individual. Then the claim would be that however complex the condition might be, it holds contingently of the individual to which it is supposed to apply. But for the sake of simplicity, we can omit this complication and stick with the plain case of a simple accident involved in a description.

The reason that leads one to ascribe Aristotle the view that every definite description of a particular is accidental in the sense described earlier is that Aristotle endorses the view that individuals cannot be defined.²⁹ According to him, a definition is meant to answer the question τί ἐστὶ ("what is it?") with reference to something by picking out a complex of attributes that can apply separately to many things but altogether apply only to the thing defined.³⁰ To count as a definition, the complex of attributes must identify the *definiendum* in a permanent and stable way by separating it from all other things. If we are going to define "man," a *definiendum*, we have to look for a *definiens* that, by applying only to men, divides them from all other animals and, in general, from all sorts of things that are not a man. This cannot happen in the case of an individual identified by a definite description ἀπὸ τοῦ συμβεβηκότος, because such a description isolates the individual at a certain moment and in a certain context but not permanently and from all other individuals. Were there a complex of attributes such that a particular were identifiable in a stable way when applied to it, we could conclude that there are definitions for individuals. Suppose that there were such a permanent definite description of *a*. Let us call it " $\iota xK(x)$." By hypothesis, $\iota xK(x)$ refers to a property (or a group of properties) that is not contingently possessed by *a*. Being not contingently possessed by *a*, such a property belongs to *a* necessarily. Therefore, description $\iota xK(x)$: (i) applies uniquely to *a*, and (ii) applies necessarily to it. Thus, description $\iota xK(x)$ is able to discriminate *a* from any other individual different from *a* under any circumstance. Insofar as $\iota xK(x)$ is supposed to satisfy these conditions, why should we not take it as the correct answer to the question "what is *a*?"? Therefore, description $\iota xK(x)$ counts as a definition of *a*. But there is no definition of *a*; consequently, there is no such description.

It should be clear that Aristotle's refusal of the possibility of defining particulars does not imply a denial of the view that, given two particulars *a* and *b*, it is always possible to find a definite description of one of them that is able to discriminate it from the other. His position is consistent with the view that if $a \neq b$ then there is at least one attribute of *a* that does not belong to *b* or an attribute of *b* that does not belong to *a*. This means that we are able to discriminate by description *a* from *b*, but this does not entail that we can get a definitory description of *a*, because a

definitory description should isolate *a* not only from *b* but also from whatever else is not *a* in a permanent and stable way.

IV

So far we have seen that every statement of identity concerning an individual and expressed by (at least) one definite description is a contingent statement. Can we extend this claim to any proposition in which identity between individuals is truly asserted? Put in this generous form the question has an obvious negative answer. For instance, $a = a$ cannot be taken as a sentence expressing contingent identity. It is impossible for an individual to be different from itself: $a \neq a$ implies a contradiction.

But how should we consider a proposition such as “Caravaggio is Michelangelo Merisi” or, in the style of (5), “Caravaggio = Michelangelo Merisi,” where apparently two names for one and the same thing are used? The answer depends on the way in which we consider the expressions “Caravaggio” and “Michelangelo Merisi.” There are several options at hand. One might claim that proper names are disguised descriptions and that the appropriate way to treat “Caravaggio” and “Michelangelo Merisi” is to expand them into “the one called ‘Caravaggio’” and “the one called ‘Michelangelo Merisi.’” If we proceed along this path, it is easy to see that our “Caravaggio = Michelangelo Merisi” is a contingent proposition no more and no less than (5), because “being called ‘Michelangelo Merisi’” is surely an accidental property of the painter called “Caravaggio.” The same conclusion is reached if one thinks that “Michelangelo Merisi” is a proper name in no way reducible to a definite description, whereas “Caravaggio” is in fact a disguised definite description. This may be true but it only shows that our example is ill-chosen. To make a general point, one should claim that one individual can get only one proper name, and its alleged synonymous are in fact definite descriptions. Therefore, of the two ways of referring to the painter, at least one must be a definite description and, consequently, we are again in the situation of example (5), for which we have proven that it is a contingent statement.

Let us say that the claim that there is only one proper name for an individual looks quite difficult to defend and has no evidence, as far as I know, in the Aristotelian corpus. Therefore, the easiest way to hold that “Caravaggio = Michelangelo Merisi” is a contingent statement consists in supposing that both names it contains are in fact definite descriptions according to the general assumption that proper names must be reduced to definite descriptions. However respectable this view may be, I do not think that Aristotle shares it. Evidence for this point is found in Aristotle’s way of describing another type of numerical identity in our text (C). As we have seen, Aristotle maintains that a case of numerical identity is given when it is stated by means of two synonymous words. Take, for instance, “cloak” and “mantle.” If we say “cloak = mantle,” then we have an identity statement comparable to the statement we get when we relate a *definiens*, call it “ Δ_α ,” to a *definiendum* “ α ” and we obtain $\Delta_\alpha = \alpha$. If the type of identity that is appropriate for the proposition “cloak = mantle” is of the same sort as identity

that occurs in the attribution of a *definiens* to a *definiendum*, it is difficult to believe that “cloak = mantle” is a contingent statement of identity, because a definition in the Aristotelian perspective cannot be classified as contingent. Therefore, “cloak = mantle” is a necessary statement of identity. Now, not only in the case of the attribution of a *definiens* to a *definiendum* but also in a proposition such as “cloak = mantle” a relation is established between entities that are supposed to be universal or general, whatever this may in fact mean. If we were thinking in modern semantic terms, we could say that here “cloak” stands for the class of the individuals that are cloaks, or one might imagine that “cloak” and “mantle” are names of concepts. Whatever the ontology of these things may be, it is hard to reject the idea that “cloak” and “mantle” refer to special entities, being names, or better, proper names for them.

Suppose now that these names are not directly referring, but must be expanded into descriptions, “the object called ‘cloak’” or “the object called ‘mantle,’” in the same way in which we imagined that “Caravaggio” and “Michelangelo Merisi,” taken as descriptions, had to be expanded into “the one called ‘Caravaggio’” and “the one called ‘Michelangelo Merisi.’” It would be plausible to maintain that, in the object called “cloak” (or “mantle”), the property of being called “cloak” (or “mantle”) is accidental with respect to the object referred to in the description. After all, according to Aristotle, names are assigned by convention and not by nature.³¹ Therefore, if “Caravaggio = Michelangelo Merisi” is a contingent statement of identity under the assumption that “Caravaggio” and “Michelangelo Merisi” stand for definite descriptions, because these descriptions involve an accidental property of the thing designated, the same should be true for “cloak = mantle” because also in this case descriptions referring to accidental properties are used. But we have seen that “cloak = mantle” cannot be taken as a contingent statement of identity because it is put on the same footing as definitional identity. The conclusion seems to be compelling. To be faithful to Aristotle, we must give up the claim that “cloak” and “mantle” function as definite descriptions. But if so, we do not see why “Caravaggio,” and “Michelangelo Merisi,” being proper names for an individual, should be conceived of as definite descriptions.

If our analysis is correct, we are in a position to make a general claim about statements of identity concerning general or individual entities. Not only propositions such as “cloak = mantle,” namely statements involving proper names for general entities, but also propositions such as “Caravaggio = Michelangelo Merisi,” where proper names for individuals are used, should be taken to be necessary. How can we explain this position? A possible way is the following. If “cloak = mantle” holds, then “cloak” and “mantle” are coreferential according to condition (iv) of definition (1). Let μ be the object to which both “cloak,” and “mantle” refer, and remember that identity is a relation between things, πράγματα. Therefore, “cloak = mantle” simply tells us that the object denoted by “cloak,” namely μ , is the same as the object indicated by “mantle,” namely that μ is the same as μ , and this is a necessary truth. The same happens with “Caravaggio = Michelangelo Merisi.” Take a as the object denoted both by “Caravaggio” and “Michelangelo Merisi.” What “Caravaggio = Michelangelo Merisi” states is that a is a , again a necessary proposition. By

generalization, we are allowed to claim that where “ μ ” and “ ξ ” name the same thing, then “ $\mu = \xi$ ” is necessarily true:

$$(13) \quad \mu = \xi \rightarrow \Box \mu = \xi.$$

This interpretation is based on the idea that a statement such as “ $\mu = \mu$ ” is a necessary one. It is exactly this assumption that gives a rationale to the introduction of (13). There is some evidence that Aristotle shared this position. First of all, “ $a = a$ ” (or “ $\mu = \mu$ ”) is treated by him as a sound statement. In *Metaphysics* 5.9, 1018a8–9, that a thing is the same as itself (αὐτὸ αὐτῷ ταὐτόν) is cited as an obvious case of numerical identity, and in *Metaphysics* 10.3, 1054 a 35 “you are the same as you” (σύ σαυτῷ) is offered as an evident example of numerical oneness. Moreover, in a passage of the *Sophistical Refutations*, a principle of identity is used to build up a sophistical argument concluding that one and the same thing is the same as itself and different from itself,³² and the explanation of the paradox does not involve any restriction on the principle of identity itself. Thus, everything is self-identical. Finally, the idea that a proposition such as “ $\mu = \mu$ ” constitutes a philosophical principle is suggested in the following passage:

(D) To inquire why a thing is itself is not a real inquiry, since for it the fact or that the thing is must already be evident (e.g., that the moon is eclipsed), but the fact that a thing is itself is the only reason and the only cause to be given for all such questions as why man is man or musical is musical; unless one were to say that each thing is indistinguishable from itself (ἀδιαίρετον πρὸς αὐτὸ ἕκαστον) and its being one just meant this but this is common to all things and is a short and easy way with the question.

(*Metaphysics* 7.17, 1041a14–20; ROT, slightly modified)

There is a standard and, I believe, correct interpretation of this text.³³ A scientific inquiry trying to establish the reason according to which man is man or musical is musical is pointless. In general, when we ask why a predicate belongs to a subject, the fact that the predicate belongs to that subject must be already known. But in the case of “ $\mu = \mu$,” stating that $\mu = \mu$ is self-explanatory and we do not need to look for a reason for it. A possible justification that $\mu = \mu$ does not depend on the nature of μ but, at most, is based on the fact that μ cannot be distinguished from itself and on the assumption that indistinguishability is what characterizes identity. It is therefore clear that for Aristotle the fact that one thing is the same as itself is something that applies to everything for reasons which are not tied to the particular nature of one thing or another. We are entitled to infer from this that $\mu = \mu$ is a general principle of philosophy and, as such, a necessary statement.

V

We are faced with an obvious problem. As a special case of (13) we can state that

$$(13.1) \quad a = b \rightarrow \Box a = b.$$

Now, if we allow a nonrestricted substitution rule for individual terms, one might think that (13.1) conflicts with (12).³⁴ It is sufficient to substitute “ $\iota xF(x)$ ” for b in (13.1), by which we get:

$$(14) \quad a = \iota xF(x) \rightarrow \Box a = \iota xF(x).$$

This yields, together with (12), the implausible conclusion that no proposition like (5) can be true. To avoid this unpleasant consequence, two ways out are left: Either one restricts the substitution rule for individual terms (for instance, to non-modal contexts) or one considers the use of the ι -operator with modalities to be misleading.

I will not discuss in detail the first possibility. I will concentrate on the second. The idea that the ι -descriptor is inadequate when we have to deal with modalities is well known to logicians. They have repeatedly pointed out that by using it we are not able to exactly determine the scope of the modal operators.³⁵ Let us adopt the standard way to get rid of descriptions introduced by means of the “ ι ” operator. As everybody knows after Russell, a sentence such as (6) can be rephrased in the following way:

$$(15) \quad F(a) \wedge \forall x(F(x) \rightarrow x = a).$$

Assume now that the property on which a description of an individual is based belongs accidentally to the individual in question. Starting from the obvious thesis that

$$(16) \quad (F(a) \wedge \forall x(F(x) \rightarrow x = a)) \rightarrow F(a)$$

by the same argument we produced before to prove that (5) is contingent, we can easily derive

$$(17) \quad \Diamond \neg (F(a) \wedge \forall x(F(x) \rightarrow x = a)).$$

Statement (17) is true if (15) and (7) are true. Statement (17) is actually implied by any generalization of (5) if we stick to the view that (5) includes a description ἀπὸ τοῦ συμβεβηκότος.

But admitting (17) does not entail that we must reject the view that identity is conceived by Aristotle as necessary. In fact, (17) does not conflict with (13.1). When we claim that (17) holds, we are giving up the idea that propositions such as (5) should be considered necessary, because (17) rules out

$$(18) \quad \Box (F(a) \wedge \forall x(F(x) \rightarrow x = a)).$$

But why should one take an identity statement such as (5) as implying a necessary proposition in the sense of (18)? When we say that Socrates is the one sitting, we are not thereby committed to the claim that necessarily Socrates is the only sitting object. It might have been that Socrates is not sitting or that he is not the only one

to be sitting. We can more plausibly take (5) to mean that one and the same person, Socrates, is described as the only one who is sitting and addressed as “Socrates.” It may be that the description is based on an accident of Socrates, but Socrates, however described, is always the same individual as the one named “Socrates,” and this is a necessary fact.³⁶

In this perspective if we would like to make explicit the kind of necessity involved in the use of identity in a statement such as (5), when (5) is unpacked as (15) and identity is supposed to satisfy (13), we should state something like:

$$(19) \quad F(a) \wedge \forall x(F(x) \rightarrow \Box x = a)$$

which corresponds exactly to our intuition about (5): The individual named Socrates is necessarily the same individual contingently described as the one sitting. If we adopt the view that identity is necessary, (19) is an immediate consequence of (15). But (19) is in no way inconsistent with (17). Therefore, (5) can be taken to be a contingent statement as Aristotle does, but this claim does not commit us to attribute to him the view that identity has to be considered contingent.³⁷

Our interpretation is clear. If we adopt the view that identity according to Aristotle is necessary, namely that (13) holds for him, we are entitled to claim that the kind of necessity involved by propositions such as (5) is, at most, the necessity expressed by (19). This conclusion is confirmed by considering the opposite view, according to which the contingency of (5) would imply that Aristotle admitted cases in which identity is contingent. We say that $\mu = \xi$ is an instance of contingent identity if it satisfies

$$(20) \quad \mu = \xi \rightarrow \Diamond \mu \neq \xi.$$

Needless to say, if we accept that (20) holds in some cases, we must put a restriction on (13), because (20) and (13), taken without qualification, cannot hold together. Not every identity statement is necessary if some of them are contingent. Suppose now that a statement such as (5) is a statement of contingent identity. To make this claim plausible, we cannot take the one sitting as a definite description that contingently applies to Socrates. We must imagine that the one sitting is the name for an individual, the one who is sitting, who is identified with different particulars in different possible situations. In these circumstances, the one who is sitting is Socrates, and in other circumstances he could be different. In general, we need to postulate the existence of special individuals such as the one sitting, the winner, the runner, and so on, which are intensional objects different from individuals such as Socrates, Cleon, and Callias. “Socrates” denotes always and everywhere the same individual. “The winner,” on the other hand, can refer to different individuals in different situations.³⁸ If we take this to be the ontology that allows us to think that (20) can be satisfied in some cases, one should be confident enough to claim that this ontology is not the Aristotelian one. As is well known, the only primary existent things that Aristotle admits

are individual substances, and the one who is sitting, or the winner, is not a substance.

However, one might observe that in the domain in which the bearers of Aristotelian identity are included there are not only individual substances but also abstracts, so one might conclude that there is no reason to rule out from this claim intensional objects such as the one sitting, or the winner, while including general terms such as man, cloak, mantle, or a *definiens*. But there is a difference. Because of the Aristotelian ontology, we are tempted to claim that in his perspective we can always replace a statement such as “ $\mu = \xi$,” where “ μ ” and “ ξ ” are names for general entities, with an equivalent statement where only “normal” individuals are the *denotata* of the arbitrary names and variables that represent the logical subjects of the propositions. The most obvious example is constituted by *propria*. “Man is capable of knowledge” is viewed as an identity statement in text (C), but when Aristotle gives the official definition of this notion³⁹ he explains our proposition as an equivalence: “everything is a man if and only if it is capable of knowledge.” This is not what happens with the intensional objects such as the one who is sitting or the winner. If their introduction answers the need to offer a justification for contingent identity, they cannot be replaced or reduced to something else (for instance, to definite descriptions). We have to include them in the domain in which the bearers of the identity relation are. But such individuals are not part of the furniture of the Aristotelian world.

One might challenge this last claim by pointing out that some scholars believe that the universe of Aristotelian objects includes not only individual substances but also what are called accidental compounds. According to them, the truth of a proposition such as “Socrates is pale” commits us to assert the existence of the accidental compound Socrates+pale, which is ontologically dependent on the substance out of which it is generated, Socrates.⁴⁰ By adopting this general view one might try to adapt it to the case of contingent identity and claim that the intensional objects we are considering are special cases of accidental compounds. I cannot discuss here the complicated issues connected with the idea of considering accidental compounds as Aristotelian individuals. What is sufficient for our purposes is to point out that these individuals cannot play the role that intensional objects do in the justification of contingent identity. Accidental compounds are intrinsically dependent for their existence on the individual substances to which they are connected. Socrates+pale is parasitic to Socrates. When Socrates ceases to be pale, Socrates+pale no longer exists. That is not what is expected from our intensional object, the one sitting. When Socrates is no longer sitting, the one sitting can still be in existence, being identified with, for instance, Coriscus (if Coriscus is sitting). Moreover, the relation between Socrates and the one sitting is an identity relation. By contrast, the relation between an accidental compound and its cognate substance is not an identity relation, being irreflexive and nontransitive.⁴¹ Therefore, we can conclude that Aristotle does not share the view that identity is contingent, as the lack of an ontology supporting its introduction shows.

VI

Let us come back to text (C). Various cases of identity are mentioned, and a hierarchy among them is established. We have already considered two of these cases. One occurs when identity is expressed by attributing a definite description to an individual; the other is the case of two synonyms for the same general entity. As we have seen, it is legitimate to equate to the latter the case of two synonyms for the same individual, namely the case of identity expressed by propositions such as “Caravaggio = Michelangelo Merisi.” After all, in a passage parallel to text (C) Aristotle mentions the proposition “you are you” as an example of the strongest case of identity, and this fact shows that he does not distinguish between the situation of general entities and individuals.⁴²

We must briefly explore the two remaining cases: the case of the attribution of a *proprium* to its subject and the case of definition. Here, once again, Aristotle is applying identity to entities denoted by general terms, taking the latter as proper names for the former. In this perspective, we must consider the *definiendum* that appears as a member of definitional identity as expressing that the name of a class or concept, no more and no less than “Coriscus,” is the name for an individual and a way to indicate it. If we take this point of view seriously, we can think of a *definiens* as a special kind of definite description. As we have seen, “the one sitting” is a definite description of Socrates ἀπὸ τοῦ συμβεβηκότος. In the same way, we can consider “two-footed terrestrial animal” as a definite description of what is designated by the name “man.” The only difference with respect to “Socrates” and “the one sitting” is that “two-footed terrestrial animal” is not ἀπὸ τοῦ συμβεβηκότος with respect to man, being not based on an accidental attribute of man, but it expresses what answers the question “what is a man?” In other words, it picks out the essence of man. We can interpret the case of *proprium* along the same lines. A *proprium* is an attribute that applies to one subject and no other. “Capable of knowledge” is a *proprium* of man because only men are capable of knowledge. Consider, for instance, the statement “man = capable of knowledge,” and, as before, take “man” as a name of the entity designated by “man.” Then “capable of knowledge” can be considered a definite description of this entity. A definite description construed from a *proprium* differs, however, from a definite description based on a definition, because a *proprium*, according to Aristotle, does not reveal the essence of the object to which it is applied. On the other hand, a definite description based on a *proprium* differs from a definite description ἀπὸ τοῦ συμβεβηκότος, because a *proprium* is not an accidental attribute of the thing to which it applies.

We are now in a position to explain what the cases of identity considered by Aristotle are. It should be clear that it is not a difference in degrees of identity: *a* and *b* cannot be more (or less) identical than *c* and *d* are. If they are identical, they are simply the same. Nor can it be a question of strong versus weak identity, namely necessary versus contingent identity. As we have seen, identity in Aristotle’s view is always strong in the sense that it is necessary, a notion of contingent identity being outside his conceptual framework. I think that the cases

described by Aristotle in text (C) refer to different ways in which identity can be expressed. We can express identity by picking two names for the same object, or we can use a definite description. As we have seen, definitions, *propria*, and definite descriptions for individuals all fall in the latter case, although they differ from one another. In the case of a definition, a description is offered that reveals the essence of the *definiendum*. In the case of a *proprium*, we have a description based on a necessary but not essential attribute. Finally, in the case of individuals, the description is construed by means of an accident, which is neither essential nor necessary to the particular referred to.

We can now face our last question, namely the question of the order in which Aristotle puts his various types of identity statements. The order constitutes a hierarchy. At the top we find identity statements that are expressed by definitions and synonyms; at the bottom we find definite descriptions for individuals; finally, at the intermediate level, we find identity statements that are expressed by means of *propria*. Why this order? My hypothesis is that Aristotle was driven by the idea that any identity statement exhibits a slightly embarrassing feature, because it expresses sameness by means of different terms.⁴³ When we state, for instance, “Caravaggio = Caravaggio,” we use two different occurrences of the same name for one object, whereas in “Caravaggio = Michelangelo Merisi” two *different* names are employed to designate one and the same particular. Thus, one could think that the various types of identity statements are ordered according to the grade of *difference* of the terms by which the identity is expressed. Aristotle does not here mention the strongest case, namely $a = a$, or more generally $\mu = \mu$, though he does so elsewhere.⁴⁴ In text (C), his starting point is the case of synonyms and definitions, where there is a difference between the terms through which identity is formulated, but the difference is less than the one involved by *propria* or accidents. Synonyms do not present any difference in content, and a *definiens* manifests what its *definiendum* is. A greater distance occurs between a *proprium* and its subject because they have different definitions. The last position is occupied by accidents, which attach in a loose way to their subjects, being not even necessary to them.

Notes

1 Aristotle, *Topics* 1.5, 102a7–9.

2 Aristotle, *Topics* 1.5, 102a36 ff.

3 See, for instance, Maier 1969–1970³: II.b, 78; Hambruch 1904: 3 ff.; Solmsen 1929: 194; Ross 1939: 251–272; Bochenski 1968⁴: 22–24; Düring 1966: 69 ff.; Barnes 1981: 17–59.

4 I use “sameness” and “identity” as synonyms. Therefore, by attributing a theory of sameness to Aristotle, I intend to ascribe him a theory of identity – *pace* White 1971: 177–197. See also Miller 1973: 483–490.

5 For example, *Metaphysics* 5.9; 10.3, 1054a32 ff.

6 Texts in Bonitz 1955²: 125a57 ff.

7 {When “if” is italicized, it is meant to be definitional.}

8 Aristotle, *On Interpretation* 2, 16a19–21; λόγος is defined at 4, 16b26–28.

9 Aristotle, *Topics* 1.7, 103a25–27 (see text (c)).

- 10 Aristotle, *Topics* 1.7, 103a29–31 (see text (c)). At 103a37 the example of “the one who is conversing” (τὸν διαλεγόμενον) is proposed.
- 11 I am grateful to Professor Enrico Martino for having made me aware of the importance of this point for the interpretation of Aristotle’s position.
- 12 Aristotle, *Metaphysics* 10.3, 1054a32–35.
- 13 See, for instance, *Metaphysics* 10.3, 1054a34–b3 where the unity, that is, identity of a definition, is in question.
- 14 Aristotle, *Sophistical Refutations* 1, 165a10–13.
- 15 {All possible combinations of a finite number of names are themselves finite, provided that we do not allow for the repetition of a name.}
- 16 I owe this point to Paolo Crivelli.
- 17 Alexander of Aphrodisias, *On Aristotle’s Metaphysics* 615.20 ff.
- 18 The same view in Ross 1953³: II, 287.
- 19 See, for instance, *Metaphysics* 5.9, 1018a4–9.
- 20 Aristotle, *Metaphysics* 5.9, 1018a7–9.
- 21 It should be kept in mind that according to Aristotle, relations different from identity are treated as properties, so that in his language there are no n -place predicates different from identity where $n > 1$.
- 22 Aristotle seems to consider unity in terms of indivisibility, for instance, at *Metaphysics* 5.6, 1016a32ff; 7.17, 1041a14–20; 10.1, 1052a29ff. But it is not clear to me whether we must confer a deep, and philosophically relevant, sense to this way of speaking.
- 23 Aristotle is well aware of the fact that numerical identity implies the corresponding specific and generic one, whereas specific identity implies generic identity but not vice versa (*Metaphysics* 5.6, 1016b35ff).
- 24 Geach 1972: 239 ff.; Wiggins 1980: 15 ff.
- 25 A proposition A is contingent if $(A \rightarrow \Diamond \neg A) \wedge (\neg A \rightarrow \Diamond A)$ is true.
- 26 Russell and Whitehead 1927²: I, 173 ff.
- 27 In a formal system as strong as T we can deduce (9) from (8) by means of the necessitation rule (see Hughes and Cresswell 1996: 23 ff.).
- 28 Aristotle proves the cognate law of distribution of possibility along implication (see *Prior Analytics* 1.15, 34a5 ff.; *Metaphysics* 9.4, 1047b14 ff.).
- 29 See, for instance, Aristotle, *Metaphysics* 7.15, 1039b27 ff.
- 30 Aristotle, *Posterior Analytics* 2.13, 96a24 ff.
- 31 Aristotle, *On Interpretation* 2, 16a19–20.
- 32 Aristotle, *Sophistical Refutations* 30, 181b13 ff.
- 33 See, for instance, Ross 1953³: II, 206–207; Reale 1993: III, 396–397; Frede and Patzig 1988: II, 310–312.
- 34 A rule of substitution for identicals can be expressed as follows: (SR) $a = b \rightarrow (A \leftrightarrow B)$ (B differing from A only in having b in 0 or more occurrences of a in A). (SR) is notoriously at odds with systems of contingent identity. They normally adopt a nonmodal version of it, in the sense that (SR) is supposed to hold if A and B do not contain modal operators (see Hughes and Cresswell 1996: 332–334).
- 35 Russell was aware of the ambiguity involved by the use of the “ i ” operator, and to avoid it he introduced the notion of scope of a description (Russell and Whitehead 1927²: I: 173). The ambiguity of “ i ” increases in a modal context (see Hughes and Cresswell 1996: 324–325).
- 36 See Hughes and Cresswell 1996: 318 ff.
- 37 I am aware of the fact that using formulas with arbitrary names in a modal context can raise difficulties (on them see, for instance, Hughes and Cresswell 1996: 274ff). However, I do not think that I must take care of them at the level of analysis in which I am moving.
- 38 See Hughes and Cresswell 1996: 331 ff.
- 39 For the definition of *proprium*, see *Topics* 1.5, 102a18–24.

- 40 The theory of accidental compounds has been developed by Lewis 1991. See also Matthews 1982: 223–240.
- 41 See Lewis 1991: 103–105.
- 42 Aristotle, *Metaphysics* 10.3, 1054a34–35.
- 43 This idea is implied by *Metaphysics* 5.9, 1018a7–9. See also Wittgenstein 1922: 5.5303: “*Beiläufig gesprochen: Von Zwei Dingen zu Sagen, sie seien identisch, ist ein Unsinn, und von Einem zu sagen, es sei identisch mit sich selbst, sagt gar nichts.*”
- 44 Aristotle, *Metaphysics* 5.9, 1018a7–9; 10.3, 1054a33–35.

7 Aristotle on universals and particulars

I

In this chapter, I do not examine all the problems arising from *On Interpretation* 7, nor am I committed to a *commentarium perpetuum* of the text. Rather, I pick up some of the philosophical questions involved in the text and try to discuss them in a fairly systematic way.

The opening of the chapter contains the well-known definitions of καθόλου and καθ'ἑκαστον. A possible translation of the passage goes as follows:

(A) Of things (πράγματα) some are universal (τὰ μὲν καθόλου), others particular (τὰ δὲ καθ'ἑκαστον). I call universal that which is by its nature predicated (πέφυκε κατεγορεῖσθαι) of many, and particular that which is not; for instance, man is among the universals and Callias among the particulars.

(Aristotle, *On Interpretation* 7, 17a38-b1; ROT, slightly modified)

This text presents many problems and, because they are connected, it is better to list them together before trying to offer a solution to them. First of all, it is not obvious what “καθόλου” and “καθ'ἑκαστον” refer to. The temptation is to think that “καθόλου” and “καθ'ἑκαστον” are said of terms, ὅροι, the constituents of propositions. But Aristotle says that πράγματα, things, are καθόλου and καθ'ἑκαστον, and this is surely not an unfortunate or careless way of speaking because, in a parallel passage to be considered later on, the characteristics of being universal or being singular are attributed to things that are, τὰ ὄντα.¹ Jan Łukasiewicz, in his famous book on Aristotle's syllogistic, promptly remarked that this is a category mistake, because terms, not things, can be said to be universal or singular.² But this criticism does not really hit the mark. If the referent of a proper name can rightly be called an “individual” or a “particular,” why should we not be allowed to call a universal what represents the reference of a general term? The real difficulty is this: If we adopt this explanation for Aristotle's terminology, we are compelled to attribute him an ontology that he surely refused. If we think of his criticism of Plato's theory of ideas and his claim that the οὐσία of a thing is particular, it is difficult to believe that he would have admitted the existence of universal things, such as man or butterfly, as both constituting the reference of the words “man”

and “butterfly” and being a complement of the existence of individual things corresponding to the proper names “Callias” and “Cleon.”

A second, and indeed more difficult, problem is constituted by the definitions of καθόλου and καθ’ἕκαστον themselves. A little reflection shows that these Greek expressions are ambiguous. Καθ’ἕκαστον is defined by the negation of καθόλου, and it is not easy to understand what the negation amounts to. By paraphrasing Aristotle, a καθόλου is a πρᾶγμα that by its nature is such as to be predicated of many. We may think of a particular either as a πρᾶγμα that is not by its nature such as to be predicate of many things, or as a πρᾶγμα that is by its nature such that it is predicated of not many but only one thing. This latter possibility is more than a theoretical one because it has been the standard interpretation adopted by the Neoplatonic commentators.³ Needless to say, this alternative is philosophically relevant. If we adopt the Neoplatonic interpretation, naming is reduced to a special act of predicating, and proper names do not play a role logically different from that of common names, the only difference being that the former are true of only one individual, whereas the latter may be true of many. The idea that “Cleon” must be treated as a predicate that uniquely belongs to an individual reminds us of Quine’s analysis of proper names: Cleon has to be considered the individual who cleonizes,⁴ and this should be sufficient to show how deeply our interpretation is involved in difficult questions concerning philosophy of language.

A third problem concerns the relation that the Aristotelian definition of καθόλου has to the notion of predicate. Shall we identify καθόλου and predicate, at least in the sense that every predicate is a καθόλου and every καθόλου may play the role of a predicate? Of course, a positive answer to this question implies that we have already decided that proper names cannot be treated as predicates. But even if we make this move, there is a further problem tied up with the way in which the expression “πέφυκε κατεγορεῖσθαι” in the definition of καθόλου is interpreted. Suppose that we have a predicate that is true of nothing as, for instance, *being a witch*, which is what we claim in our post-Enlightenment world. Shall we count this predicate as a καθόλου according to the Aristotelian definition or not? One might assert that being a witch is not a universal, because a universal must be true of many, and our predicate (by our post-Enlightenment hypothesis) is true of nothing. For a similar reason, one might doubt that a predicate that is true of just one thing, for instance, *being a portrait of Benvenuto Cellini* (if there is only one such portrait), is universal. Even in this case the condition of being true of many is not satisfied, and therefore one might deny that this predicate is a καθόλου in the Aristotelian sense.

There is an obvious objection to the attempt to keep universals and predicates separate. It might be pointed out that Aristotle does not say that *a* is a καθόλου if *a* is true of many. What he says, rather, is that *a* must be such that it πέφυκε κατεγορεῖσθαι of many. If a modal implication is supposed to be involved in “πέφυκε κατεγορεῖσθαι,” then one might maintain that *being a portrait of Benvenuto Cellini* and *being a witch* are universal. Although there is (by hypothesis) only one portrait of Benvenuto Cellini and there are no witches, it might be the case that more than one portrait of Benvenuto Cellini was made, and it might be

the case that there are many witches in the world. This possibility would warrant that predicates such as *being a portrait of Benvenuto Cellini* and *being a witch* are universal in the Aristotelian sense.

However, it must be clear that exploiting the possibility involved in the use of the expression “πέφυκε κατηγορεῖσθαι” is not enough to ensure the equivalence between the notion of καθόλου and the notion of predicate. To be convinced of this, it is sufficient to think of a predicate that *cannot* be true of anything, as, for instance, *being different from itself*. Not only is there nothing that is different from itself, but there cannot be anything of which we can say that. Therefore, *being different from itself* is not a καθόλου.

There are various ways to react to this difficulty. One is to interpret the verb “κατηγορεῖσθαι” in “πέφυκε κατηγορεῖσθαι” in a nonstandard way. The idea would be that “κατηγορεῖσθαι” means not “being predicated affirmatively,” but only “being a predicate of” or “having the function of a predicate.” In this way, *being different from itself* would be a καθόλου because it can be predicated of everything although it is true of nothing. In the true sentences “*a* is not different from itself” and “*b* is not different from itself,” *different from itself* appears as a predicate, and therefore it satisfies the definition of καθόλου.

I am doubtful about this solution. Although there are passages in which Aristotle uses “κατηγορεῖσθαι” in this special sense,⁵ it is difficult to believe that in the formal definition of καθόλου, the term “κατηγορεῖσθαι” takes this meaning without no warning on his part. We should expect a comment explaining to us that here “κατηγορεῖσθαι” means “being a predicate (either affirmative or negative) of” or an additional remark to cover the case of negative propositions. But we find nothing of the kind both here and in the parallel passage of *Metaphysics* 7.13, where the same definition is given, the only difference being that “πέφυκε κατηγορεῖσθαι” is substituted for “πέφυκε ὑπάρχειν.”⁶ We *do* find places where “ὑπάρχειν” is taken to cover both affirmative and negative predications,⁷ but this way of speaking is rather unusual for Aristotle and therefore it would be surprising if he resorted to it in a formal definition. Moreover, by admitting negative propositions, I do not see how universal and singular terms can be distinguished from Aristotle’s point of view. Consider, for instance, the two following propositions:

(1) this Athenian is not Callias

and

(2) this Spartan is not Callias.

Under the assumption that Callias is a Boeotian, these propositions are both true, and in the Aristotelian perspective *Callias* has the function of a predicate in both. Therefore, it satisfies the definition of καθόλου and it must be counted among the terms of this kind. I have the feeling that we must stay at the obvious meaning of “κατηγορεῖσθαι” and must look for a different way out to our problem.

II

Let us begin by focusing on the claim that πράγματα, things, are καθόλου and καθ'ἑκάστα. As scholars have often remarked, the word “πρᾶγμα” is used in different senses by Aristotle.⁸ For instance, at the outset of the *Sophistical Refutations*, he observes that one reason for possible confusion in an argument is that we cannot construe it by bringing in πράγματα themselves but we are compelled to use ὀνόματα instead.⁹ Here “πρᾶγμα” stands for thing, namely a piece of the furniture of the external world, something that is well exemplified by a space-temporal object. Again, at the beginning of the treatise *On Interpretation*, things, πράγματα, are distinguished from spoken sounds, on the one hand, and affections in the soul, on the other, and this points toward an interpretation of πραγματά in terms of external things.¹⁰

There are other passages in which “πρᾶγμα” is simply opposed to “ὄνομα,” and in this sense it denotes the content of a verbal expression, namely its meaning. In *Metaphysics* 4.4 the point is made that a name such as “man” would be meaningless if it meant infinitely many¹¹ From the context, one is allowed to fill the dots by supplying the word “things,” “πράγματα,” because Aristotle says that “it is not possible even to conceive ἓν if one is not conceiving ἓν and, if it is possible, one name should be assigned to the πρᾶγμα.” This way of speaking suggests that we are allowed to call “πρᾶγμα” what is expressed by a name or, in general, by a verbal expression. In light of these considerations, when we are told, in *On Interpretation* 1, that the name “goat-stag” signifies something (σημάνειν μὲν τι),¹² we may claim that what is meant by this word is a πρᾶγμα, and this claim is supported by several other passages.¹³ Therefore, the use of the word “πρᾶγμα” does not commit us to claiming that the Aristotelian definitions of καθόλου and καθ'ἑκάστον concern pieces of the furniture of the world. Rather, this use suggests that Aristotle consciously refuses to treat καθόλου and καθ'ἑκάστον as grammatical items that have to do with words. What we say to be either universal or particular is not a verbal expression, but what is meant by this expression, whatever this may be. In this sense we may claim that Aristotle's distinction is not a grammatical but rather a logical distinction. To underline this point, let us say that what is universal or particular is a *term* in the sense that it is not a name, but rather what is meant by a name.

Against this interpretation, two objections may be raised. The first is methodological: Are we allowed to attribute to Aristotle a distinction between grammar and logic that is not theorized by him? I do not think that what I have said about the meaning of “πρᾶγμα” implies that Aristotle should have had a clear-cut theory of the distinction between grammar and logic. What I mean is that what Aristotle says leads us, who have a precise distinction in mind between the two disciplines, to place his definition of universal and particular more on the side of logic than on the side of grammar.

There is, however, a second, more interesting, objection. In the *Prior Analytics*, Aristotle makes a distinction between ὄντα that are predicated of nothing and ὄντα that may be predicated of something.¹⁴ One might take this

distinction as reformulating the distinction between καθόλου and καθ'ἑκαστον of the *On Interpretation*, and so one might point out that in the *Analytics* universality and particularity are attributed not to πράγματα, but to ὄντα, things which are. So “πράγματα” and “ὄντα” should be taken as semantically equivalent. But it looks odd to say that what is meant by “goat-stag” is an ὄν, because its status is more properly described as a μὴ ὄν. Therefore, we cannot qualify it as a πράγμα either.

I must confess that I am not impressed by this difficulty. As a preliminary and general answer, I would say that to qualify what is meant by a word, or what is the subject of a predication as ὄν, does not necessarily imply an odd Meinongian ontology or a departure from our everyday way of speaking. Therefore, I am inclined to think that if Aristotle uses the word “ὄν” where “πράγμα” might be expected, and “πράγμα” is supposed to mean something nonverbal, there is no reason to conclude that this meaning has to be revised or that it is incompatible with its equation to “ὄν.” This answer is based on the idea that “ὄντα” does not necessarily mean things that exist. However, in the particular case of the *Prior Analytics*, one might also maintain that “ὄντα” has an existential meaning. As we will see in a moment, in this text Aristotle is not concerned with a general distinction of types of terms as in the treatise *On Interpretation*. Rather, his point is connected to a thesis that is proven in the *Posterior Analytics*, namely that a series of predicates ἐν τῷ τί ἐστι terminate up and down. It is about the elements of such series, or συστοιχίαι, that Aristotle is speaking here. It is therefore quite natural that he uses the word “ὄντα” to designate these elements. They are terms of which there are always instances that are parts of the furniture of the world. In this sense what is indicated by “goat-stag” is a πράγμα but not an ὄν, whereas what is expressed by “man” is both a πράγμα and an ὄν.

III

Let us turn to the second of the problems listed at the beginning. Can we really interpret the distinction between καθόλου and καθ'ἑκαστον as a distinction between kinds of predicates, namely predicates that may be true of many individuals and predicates that are necessarily true of only one individual? To answer this question one may consider passages such as the following one:

(B) For in some cases learning occurs in this way, and the last term does not become familiar through the middle – in cases dealing with what are in fact particulars and not said of any underlying subject (μὴ καθ'ὑποκειμένον τινός).

(Aristotle, *Posterior Analytics* 1.1, 71a21–24; ROT translation)

I think that it would be hazardous to conclude from this passage that the Neoplatonic interpretation must be rejected. The expression “μὴ καθ'ὑποκειμένον τινός,” to which “λέγεται” must be added, immediately reminds us of the distinction that is found in the *Categories* between “καθ'ὑποκειμένον τινὸς λέγεσθαι” and “ἐν

ὑποκειμένῳ τινὶ εἶναι.” This association is reinforced by the definition of ἄτομα, another name for καθ’ ἕκαστα, that is found there:

(C) Things that are individual and numerically one are, without exception, not said of any subject (καθ’ οὐδενὸς ὑποκειμένου), but there is nothing to prevent some of them from being in a subject – this particular grammatical knowledge is one of the things in a subject.

(Aristotle, *Categories* 1, 1b6–9; ROT translation, slightly modified)

It is clear from the fact that an individual *a* is not said of any subject we are not allowed to infer that *a* is not true of anything, that is, is predicated of nothing, because “*a* is said of *b* as of a subject” is not the same as “*a* is predicated of *b*.” Evidence for this claim is offered by the way in which Aristotle conceives of the distinction between καθ’ ὑποκειμένον τινὸς λέγεσθαι and ἐν ὑποκειμένῳ τινὶ εἶναι. He claims that it may be that *a* is in *b* as in a subject and *a* be such that it is predicated of *b*.¹⁵ Therefore, there are things that are not said of something as of a subject and that are in a subject, and this situation does not prevent them from being predicated of the subject in which they are. The conclusion is that we can extract no decisive evidence from texts (B) and (C) against the Neoplatonic view.

A passage of the *Prior Analytics* that we have already quoted looks more promising. Aristotle says:

(D) (a) Of all things which are some are such that they are not predicated of anything else truly and universally (as, for instance, Cleon and Callias, i.e., the particular and sensible), (b) but other things are predicated of them (for each of them is both man and animal); (c) some things are themselves predicated of others, but nothing prior is predicated of them; (d) some things are predicated of others, and yet others of them, e.g., man of Callias and animal of man. (e) It is clear that some things are by their nature said of nothing, (f) for as a rule, each sensible thing is such that it is by its nature predicated of nothing, (g) save incidentally; for we say that that white object is Socrates and that that which approaches is Callias.

(Aristotle, *Prior Analytics* 1.27, 43a25–36; ROT translation, modified)

On a first reading, one might be tempted to say that it is pretty clear that a singular term such as Cleon or Callias cannot be predicated of anything and that the Neoplatonic interpretation must be rejected. But things are not so easy. The predication that is alluded to here is not any predication, but rather a generic or ἐν τῷ τί ἐστὶ predication; otherwise, it would be impossible to understand how it may happen that there are predicates of which nothing can truly be said – that is, the categories – as most interpreters think. Needless to say, we can truly attribute many predicates to a category, but these predicates cannot play the role of a genus or the role of an ἐν τῷ τί ἐστὶ predicate. For instance, I can say of a category that it is one, but *one* is not the genus, or part of the genus, of the category. In other words, we have to do here with series of terms, each of

which has a wider extension with respect to its subordinates and is connected by an essential relation to them. According to Aristotle, a series of such terms is finite and terminates in both directions. The top element is constituted by a category and the bottom term by a singular term. Now it is true that Aristotle in clause (e) asserts that there are terms that cannot be predicated of anything, and in (f) he adds that these terms are individual terms. However, a little reflection shows that this claim is not sufficient to reject the Neoplatonic view about singular terms. It may be that Cleon is predicated of nothing as a genus, but this is consistent with the view that Cleon is uniquely predicated of one particular. This squares with the last part of the passage where Aristotle says that in a proposition such as

(3) the approaching object is Cleon

the predicate *Cleon* is incidentally predicated of its subject, because this statement may be interpreted as the claim that *Cleon* is not predicated ἐν τῷ τί ἐστι of its subject. Moreover, (a) has a pretty natural interpretation in the Neoplatonic view. The expression “κατὰ μηδενὸς ἄλλου κατεγορεύσθαι ἀλεθῶς καθόλου” is difficult. It is plausible to take καθόλου as an adverb modifying κατεγορεύσθαι. According to the terminology of *On Interpretation*, “καθόλου ἀποφάναι ἐπὶ τοῦ καθόλου” qualifies the situation of a predicate that is attributed affirmatively or negatively to a universal subject universally quantified.¹⁶ Of course, here “καθόλου κατεγορεύσθαι” is not referring to the universal quantification of a proposition, and therefore one might reasonably claim that it indicates what is predicated as a universal of a subject. In this perspective it becomes natural to take ἀλεθῶς as reinforcing καθόλου, so the point would be that there are things that cannot be predicated of something else as a true universal, namely as something that really is or may be true of many. Once more, the Neoplatonic proposal is safe: A singular term, although it is a predicate, is not really true of something as a proper universal is because it cannot be true of many.

I do not claim that this interpretation is the correct one. My point is only that it is consistent with the Neoplatonic view and there are no obvious elements in the passage to disprove it.

IV

Our third problem concerns the relation between universal terms and predicates. On the one hand, thinking of predicates such as *different from itself*, which can be true of nothing, brings us to the view that not every predicate is universal, so that the class of καθόλου terms is a subclass of the class of predicates. On the other hand, it is clear that a predicate such as *different from itself* cannot be considered a καθ'ἑκάστων either, because it would be odd to put Callias and predicates true of nothing in the same class. However, the Aristotelian distinction seems to be thought of as exhaustive: A πρᾶγμα is either universal or particular. *Tertium non datur*. But *different from itself* is a πρᾶγμα, because the expression “different from

itself” has a meaning that is easily understood by everybody. Therefore, it is a $\pi\rho\tilde{\alpha}\gamma\mu\alpha$ that is neither universal nor singular.

To make the difficulty clearer, let us consider again the meaning of the expression “πέφυκε κατηγορεῖσθαι,” which is part of the Aristotelian definition of $\kappa\alpha\theta\acute{o}\lambda\omicron\upsilon$. So far we have taken it as implying a modal commitment in the sense that we have assumed that if a πέφυκε κατηγορεῖσθαι of b , then a may be true of b . It was exactly this assumption that made a predicate such as *different from itself* not a $\kappa\alpha\theta\acute{o}\lambda\omicron\upsilon$. To see the point, we may translate Aristotle’s definition of $\kappa\alpha\theta\acute{o}\lambda\omicron\upsilon$ as follows:

$$(4) \quad U(a) \leftrightarrow \Pi \exists x \exists y (x \neq y \wedge K(a, x) \wedge K(a, y))^{17}$$

where, as it is easy to guess, “ $U(x)$ ” stands for “ x is universal,” “ $K(x, y)$ ” for “ x is (affirmatively) predicated of y ,” and “ $\Pi(A)$ ” for “it is by nature that A .” We can safely suppose that

$$(5) \quad \Pi(A) \rightarrow \Diamond A.$$

The reason is that in a well-known passage, Aristotle explicitly states that $\tau\acute{o}$ πεφυκὸς ὑπάρχον is a special kind of ἐνδεχόμενον, namely of possibility.¹⁸ Therefore, from (4) and (5), we can easily derive

$$(6) \quad U(a) \leftrightarrow \Diamond \exists x \exists y (x \neq y \wedge K(a, x) \wedge K(a, y))$$

and can conclude that *different from itself*, as well as all predicates of this sort, cannot be taken to be a $\kappa\alpha\theta\acute{o}\lambda\omicron\upsilon$.

There is something strange in this reasoning, because the logical status of these neither universal nor particular predicates is unclear. I propose to make a fresh start. Let us forget, for a moment, predicates such as *different from itself* or *goat-stag*. Let us consider, instead, *man* or *animal*. First of all, because in a proposition such as (3) Aristotle considers *Cleon* a predicate, we must conclude that in his view not all predicates are $\kappa\alpha\theta\acute{o}\lambda\omicron\upsilon$. What seems to make a predicate a $\kappa\alpha\theta\acute{o}\lambda\omicron\upsilon$ is its possibility of having instances, which is ruled out for those predicates that can take the role of predicates only in identity propositions. Therefore, what makes a predicate have instances is the fact that it is true or can be true of at least two different subjects. From this point of view, *man* is a universal because it is true of Callias and Cleon, who are different entities, and *Cleon* is a particular because it has no instance.

If we adopt this interpretation, we are in a position to evaluate the Neoplatonic proposal. Suppose that what makes a predicate a $\kappa\alpha\theta\acute{o}\lambda\omicron\upsilon$ is its having (possible) instances. It is reasonable to infer that a particular is primarily characterized by not having instances, namely not being true of different subjects. That a particular can play the role of predicate only in an identity proposition may be taken as a consequence of its not having instances, but this is not something that specifies the nature of its being particular. In other words, it is because it has no instance that

a particular is a particular and not because it is true of only one subject. Therefore, the Neoplatonic interpretation is not required by the text, and it represents a development of Aristotle's view, which is interesting and not inconsistent with it. In particular, there is no reason for attributing to Aristotle a reductionist view about proper names because they do not need to be treated as a special kind of predicates.

Let us now return to *goat-stag* or *different from itself*. As far as I can see, there are at least two ways to deal with the difficulty of being compelled to admit empty terms that are neither universal nor particular. The first one consists in avoiding (6) by interpreting “πέφυκε κατηγορεῖσθαι” as implying a sort of conditional necessity. We may suppose that by saying that a universal term *a* is such that it is by nature true of many Aristotle means that *a* is such that, if it is true of something, then it may be true also of something else. Therefore, we can state:

$$(4^*) \quad U(a) \leftrightarrow \forall x(K(a, x) \rightarrow \Diamond \exists y(x \neq y \wedge K(a, y))).$$

On the one hand, one might easily claim that not only *goat-stag* but also *different from itself* satisfy this definition of καθόλου because the antecedent of the implication is by hypothesis false. On the other hand, no singular term can be placed among the universals because *Cleon* is true of something, namely the approaching object, but it cannot be true of anything different from Cleon.

I have two worries about this analysis. The first is that if the definition of particularity is given by negation of the second element of equivalence (4*), it is difficult to avoid the Neoplatonic interpretation. A singular term is exactly what is true of only one thing. This is not a real difficulty because the reason we have put forward to reject the Neoplatonic view is quite weak, and we have seen that the Aristotelian texts do not rule it out. The second is that Aristotle allows identity to hold not only between individuals but also between general terms. For instance, the relation between a *definiendum* and its *definiens* is an identity relation.¹⁹ But as we have seen, an element of an identity relation is taken by Aristotle as a predicate of the other. One might consider any description whatsoever of an impossible term to state that this term is true of it and conclude that the term in question must be taken as a particular. Suppose, for instance, that *b* is a description of being different from itself. Then, *being different from itself* is true of *b*. Therefore, the antecedent of the right side of equivalence (4*) is fulfilled, whereas its consequent cannot be. Thus, *being different from itself* must be placed among the particulars, and our initial uneasiness returns.

I would like to propose a bolder thesis, namely that not only *man* and *animal* but also *goat-stag* and *different from itself* are universal because all of them *do* have instances. This view, of course, requires an explanation because everyone is ready to say that *goat-stag* has no instance in the present world and *different from itself* in every possible world. However, suppose that $a \neq a$ and $b \neq b$. In these cases we are allowed to say both that *a* is different from itself and *b* is different from itself and place *different from itself* among the universals. It is sufficient that we can conceive of instances of *different from itself* to conclude that *different from itself* has instances. It does not matter whether these instances exist in this

or in a possible world, and it is not even relevant whether these instances cannot exist at all. The important point is that we can think of them. Even if we all agree that nothing can be different from itself, I do not think that we can deny that $a \neq a$ and $b \neq b$ are special cases of things different from themselves, no more and no less than we all are ready to admit that $P \wedge \neg P$ and $(P \rightarrow Q) \wedge \neg(P \rightarrow Q)$ are two cases of contradiction. Although we agree that no contradiction is true, we understand quite clearly what a contradiction in general is and we recognize its instances. The same cannot happen with particulars. I cannot think of an instance of Cleon because it would be a category mistake to imagine it. I do not claim that this proposal is clear. I will come back to it later. For the moment, let us consider other parts of our chapter.²⁰

V

A different question is raised by 17b12–16, where Aristotle takes a position with regard to the possibility of quantifying the predicate in a proposition. The text is unfortunately doubtful. In the critical edition established by Lorenzo Minio Paluello, it reads as follows:

(E_m) (a) ἐπὶ δὲ τοῦ κατεγορευμένου τὸ καθόλου κατηγορεῖν καθόλου οὐκ ἔστιν ἀλεθές: (b) οὐδεμία γὰρ κατάφασις ἔσται, (c) ἐν ᾗ τοῦ κατεγορευμένου καθόλου τὸ καθόλου κατεγορηθήσθαι,²¹ (d) οἷον ἔστι πᾶν ἄνθρωπος πᾶν ζῷον.

For clause (a), there is a different version adopted by both Emmanuel Bekker and Theodor Waitz. In this version, the article τό is moved from the first to the second καθόλου. Moreover, (b) is differently read by Bekker and Waitz, because they add ἀλεθής after κατάφασις. Therefore, the Bekker-Waitz text is:

(E_b) (a) ἐπὶ δὲ τοῦ κατεγορευμένου καθόλου κατηγορεῖν τὸ καθόλου οὐκ ἔστιν ἀλεθές: (b) οὐδεμία γὰρ κατάφασις ἀλεθής ἔσται, (c) ἐν ᾗ τοῦ κατεγορευμένου καθόλου τὸ καθόλου κατεγορηθήσθαι, (d) οἷον ἔστι πᾶν ἄνθρωπος πᾶν ζῷον.

Finally, Hermann Weidemann has proposed a new reading of our passage based on Stephanus and Boethius, which is as follows:²²

(E_w) (a) ἐπὶ δὲ τοῦ κατεγορευμένου καθόλου τὸ καθόλου κατηγορεῖν καθόλου οὐκ ἔστιν ἀλεθές: (b) οὐδεμία γὰρ κατάφασις ἀλεθής ἔσται, (c) ἐν ᾗ τοῦ κατεγορευμένου καθόλου τὸ καθόλου <καθόλου> κατεγορηθήσθαι, (d) οἷον ἔστι πᾶν ἄνθρωπος πᾶν ζῷον.

It is clear that if we adopt either (E_m) or (E_w) for (a) we must take “κατεγορευμένου” as a passive voice, “that of which something is predicated,” so that the expression indicates the subject of a proposition and not, as usual, the predicate. Although rare, this use is not without parallels in the Aristotelian corpus, and there is at least one sure example in *Prior Analytics* 1.32, 47a40–b2. This choice has the

advantage that it fits better with the way in which Aristotle makes his point in the previous lines. Universal predication is expressed by him by the formula “ἐπὶ τοῦ καθόλου ἀποφαίνεσθαι καθόλου,”²³ which corresponds to some extent to “ἐπὶ δὲ τοῦ κατεγορευμένου τὸ καθόλου κατηγορεῖν καθόλου” in (E_w). This is an argument for abandoning the (E_b) reading for (a).

I think that there are also good reasons for rejecting the suggestion made by Weidemann. First of all, the addition of καθόλου in (c), which is required by his choice in (a), has no evidence in the manuscript tradition. Moreover, the authority of the commentators is not so strong. Stephanus adds “καθόλου” in the pericope of (a),²⁴ but in the pericope of (c) he makes no mention of “καθόλου.”²⁵ We find the same situation in Boethius.²⁶ On the other hand, Ammonius’s pericope favors the Bekker–Waitz reading²⁷ and no trace of καθόλου is found in (c).²⁸ Apart from the dubious status of the pericopes, in this particular case I would be hesitant to confer a great weight to the testimony of the commentators because they are clearly uncomfortable with the unusual meaning of “κατεγορευμένου” in (a), which they tend to understand in the sense of “predicate.”²⁹

Moreover, and most importantly, by adopting Weidemann’s reading of (a), the kind of propositions that Aristotle for some reason would have rejected would be propositions universally quantified in the subject and in the predicate, as the example mentioned in (d) is. In other words, by adopting the (E_w) reading Aristotle would have ruled out all propositions of the form:

(S*) every --- is every +++

where “---” and “+++” may be substituted for general terms. On the other hand, according to (E_m), all propositions of the form

(S) ••• is every +++

would be rejected, where “•••” stands for any appropriate subject, namely a universally quantified general term or a singular term.³⁰ Now in a parallel passage where Aristotle takes once more a position against propositions with universally quantified predicates, he mentions, together with the example of (d), an example where the subject is not quantified

justice is every good.³¹

This makes it plausible to think that the propositions he rejects here are of the form of (S) and not only those of the form of (S*). Consequently, (E_m) appears preferable to (E_w) as far as (a) and (c) are concerned.

There is a more crucial difference in the various readings of our text. This difference concerns (b). As we have seen, both (E_b) and (E_w) add ἀλεθής to κατάφασις on the basis of Ammonius’s commentary. This addition makes a relevant difference, because if we read (b) in this way, it is quite clear that Aristotle states that all propositions of the form of (S) are false. On the other hand, if we adopt text (E_m), it does not follow

necessarily that propositions such as (S) must be considered false. In the (E_m) perspective it depends on the way we read (a) and we interpret “οὐκ ἔστιν ἀληθές.” We may take this expression to mean that the result of attributing a predicate universally quantified to a subject is false and therefore that all propositions of this type are false. But we may also interpret it in the sense that the act of attributing a universally quantified predicate to a subject is false, and this may mean that such an act is incorrect for some reason. In this case, the propositions resulting from this attribution are not true or false but rather ill-formed or ungrammatical. This hypothesis is, of course, ruled out if we read ἀληθής in (b) because in that case the ambiguity of (a) would be solved in the sense of the falsity of the propositions with universally quantified predicates.

Although there is a tendency to privilege the testimony of the commentators against the manuscript tradition, I do not think that it is a good idea to do so in this particular case. Ammonius has a very long discussion of propositions with quantified predicates. He considers all possible cases – that is, not only universally but also particularly quantified propositions. His view is that all affirmative propositions in which the predicate is universally quantified are always false, whereas the corresponding negative propositions in which the predicate is existentially quantified are always true but useless. This view is probably based on the idea that every general term must be true of at least two individuals.³² On the one hand, there is no evidence that Aristotle would have shared this way of thinking of universals and predication. On the other, it is easy to guess that Ammonius wanted to get rid of some developments of the logic of the quantification of predicates elaborated despite the Aristotelian condemnation. Therefore, Ammonius and the Neoplatonists read Aristotle with an eye to the subsequent debate on the quantification of the predicates, and this may have brought them to interpret him in light of this more recent discussion. For this reason, I think that we must take the evidence coming from the commentators with a critical attitude, and I prefer to keep to Minio-Paluello’s text. This makes the Aristotelian doctrine ambiguous.

There is, however, another passage where Aristotle criticizes the quantification of a predicate, which goes as follows:

(F) That which follows one must not suppose to follow as a whole, e.g. that every animal follows man or every science music, but only that it follows without qualification, as indeed we state it in a proposition. For (a) the other statement is useless and impossible, (b) e.g., that every man is every animal or justice is every good.

(Aristotle, *Prior Analytics* 1.27, 43b17–21; ROT translation)

“That which follows” (τὸ ἐπόμενον) is the predicate of a proposition, and “to follow as a whole,” (ὅλον ἔπασθαι) means that the predicate is universally quantified and attributed to a subject. We have given the Oxford translation of the passage. The Greek for (a) and (b) is:

(F_g) (a*) καὶ γὰρ ἄχρηστον θάτερον καὶ ἀδύνατον, (b*) οἷον πάντα ἄνθρωπον εἶναι πᾶν ζῷον ἢ δικαιοσύνην ἅπαν ἀγαθόν.

Here the text is not disputed but is nonetheless ambiguous. A first point concerns the meaning of “θάτερον” in (a*). One might suppose that it means something like “the second way of choosing the predicates,” namely as universally quantified, in contrast with the former way of selecting them, namely without quantification at all. In this case, it does not follow from what Aristotle says that the propositions in which the predicate is universally quantified are impossible, and thus always false. But “θάτερον” could also be taken to mean “the second way of quantifying a proposition” in contrast with the normal way of putting the quantifier only near the subject. In this case it is the operation of quantifying the predicate that is said to be useless and impossible. To understand what this claim means, we must consider “οἷον” at the beginning of (b*). This word may simply be taken to introduce an example of propositions in which the predicate is universally quantified in order to show what the result is if one performs the meaningless act of quantifying the predicate. In this case, it does not follow that what results from an impossible operation is in itself impossible, in the sense that it has a truth value that is always false. But it may be also taken to exemplify the uselessness and impossibility of the act of quantifying universally the predicate in the sense of showing that what results from this act is in itself useless and impossible. In this case, we should conclude that “ἀδύνατον” has the usual modal meaning and that the propositions themselves are impossible, that is, never true.

Although the tradition, as we have seen, has not hesitated to interpret Aristotle’s words as implying that the propositions of the form of (S) are impossible, this is not the only way to present his view. Surely Aristotle rejected propositions of the form of (S), but the dubious point concerns the reasons that may have led him to make such a move. According to one reading of texts (E) and (F), (S) is ruled out because its instances are not propositions in the sense that they infringe on the usual way in which we state propositions, or at least the propositions on which the Aristotelian logic is based. But, on the other hand, there is no good reason to deny that (S) is dropped by Aristotle because all its instances are false, although this statement is far from clear, especially if one thinks that according to this interpretation the instances of (S) are said to be useless in text (F). A contradiction is surely always false, but it is not useless in arguments and demonstrations. But there may be a more charitable way of taking Aristotle’s words. Perhaps the rejection of the instances of (S) does not amount to ruling them out of logic or condemning them to a useless limbo, but simply to a warning that they must not be considered capable of expressing true propositions and telling us something about the world, as is the case with normal elementary propositions.

VI

Our chapter contains the beginning of the theory that the tradition has called the doctrine of the logical square. It concerns the logical relations between the four kinds of propositions that constitute the core of Aristotle’s logic, namely universal and particular affirmative propositions (*a*- and *i*-propositions) and universal and particular negative propositions (*e*- and *o*-propositions).

On the one hand, *a*- and *o*-propositions and *e*- and *i*-propositions are said by Aristotle to be opposed in a contradictory way (ἀντιφαντικῶς), and they are

usually called contradictory propositions. On the other hand, *a*- and *e*-propositions are opposed in a contrary way (ἐναντίως), and they are called contrary propositions. Aristotle does not assign a name to the pair of *i*- and *o*-propositions. The tradition has called them “subcontraries,” and a similar gap has been filled in by calling an *i*-proposition the subaltern of the corresponding *a*-proposition; the same holds for an *o*-proposition with respect to its *e*-proposition.

Aristotle states the rules that are supposed to define from a formal point of view the logical status of these pairs. As far as contradictory propositions are concerned, he says:

(G) Of contradictory statements about a universal taken universally it is necessary for one or the other to be true or false.

(*On Interpretation* 7, 17b26–27; ROT translation)

The point is clear and simple. We may put it this way: If one of two contradictory propositions is true, the other is false. For the sake of simplicity, we can drop any reference to the truth predicate and say that if one of two contradictory statements is the case, the other is not the case. Formally, we have:

(CT₁) $AaB \vdash \neg AoB$

(CT₂) $\neg AoB \vdash AaB$

(CT₃) $AeB \vdash \neg AiB$

(CT₄) $\neg AiB \vdash AeB$.

The situation of contrary propositions is different:

(H) I call the universal affirmation and the universal negation contrary opposites, e.g. every man is just – no man is just. So these cannot be true together, but their opposites may both be true with respect to the same thing, e.g., not every man is white – some man is white.

(Aristotle, *On Interpretation* 7, 17b20–26; ROT translation)

By adopting the same simplification we have used before, we may restate Aristotle’s point as follows: If one of two contrary propositions is the case, the other is not the case, but not vice versa. Formally, we have:

(CR₁) $AaB \vdash \neg AeB$

(CR₂) $AeB \vdash \neg AaB$.

On the other hand, the fact that the opposite of two contrary propositions may be true together implies that one of two contrary propositions does not follow from the negation of the other:

(CR₃) $\neg AaB \nvdash AeB$

(CR₄) $\neg AeB \nvdash AaB$.

As is easy to see, (CR₃) and (CR₄) make contrary propositions different from contradictory propositions. To make the point in Aristotelian terms, whereas a pair of contradictory propositions cannot be both true and both false, contrary propositions cannot be both true but they may be both false.

Aristotle does have something to say also about subaltern propositions in the *Topics*. He states:

(I) The methods of establishing and overthrowing a view universally are common to both kinds of problems <sc. universal and particular problems>; for when we have proven that a predicate belongs in every case, we shall also have proven that it belongs in some cases. Likewise, also, if we prove that it does not belong in any case, we shall also have proven that it does not belong in every case.

(Aristotle, *Topics* 2.1, 109a1–6; ROT translation, slightly modified)

This passage is commonly taken as evidence of Aristotle's acceptance of what the tradition has called the rules of subalternation:

- (SU₁) $AaB \vdash AiB$
 (SU₂) $AeB \vdash AoB$.

To be precise, what Aristotle says is less general. His claim is that if we have proven that a universal proposition holds, we have also proven that the corresponding particular holds, and this may be taken to mean:

- (SU₁*) $\vdash AaB$
 $\hline \vdash AiB$
 (SU₂*) $\vdash AeB$
 $\hline \vdash AoB$.

However, we might think that he would have accepted (SU₁) and (SU₂) for the same reason that led him to state (SU₁*) and (SU₂*). Moreover, it is easy to see that (SU₁) and (SU₂) can easily be derived from (CR₁) and (CR₂) by means of the contrapositives of (CT₄) and (CT₂).³³

To complete the doctrine of the logical square, we should also state the rules that govern subcontrary propositions. What Aristotle says about these propositions is contained in text (H) and consists of the statement that the opposites of two contrary propositions can be true together. A way of making this point is by asserting (CR₃) and (CR₄).³⁴ An equivalent assertion is that two subcontrary propositions can be true together, and this means:

- (SC₁) $AiB \nvdash \neg AoB$
 (SC₂) $AoB \nvdash \neg AiB$.

Aristotle does not express a positive view about this kind of proposition, but it is easy to see that the rule concerning them is implied by (CR₁) and (CR₂). It is also easy to derive from them

$$(SC_3) \quad \neg AiB \vdash AoB$$

and

$$(SC_4) \quad \neg AoB \vdash AiB.$$

In this way, the whole traditional doctrine is completed.³⁵

VII

It has been pointed out that the Aristotelian doctrine of the logical square is inconsistent with the standard first-order predicate calculus, and possibly with Aristotle himself.³⁶ To see the point consider, for instance, the rules of subalternation, namely (SU₁) and (SU₂), and interpret AaB , the universal affirmative proposition, as

$$(8) \quad \forall x(B(x) \rightarrow A(x)).$$

Formula (8) means the same as

$$(9) \quad \neg \exists x(B(x) \wedge \neg A(x)).$$

Saying, for instance, that every man is mortal is taken to mean that there is no immortal man. It follows immediately that if B is true of nothing or denotes an empty class,³⁷ the proposition saying that every B is A is true, because if there is no B , then there is no B that is not A either. Therefore, we have a counterexample to the subalternation rules. Take, for instance, the proposition

$$(10) \quad \text{every witch is nasty.}$$

Because we are all inclined to believe that there are no witches, (10) is true. But, for the same reason, it cannot be true that

$$(11) \quad \text{some witch is nasty}$$

because we have agreed that witches do not exist. In this way, we obtain a counterexample to the first subalternation rule because in at least one case its premise is true and its conclusion is false. Because the rules for contrary and subcontrary propositions are interderivable with the rules of subalternation, if the latter are not valid, the former cannot be valid either. The whole theory of the logical square is destroyed with the only exception of the contradiction rules.

Of course, in order to create the counterexample, it is not relevant whether or not we agree about the nonexistence of witches. Two important assumptions are necessary to generate the counterexample. The first is that we must allow the possibility that there are terms in our language that are true of nothing or refer to empty classes. In the standard semantics for first-order predicative calculus, the point is clear. In each model with respect to which we evaluate a formula, we have a domain of individuals on which the individual variables range and subsets of the domain to which predicative letters can be associated. Not every subset is supposed to have members. Therefore, predicative letters can be associated with empty subsets of the domain. Although one might think that the distinction between what is real and what is unreal is to some extent arbitrary and depends on the context, it is difficult to deny that there is such a distinction and that our language has terms to address nonexistent entities.

The second assumption necessary to generate the counterexample is that we interpret expressions such as “some” as implying an existential commitment. If we take (11) to assert that there exists at least one nasty witch, it is clear that we must admit that it is a false assertion, because witches by hypothesis do not exist. Because universal negative propositions are negations of particular affirmative propositions, if the latter have an existential import, the former must deny such an import. Therefore, universal propositions have no existential commitment. In this way, an asymmetry is created between universal and particular quantification, and precisely this asymmetry is responsible for invalidating the rules of the logical square.

Connected with this way of thinking of particular quantification is the idea that existence is not a predicate. This is a dogma of the philosophy of logic of the twentieth century. As Quine has put the argument: “To say that something does not exist, or that there is something which is not, is clearly a contradiction in terms.”³⁸

The argument is obviously correct on the assumption that “something” means “an existing x .” Then to say that something does not exist implies that there is an existing x that does not exist. The Quinean solution is that the statement that a exists must be taken to mean that there is something that is the same as a , namely

$$(12) \quad \exists x(x = a).$$

According to the famous slogan that to be is to the value of a variable.³⁹

It may be interesting to observe that the doctrine of the logical square is sometimes said to be inconsistent not only with modern logic but also with the views of Aristotle himself, because he seems to accept both the presuppositions that make the counterexample possible. On the one hand, Aristotle surely admits the possibility of speaking of what is not: “τραγέλαφος” – “goat-stag” – is one of his standard examples for an empty term. On the other hand, his definition of universal predication suggests an interpretation in terms of negated existence. At the beginning of the *Prior Analytics*. Aristotle says:

(J) We say that one term is predicated of all of another, whenever nothing of the subject can be found of which the other term cannot be asserted. “To be predicated of none” must be understood in the same way.

(Aristotle, *Prior Analytics* 1.1, 24b28–30; ROT translation)⁴⁰

This statement is taken to mean that there is no x which is B and not A . This is supposed to imply that no existential import is assigned to universal propositions. Therefore, the logical form of a universal Aristotelian proposition is adequately represented by (12), and particular propositions must carry an existential commitment, being the negations of the corresponding universal. It seems to me doubtful that text (J) offers evidence for such an interpretation of predication. We will see that it is instead consistent with other possible views about quantification and that there are other possible interpretations of Aristotle’s position.

However, the attempts that have been made to recover the Aristotelian doctrine of the logical square consist normally in eliminating the first presupposition that generates the counterexample, because it is the possibility of empty terms that is taken to be inconsistent with the Aristotelian theory. The most elegant way of embedding this idea in a formal system has been presented by Timothy Smiley, who uses a many-sorted system.⁴¹ There is no need to present Smiley’s system here. What is important is to stress that this system is able, in a logically sound way, to represent the doctrine of the logical square. But, of course, there is a price to pay for such a nice result: We have to restrict the expressive power of our language. We cannot speak of witches and goat-stags, of elves and unicorns, and our logic is no longer able to account for square-circles or mountains bigger than the Everest. In this sense, Aristotelian logic might no longer be presented as an attempt to formalize arguments of natural language, only a small number of which are reducible to the formal patterns of this logic.

VIII

What worries me about this way of putting things is the feeling that Aristotle cannot be completely placed in the frame of the presuppositions that govern the standard approach to quantification. In a system where (SU_1) and (SU_2) hold, one should expect that the same happens with a weaker statement such as

(EC) $\Phi(a) \vdash \exists x(x = a)$

where “ $\Phi(a)$ ” is supposed to stand for any formula containing a . In standard logic, (EC) is supposed to hold also when (SU_1) and (SU_2) do not, and this means that if there are reasons to believe that (SU_1) and (SU_2) are valid, there are more reasons to do the same for (EC).

This expectation is not fulfilled by Aristotle’s system. In a well-known passage, Aristotle appears to reject the validity of (EC):

(K) (a) It is true to speak of the particular case even without qualification; e.g., to say that some particular man is a man or some particular white man

white. Not always, though. (b) When in what is added some opposite is contained from which a contradiction follows, it is not true but false (e.g., to call a dead man a man); (c) but when no such opposition is contained, it is true. (d) Or rather, when it is contained it is always not true, but when it is not, it is not always true. (e) For example, Homer is something (say, a poet). Does it follow that he is? (f) No, for the “is” is predicated accidentally of Homer; for it is because he is a poet, not *per se* (καθ’αυτό), that the “is” is predicated of Homer. (g) Thus, where predicates both contain no contrariety if definitions are put instead of names and are predicated *per se* (καθ’αυτά) and non-accidentally, in these cases it will be true to speak of the particular thing even without qualification.

(Aristotle, *On Interpretation* 11, 21a18–32; ROT translation, modified)

The passage is controversial but perhaps less than some interpreters have thought. In (a) the main thesis is put forward; it may happen that the component parts of a composite predicate are separately true of the subject of which the composite predicate is true. As Aristotle says, if it is true to say that that *a* (this particular man) is a white man, it is also true to say that *a* is a man and that *a* is white. By generalizing the point, we can say that

(DT) $a \text{ is } F + G \vdash a \text{ is } F \text{ and } a \text{ is } G$

is satisfied in some cases but not in all.⁴² What follows the main thesis is supposed to make it clear in what cases (DT) can and in what cases it cannot be fulfilled. Sentence (b) expresses a case in which (DT) cannot be satisfied. It is the case in which *F* and *G* are such that they imply an inconsistency. As is explained in (g), the implied inconsistency is revealed if their definitions are substituted for *F* and *G* and the definitions are inconsistent. The example mentioned by Aristotle is clear. The proposition

(13) $a \text{ is a man}$

does not follow from

(14) $a \text{ is a dead man.}$

In fact, the definitions of man and dead are inconsistent. The idea behind this claim is probably that because the definition of “man” is inconsistent with the definition of “dead,” the meaning of “man” in “dead man” is not the standard one so that we are not allowed to detach “man” from “dead man” in predication. Clause (c) seems to suggest that inference (DT) is allowed in all cases in which there is no inconsistency between the parts of a composite predicate. But this view is promptly corrected by (d), where it is said that if there is an inconsistency it never happens that (DT) is satisfied, whereas in the cases in which there is no such inconsistency it *may* happen that it is satisfied. Therefore, the point is that

(DT) does not hold in general even if there is no inconsistency between the parts of composite predicates. With (e) the critical part of our text begins. Aristotle considers the proposition

(15) Homer is a poet.

He wonders whether from this proposition it follows that

(16) Homer is.

Clause (f) makes it clear that the answer is negative. Therefore, it is safe to infer that proposition (15) is meant to be an example of a proposition with a composite predicate whose parts are not conflicting and not fulfilling (DT). It may be strange to consider “is a poet” as a predicate with as many parts as “white man” or “good cobbler.” But this is what we are compelled to admit if we read clause (e) in the context of Aristotle’s argument. The reason put forward by Aristotle to disprove the passage from (15) to (16) is contained in (f) and it is the more difficult part of the whole passage. What he seems to claim is that “is” in “Homer is a poet” is attached only indirectly to “Homer” through “poet” so that it cannot be attributed to him in its own right. In other words, Aristotle’s idea appears to be that we cannot detach a part of a compound predicate when this part can be attributed to a subject only because it is a part of that whole. “Is” is attributed to “Homer” only as a part of the complex “is-a-poet.” Therefore, there is no reason to claim that it can be attributed to “Homer” on its own.⁴³

I do not expect everyone to agree on all details of this interpretation. What is relevant for our discussion is the claim, which seems to me indisputable, that (15) is meant to represent a counterexample to (DT). How has a counterexample to a scheme of inference to be construed? By offering a case in which the premises are true and the alleged consequent is false. This is the procedure used by Aristotle himself in order to show that some combinations of premises do not yield a syllogistic conclusion. Therefore, we must suppose that, in Aristotle’s view, (15) is true and (16) is false. Thus, if we admit that “is” in “Homer is” is supposed to have an existential meaning, and we interpret it according to the pattern offered by (15), our passage states that

(17) $F(a) \vdash \exists x (x = a)$

does not hold in general.⁴⁴ But we cannot at the same time reject the validity of (17) and accept (EC). Thus, it would be very strange that Aristotle, in order to validate the subalternation laws, had refused to admit empty terms and, at the same time, had rejected (EC).

One might react to this argument in several ways. The easiest consists in denying that the “is” that is at issue in text (K) is the “is” that is expressed, as in (12), in the standard interpretation. One might distinguish temporal from pure existence and claim that “Homer” in “Homer is a poet” refers to something that has pure

existence, being an element of the domain of individuals, although it does not exist at the moment when the proposition is uttered. But it is not in the spirit of the supporters of the traditional interpretation to distinguish between different senses of existence, and no such distinction is mentioned or sketched by Aristotle. On the other hand, what immediately follows text (K), and will be discussed later on, appears to imply that “Homer” in a proposition such as (18) has to be taken as referring to a nonexistent entity and not to an existing entity that is not actualized at the moment it is uttered.⁴⁵

There is a more complicated way to resist our conclusion. It has been pointed out that the interpretation of text (K) defended here is inconsistent with the standard interpretation of a passage in the *Categories* where Aristotle allegedly claims that affirmative, but not negative, singular propositions do have an existential commitment. The text goes as follows:

(L) It might, indeed, very well seem that the same sort of thing does occur in the case of contraries said with combination, “Socrates is well” being contrary to “Socrates is sick.” Yet not even with these is it necessary always for one to be true and the other false. For if Socrates exists one will be true and one false, but if he does not both will be false; neither “Socrates is sick” nor “Socrates is well” will be true if Socrates himself does not exist at all.

(Aristotle, *Categories* 10, 13b12–19; ROT translation)

Here Aristotle is asking whether two singular propositions such as

(18) Socrates is well

and

(19) Socrates is sick

namely propositions in which two contrary predicates (*being well* and *being sick*) are attributed to the same subject, may be treated as contradictory statements. His view is that they cannot be such that if one of the two is true the other is false, because this happens only if Socrates exists. When Socrates does not exist, they are both false and, consequently, they do not act as contradictory statements. One might think that Aristotle here rejects the truth of (18) and (19) under the hypothesis that Socrates does not exist on the basis of (EC). But what follows our passage clearly shows that this is not the case. Aristotle says:

(M) But with an affirmation and negation one will always be false and the other true whether he exists or not. For take “Socrates is sick” and “Socrates is not sick;” if he exists it is clear that one or the other of them will be true or false, and equally if he does not; for if he does not exist “he is sick” is false but “he is not sick” true.

(Aristotle, *Categories* 10, 13b27–33; ROT translation)

What is stated here is that it is false to assert that Socrates is sick when he does not exist. Therefore, the existence of its subject is not a necessary condition for the truth of a singular proposition. For this reason we cannot take text (K) as an evidence for (EC), because (L) disproves existential commitment in the case of negative singular propositions. Moreover, text (L) allows us to state that both the negations of “Socrates is well” and “Socrates is sick” are true when these propositions are found to be false. Therefore, (EC) is not true in general. If we take the example of “Socrates is sick” as a special case for any singular proposition, we must conclude that Aristotle here wants to distinguish the case of negative from the case of affirmative singular propositions. An affirmative singular proposition carries an existential commitment if it is true, whereas the corresponding negative does not. Apart from the fact that this view sounds very implausible,⁴⁶ the doctrine of the *Categories* would be inconsistent with the passage from *On Interpretation* that we have already discussed. An affirmative singular proposition is stated there – “Homer is a poet” – that is supposed to be true, although Homer does not exist.

There are two ways out of this difficulty. One consists in denying that text (K) asserts that there are true affirmative singular propositions, the subject of which is non-denoting. This path has been taken by some scholars but, as far as I can see, it leads nowhere. The other way out is much easier and depends on reconsidering the passage from the *Categories*. As we have seen, they conflict with the text of *On Interpretation* only if we assume that what is said in the *Categories* about the pair “Socrates is well” and “Socrates is sick” can be extended to any singular affirmative proposition. Because it is stated that these propositions must have an existential commitment to be true, by generalization one is led to claim that every true affirmative proposition implies such a commitment. It is because of this generalization that the problem of reconciling the *Categories* and *On Interpretation* arises. But do we really need this extension? The context makes it clear that it is not required. Aristotle’s point is to disclaim the view that any pair of singular propositions opposed as contraries cannot be false together. By means of the pair “Socrates is well” and “Socrates is sick” he offers a counterexample to this thesis. Therefore, it is not necessary to suppose that what is said about this pair of propositions can be applied to any other pair of singular propositions in the same relation. In fact, although it is quite reasonable to claim that if “Socrates is well” is true, then “Socrates exists” is true, there is no obvious reason to require that Homer exists if it is true that Homer is a poet.

One might try to insist on the idea that only singular affirmative propositions have an existential import by quoting other passages of *On Interpretation* that may support this view. If this were the case, then one might claim that text (K) is a slip on the part of Aristotle that does not correspond to his usual position. The passage I have in mind is the following:

(N) “Does not recover” and “does not ail” I do not call verbs. For though they additionally signify time and always hold of something, yet there is a difference for which there is no name. Let us call them indefinite verbs, since they hold of anything whether existent or non-existent.

(Aristotle, *On Interpretation* 3, 16b11–14; ROT translation)

Suppose that an indefinite verb added to a (singular) subject makes the obtained proposition negative.⁴⁷ Then one could take the last part of the text as a claim that (singular) negative propositions never have an existential import and claim that this is peculiar of negative propositions. I agree with this interpretation, but I do not see how it conflicts with our general view. (Singular) negative propositions are such that they never imply an existential commitment, whereas affirmative propositions sometimes do depending on the nature of the predicates they are referring to. Predicates such as “... is well” or “... is sick” *do* imply the existence of the subject of which they are true, whereas this is not required for predicates such as “... is a poet” (at least in some circumstances). On the other hand, predicates such as “... is not well” or “... does not recover” have no existential import, although their affirmative counterparts do. This is a reasonable reading of text (N) and is not inconsistent with our interpretation.

Let us try to draw a conclusion from all this discussion. Aristotle does not seem to be committed to (EC) in his treatment of singular propositions both affirmative and negative. Shall we infer from this that he was a forerunner of the so-called free logic? One of the main tenets of free logic consists of the rejection of (EC). We are allowed to admit true singular propositions such as “Homer is a poet” without being compelled to derive from this the truth of the corresponding existentially quantified proposition like “there is somebody who is a poet.” (EC) is admitted only on the assumption that the subject of the singular proposition denotes an existing entity.⁴⁸ Although this view is to some extent consistent with texts (L) and (M), it is not consistent with the laws of the logical square. If we allow nondenoting singular terms, how can we deny the legitimacy of using empty general terms? But if we extend our language to such terms, we are compelled to refuse the status of logical laws to the relations of the logical square. Once more, the Aristotelian doctrine appears to be inconsistent, and we must recognize that all our efforts have been frustrated. On the one hand, there are reasons to believe that Aristotle was committed to the view that every general term has instances, namely that there are no empty terms. On the other, he recognized the legitimacy of nondenoting singular terms about which true affirmative and negative predications can be made. How can we reconcile these conflicting views?

IX

Before giving up any hope to recover a coherent picture of Aristotle’s position, it may be useful to pursue a different line of approach. Standard semantics and free logic have in common the view that particular quantification has an existential commitment. For this reason it is usual to represent “some” by “ \exists ” and to call it the existence quantifier. When we say

(20) some professors are clever

we suppose that *there is* – namely *there exists* – at least one professor who is clever. In my opinion, it is precisely this view that causes trouble in Aristotle’s doctrine when it is imposed on his perspective. Therefore, my proposal is as simple as this: Let us abandon the view that particular quantification has to be

expressed by means of the existential quantifier and see what happens. The result seems to be *prima facie* encouraging. Take, for instance, (SU₁). If “some” does not carry any longer an existence commitment, there is no reason to deny that “some witches are nasty” is true if the corresponding universal proposition “every witch is nasty” is supposed to be true. Therefore, (SU₁) no longer has a counterexample, and in this way the other laws of the logical square can be regained.

In a parallel way, the fact that particular quantification does not imply an existential import should lead to separate existence from quantification. Existence is a predicate, and there is no reason to express it by a quantifier. Therefore, what Aristotle says in the treatise *On Interpretation* and in the *Categories* about the existential commitment of singular propositions has nothing to do with (EC). This inference holds as well as the subalternation laws and the other rules of the logical square, because it simply tells us that what we are speaking about is something without implying anything about its existence.

Finally, the claim that according to Aristotle there are no empty terms – a claim that we have made as a result of the discussion of his definition of καθόλου – looks less extravagant than one might think if it is read in light of the separation between quantification and existence. To consider existence a predicate depends on rejecting the idea that, in logically proper form, we can only speak about what exists. In the standard perspective, the ultimate subjects of predications must be existent in order to perform their task of subjects. Therefore, individual instances of universal terms must exist as ultimate subjects of predication and, conversely, what is not true of an existing subject is true of nothing. It is precisely in this perspective that we may deal with empty terms and say that they are true of nothing, because they are true of nothing existent. However respectable and serious this view is, it is not the only possible one. If we separate quantification from existence, then we can think of instances of general terms that have no existence at all or even cannot have an existence. It is sufficient that we can conceive such instances as meeting the requirements of being instances of these terms to claim that the latter are not empty. Why should one not say that the round square cupola on Berkeley College, to use Quine’s example,⁴⁹ is an instance of a round-square cupola or of a cubic sphere? Needless to say, there is no round-square cupola and there cannot be. Nonetheless, we can think of such things no more and no less than we can think of contradictions. Does this view lead us to a Meinongian universe crowded with monstrous objects? Well, this conclusion is inescapable if we suppose that in order to speak of something we must confer an existence in some part of the universe, actual or possible. But if we drop this requirement, we do not need to suppose that particulars must be put in the world as parts of its furniture.

I am not sure that there is enough evidence to attribute this view to Aristotle – a view that is difficult to defend and full of snares. However, there is at least one passage in which a point is clearly made in this direction. As an appendix to text (K), Aristotle states in a rather sharp way:

(O) It is not true to say that what is not, since it is thought about, is something that is; for what is thought about it is not that it is, but that it is not.

(Aristotle, *On Interpretation* 11, 21a32–33; ROT translation)

This text makes two points about nonexistent entities, both of which are important. First of all, we are allowed to speak about what is not. What is not is a *δοξαστόν*, something that can be thought about, and we can take it as a subject of propositions. Of an individual that is not, for instance Brunhilda the witch, one can say that she does not exist, and perhaps something else, as for instance, that she is a witch. Both these statements are taken to be true without implying that Brunhilda exists. This view is not without parallels in other places. For instance, in his famous discussion about the different meaning of being, Aristotle points out that we say that *is* is not only a substance and its affections but also what is the negation not only of an affection or a substance and adds “that is why we assert that even what is not is a thing that is not.”⁵⁰

But, and this is Aristotle’s second point, from the fact that we are able to speak about a nonexistent thing, it does not follow that the nonexistent thing is brought into existence. The truth of “Brunhilda does not exist” does not entail that Brunhilda exists. This claim is repeated elsewhere. By placing among the sophisms the argument according to which if we say of something that is not that it is not, then we are allowed to conclude that something that is not is. Aristotle thus distinguishes between “being” (*εἶναι*) and “being something” (*εἶναι τι*) and states that we cannot infer that something is from the fact that it is true to say that it is something. Otherwise, we might conclude that a nonexistent thing is from the fact that it is true to say that it is a nonexistent thing.⁵¹

The moral is this: What is an object of our discourse, and in particular a subject of predication, does not necessarily exist. More generally, we are not compelled to claim that whatever falls in the universe of discourse is something that exists. However, whatever is in the universe of discourse is so structured that it can be qualified as universal or singular. As we have seen, both man and witch can be qualified as universals because they have instances. Of course, instances of man are on a different level than instances of witches. The former are real and the latter fictitious. But they both share the property of being in our universe of discourse, and therefore they must have the same logical features. From the mere fact that they have a different ontological status it does not follow that we must treat them differently from a logical point of view. We can easily think of logic as completely neutral with respect to ontology. In this sense, we need not first decide about the existence of witches and then apply an appropriate logic to the statements that concern them. From a logical point of view, one might think everything is on the same level, that is, the level of the universe of discourse. Whatever I am speaking about must receive the same logical treatment, because its logical features, namely the feature that depend on its being spoken about, are the same. Distinctions come later, when we try to classify things. Perhaps Aristotle would have moved in this direction had he expressed himself on the object of logic.

A reader acquainted with modern discussions about this perspective would immediately ask how Aristotle would have answered objections similar to those Bertrand Russell addressed to Alexius Meinong, namely how to avoid contradictions when impossible terms are in question.⁵² For the present, I must say that I

have found no evidence that Aristotle was aware of the problems raised by the claim that the so-called empty terms have instances. Nonetheless, I am inclined to think that Aristotle had ideas perhaps not yet organized in a full-blooded theory that find a more adequate systematization in the present interpretation than in reference to the standard semantics of modern logic.

Notes

This chapter was presented at the 13th Symposium Aristotelicum held at Certosa di Pontignano (June 23–30, 1993), which was devoted to an in-depth study of the treatise *On Interpretation*.

- 1 Aristotle, *Prior Analytics* 1.27, 43a25.
- 2 Łukasiewicz 1957²: 6.
- 3 See, for instance, Ammonius, *On Aristotle's On Interpretation* 88.29 ff.
- 4 Quine 1974: 230 ff.
- 5 See, for instance, Aristotle, *Prior Analytics* 1.1, 24b16–17.
- 6 Aristotle, *Metaphysics* 7.13, 1038b9–12.
- 7 Aristotle, *Posterior Analytics* 1.16, 80a25 and Ross' commentary (Ross 1949: 560).
- 8 The more detailed study on this subject is Hadot 1980: 309–320.
- 9 Aristotle, *Sophistical Refutations* 1, 165a6–10.
- 10 Aristotle, *On Interpretation* 1, 16a3–8.
- 11 Aristotle, *Metaphysics* 4.4, 1006b2–11.
- 12 Aristotle, *On Interpretation* 1, 16a16–17.
- 13 For instance, in *Topics* 1.7, 103a9–19, we are told that in the case of “λώπιον” and “ἰμάτιον” we have two words but only one thing. Moreover, in *Topics* 1.18, 108a18–22, we are reminded that arguments are πρὸς τὸ πρᾶγμα and not πρὸς τὸ ὄνομα. In a similar vein, in *Sophistical Refutations* 14, 174a8–9, Aristotle points out that man and white are both names and things.
- 14 Aristotle, *Prior Analytics* 1.27, 43a25 ff.
- 15 Aristotle, *Categories* 5, 2a27–31.
- 16 Aristotle, *On Interpretation* 7, 17b13 ff.
- 17 For the sake of simplicity I do not consider an alternative formalization of the Aristotelian definition of καθόλου, namely

$$(4^*) \quad U(a) \leftrightarrow \exists x \exists y \Pi (x \neq y \wedge K(a, x) \wedge K(a, y)).$$
- 18 Aristotle, *Prior Analytics* 1.13, 32b4–10.
- 19 Cf., for instance, Aristotle, *Topics* 1.9, 103a23–27.
- 20 Of course, if we keep (4) as the formal counterpart of Aristotle's definition of καθόλου we must interpret the existential quantifier in a way that is different from the standard one. We will come back to this in the last part of this chapter.
- 21 The reading κατηγορεῖται, which is found in some manuscripts, is of no philosophical interest.
- 22 Weidemann 1985: 45–56. Cf. Weidemann 1994: 212–213.
- 23 Aristotle, *On Interpretation* 7, 17b5–6. See also b3–4.
- 24 Stephanus, *On Aristotle's On Interpretation* 29.10–11.
- 25 Stephanus, *On Aristotle's On Interpretation* 31.4–5.
- 26 *De interpretatione vel Peri hermeneias*, Translatio Boethii, in Verbeke 1965: 10.15–18.
- 27 Ammonius, *On Aristotle's On Interpretation* 101.10–11.
- 28 Ammonius, *On Aristotle's On Interpretation* 101.23–24.
- 29 The Boethian translation is far from being clear to me: “*In eo quod praedicatur universaliter universale praedicare universaliter non est verum.*” It is worth noting that the second “*universaliter*” is omitted by several manuscripts.

- 30 I am doubtful that Aristotle's point may be extended to negative propositions. This is especially true if we adopt the (E_b) or the (E_w) readings because there is no reason to consider false every proposition of the form of, for example, "no F is every G ." Ammonius, *On Aristotle's On Interpretation* 104.23–26 takes all propositions of this type as true.
- 31 Aristotle, *Prior Analytics* 1.27, 43b17–22. Another similar example is music is every science. I will discuss this passage in due course.
- 32 On Ammonius's view see Mignucci 1983: 11–42.
- 33 For instance, the proof of (SU_1) can formally be reconstructed as follows:

$$\frac{AaB \vdash \neg AeB \qquad \neg AeB \vdash AiB}{AaB \vdash AiB}$$

by using the so-called cut rule.

- 34 When Aristotle says that two subcontrary propositions may be true together, he does not claim that for any pair of subcontrary propositions there is a possibility that they are true together. "Some man is white" may be true together with "some man is not white." But it cannot happen that "some triangle is a plain figure" is true together with "some triangle is not a plain figure," because "every triangle is a plain figure" is a necessary proposition so that "some triangle is not a plain figure" cannot be true. Nonetheless, "some triangle is a plain figure" and "some triangle is not a plain figure" are subcontrary propositions no more and no less than "every triangle is a plain figure" and "no triangle is a plain figure" are contrary propositions, because the first pair is an instance of a i/o pair and the second one of a a/e pair. For some of these pairs, it may happen that they are both false (or both true), and that is enough to conclude that, in general, we cannot say that the negation of an e -proposition implies the negation of an i -proposition.
- 35 To prove, for example, (SC_3), we may proceed this way:

$$\frac{\frac{\neg AiB \vdash AeB \qquad AeB \vdash \neg AaB}{\neg AiB \vdash \neg AaB} \qquad \neg AaB \vdash AoB}{\neg AiB \vdash AoB.}$$

- 36 As far as I know, the difficulty was first pointed out by Ajdukiewicz 1927; Northrop 1928: 193–210. See also Kattsoff 1934: 149–162; Henle 1935: 111–113 (*contra* Kattsoff); Popkin 1947: 670–681; Strawson 1952: 152–179; Thompson 1953: 251–265; Nelson 1954: 401–413; Thompson 1954 (*contra* Nelson); Morrison 1954–1955: 386–393; Roxon 1955: 107–111; Kneale and Kneale 1978⁷: 54–61.
- 37 I take it in a naïve way, without implying that a set can be associated with every predicate.
- 38 Quine 1962²: 150.
- 39 Quine 1961a: 1–19.
- 40 The text is not sure. Ross has eliminated τοῦ ὑποκειμένου, which is present in all the manuscripts, because of Alexander's commentary (*On Aristotle's Prior Analytics* 24.27–30). However, Alexander's text is also unsure and, at any rate, τοῦ ὑποκειμένου must be understood.
- 41 Smiley 1962: 58–72.
- 42 " $F+G$ " is meant to represent the case of a composite predicate as "white man" or "good cobbler."
- 43 More or less, this is the interpretation proposed by John Ackrill. Cf. Ackrill 1963: 148–149.
- 44 Both William Jacobs (Jacobs 1979: 282–300) and Michael Wedin (Wedin 1990: 131–150), with different and sometimes contrasting arguments, have claimed that in our

text (K) no denial of an existential commitment for singular affirmative propositions is implied. This is very hard to believe. Jacobs, for instance, thinks that our “passage merely notes that because being a poet is an accident of Homer, ‘Homer is a poet’ may be true or false” (289). It is difficult to see in what sense this alleged Aristotelian claim is relevant to the discussion of counterexamples to the derivation of simple from complex predications. According to Wedin (if I understand his point), “Aristotle would simply point out that in the proposition ‘Homer is a poet,’ ‘is’ has no bearing on Homer’s existence” (186). But if one claims that Aristotle would have maintained (EC) at least for affirmative singular propositions, as Wedin does, the “is” *does* have a bearing on Homer’s existence, being a part of the true proposition “Homer is a poet” from which “Homer exists” follows. The “is” of “. . . is a *F*,” by itself, does not mean the existence of the subject to which “. . . is a *F*” is attributed (“is” in “. . . is a *F*” has not the same meaning as “exists”) but it implies the existence because of (EC). But this is what Aristotle denies when he claims that “Homer exists” does not follow from “Homer is a poet,” because “Homer is a poet” is true and “Homer exists” is false. Therefore, we *do* have a counterexample to (EC).

45 See Aristotle, *On Interpretation* 11, 21a31–33.

46 If every affirmative singular proposition with a nondenoting subject is false and every corresponding negative is true, we should say that

(*) Socrates is dead

is false, whereas

(**) Socrates is not alive

is true – a nonsense. Alternatively, one should redefine what is for a proposition to be affirmative or negative by distinguishing logical from grammatical negation – a quite difficult task.

47 See the discussion of this point in Ackrill (Ackrill 1963: 120–121).

48 See, for instance, Leonard 1956: 49–64; Leblanc and Hailperin 1959: 239–243.

49 Quine 1961a: 4–5.

50 Aristotle, *Metaphysics* 4.2, 1003b6–10.

51 Aristotle, *Sophistical Refutations* 25, 180a32–38.

52 A discussion of Russell’s objection to Meinong can be found in Parsons 1980: 38–44. See also Griffin 1986: 375–401; Jacquette 1986: 423–438.



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Part III

Modality, time, and future contingents



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8 Aristotle's conception of the modal operators

I

After having been neglected for many years, Aristotle's modal logic is enjoying a revival of studies, especially as far as his modal syllogistic is concerned.¹ The problem I would like to examine here, although obviously linked with the interpretation of his modal syllogistic, is more general, being directed at investigating the way in which the modal operators are defined or conceived by him. Necessity and possibility can be viewed in different ways: as connected to the notion of possible worlds, as temporal operators, as syntactic items, and so on. Is there any evidence in Aristotle's works that allows us to attribute one of these views to him? Different answers have been given to this question. Jaakko Hintikka has vigorously defended a temporal interpretation of the Aristotelian modalities, and, more recently, his view has been partly endorsed by Ulrich Nortmann.² I will not discuss these positions in any detail, partly for reasons of space and partly because I do not think that Aristotle's works offer enough evidence to choose one interpretation and reject the others. We cannot find in them a precisely elaborated definition of the modal notions, and the few observations scattered in the texts often go in different directions, which all lead to a complex and multifarious intuition, where it is not clear at all whether all its parts are consistent with each other. I think that there is some evidence for a temporal interpretation, some evidence for an interpretation in terms of possible worlds, and some evidence for a syntactic interpretation of the modalities. It is with this feeling that none of these views can claim to be the true one and exclusive of the others that I propose my interpretation.

II

As most modern logicians, Aristotle treated necessity and possibility as related notions in such a way that we can characterize the one by means of the other. Therefore, we can decide to start with the notion of possibility, for which Aristotle proposes what he calls a *ὁρισμός*, that is, a definition.

Aristotle's words for "possible" are at least two, namely "*δυνατόν*" and "*ἐνδεχόμενον*." It is difficult to assess whether he uses them as synonymous or distinguishes them systematically. Sometimes, he seems to use "*δυνατόν*" to denote what is called "one-sided possibility" or "possibility in a weak sense,"

and “ἐνδεχόμενον” for the “two-sided” or “strong possibility.” I will represent a one-sided possible proposition ϕ by “ $\Diamond\phi$ ” and a strongly possible proposition ψ by “ $\varepsilon\psi$.” But there are many places in which we find “δυνατόν” where “ἐνδεχόμενον” would be expected and vice versa.³ Therefore, because we cannot rely on a terminological distinction, it is safer to take the two Greek words as synonymous and to distinguish them only when the context explicitly demands it.

There is at least one famous passage in the *Prior Analytics* where Aristotle’s purpose of offering a definition of possibility is clearly revealed by the formula “λέγω δ’ ἐνδεχέσθαι καὶ τὸ ἐνδεχόμενον.”⁴ Let us start by quoting this text:

(A) I say to be possible or possible (λέγω δ’ ἐνδεχέσθαι καὶ τὸ ἐνδεχόμενον) that which is not necessary and if it is supposed to be, there will be no impossibility in virtue of this.

(Aristotle, *Prior Analytics* 1.13, 32a18–20)

There is a standard interpretation of this passage.⁵ According to it, strong possibility is defined by stating that a proposition ϕ is strongly possible if it is both not necessary and possible:

(DE) $\varepsilon\phi \text{ iff } \neg\Box\phi \wedge \Diamond\phi.$

Equivalent ways of expressing this definition are saying that ϕ is strongly possible if it is not necessary nor impossible:

(DE*) $\varepsilon\phi \text{ iff } \neg\Box\phi \wedge \neg\Box\neg\phi$

and saying that ϕ is strongly possible if $\neg\phi$ is possible and ϕ is possible, that is

(DE**) $\varepsilon\phi \text{ iff } \Diamond\neg\phi \wedge \Diamond\phi.$

Of course, (DE), (DE*), and (DE**) are supposed to be equivalent on the basis of the traditional and obvious relations taken to hold between the \Box - and \Diamond -operators, according to which we are entitled to state

(ML) $\Diamond\phi \leftrightarrow \neg\Box\neg\phi.$

Needless to say, Aristotle’s analysis of the relations between the modal operators in *On Interpretation* 12–13 allows us to state such an equivalence. In this way, the relations holding between ε - and \Diamond -possibility are clear. For instance, we can state

(EM) $\varepsilon\phi \rightarrow \Diamond\phi$

but its converse cannot be admitted. Moreover,

(LM) $\Box\phi \rightarrow \Diamond\phi$

is surely valid, but we cannot substitute $\varepsilon\phi$ for $\Diamond\phi$ in the consequent of (LM). After Becker's work all this has become part of the standard picture of Aristotle's theory of possibility, and there is no scholar who refuses to acknowledge Becker's distinction between ε - and \Diamond -possibility.

So far, so good. From a strictly textual point of view, it is clear that the whole interpretation depends on the assumption that, for a proposition ϕ , to be weakly possible is the same as being such that nothing impossible is by virtue of ϕ . We have therefore to state:

(*) $\Diamond\phi$ iff nothing impossible is by virtue of ϕ .

On the basis of (*) we can represent text (A) by means of (DE) and conclude that (DE) and (DE*) are equivalent ways of expressing the definition of strong possibility. There are passages that offer evidence for (*). The best known to me comes from *Metaphysics* 9 (*Theta*) and runs as follows:

(B) Something is capable ($\delta\upsilon\nu\alpha\tau\acute{o}\nu$) if there will be nothing impossible in the case that the actuality of which it is said to have the capacity takes place. I mean, for instance, that if someone is capable of sitting ($\delta\upsilon\nu\alpha\tau\acute{o}\nu$ καθῆσθαι) and can sit ($\varepsilon\upsilon\delta\epsilon\chi\acute{\epsilon}\tau\alpha\iota$ καθῆσθαι), then if it happens to him to sit, there will be nothing impossible.

(Aristotle, *Metaphysics* 9.3, 1047a24–28)

This text is slightly complicated by the ambiguous use that Aristotle makes here and in the preceding lines of the word “ $\delta\upsilon\nu\alpha\tau\acute{o}\nu$,” which is best rendered sometimes by “possible” and sometimes by “capable.” However, it seems to me that his point is directed to cover both uses, as the addition of $\varepsilon\upsilon\delta\epsilon\chi\acute{\epsilon}\tau\alpha\iota$ καθῆσθαι perhaps shows. Suppose, then, that “ $\delta\upsilon\nu\alpha\tau\acute{o}\nu$ ” here means not only capable but also possible. We can safely assume that the possibility that is in question here is weak, and not strong, possibility. Therefore, here we have a statement that is very close to (*).

By comparing texts (A) and (B) it can be pointed out that the usage of the future tense in (B) – which is used to state that there is nothing impossible if ϕ is the case – does not hint at a temporal sequence but alludes at a logical or causal chain, as is usual in Aristotle's language and as $\delta\iota\alpha\ \tau\omicron\upsilon\tau\omicron$ in (A) suggests. In addition to that, it should be kept in mind that Aristotle sometimes uses the verb $\sigma\upsilon\mu\beta\alpha\acute{\iota}\nu\epsilon\iota\nu$ to express the relation between the assumption that ϕ is the case and that nothing impossible takes place.⁶ This fact, together with the use that Aristotle makes of his characterization of weak possibility in several proofs, strongly suggest that the way in which nothing impossible depends on assuming ϕ is a logical way. From supposing that ϕ is the case, nothing impossible logically follows. Therefore, we can rephrase (*) as follows:

(**) $\Diamond\phi$ iff nothing impossible follows from ϕ .

Nothing new has been said until now.⁷ One might take (*) as a terminological variance of the well-known phenomenon by which necessity and possibility are interdefinable, according to which one is taken as primitive. Aristotle is aware of this fact. Sometimes he characterizes necessity in terms of possibility as to τὸ μὴ ἐνδεχόμενον ἄλλως ἔχειν – “that which cannot be otherwise”⁸ – and in other places he qualifies possibility as the negation of necessity.⁹ All of this is perfectly clear and safe, but it does not clarify what Aristotle’s notions of possibility and necessity are.

Shall we leave the game and conclude that there is no way to understand what Aristotle meant by necessity and possibility? Not just yet. Let us consider a passage from *Physics* 7 that has reached us in two different versions. One version refers to possibility in a way that should not surprise us.

Speaking of a proposition Aristotle says:

(C) The hypothesis was assumed as possible, and if what is possible is posited nothing impossible (ἀδύνατον) results because of this.

(Aristotle, *Physics* 7.1, 243a30–31)

Here, the usual ἀδύνατον is what is said not to come out of the assumption that a possibility is the case. However, in the other version of the passage, Aristotle expresses himself in a different way:

(D) It does not matter that this has been proven by supposing something, for if what is possible is posited nothing absurd (ἄτοπον) can follow.

(Aristotle, *Physics* 7.1, 243a1–2)

The claim is the same as before, but here the word ἄτοπον is used instead of ἀδύνατον, and one could be tempted to take this terminological shift seriously by pointing out that here impossibility is specified as absurdity, and “absurdity” could be interpreted in the sense of “contradiction.” Therefore, we could consider the impossibility that does not follow from a possible proposition as a contradiction and claim that the standard Aristotelian definition of possibility is that ϕ is weakly possible if, and only if, no contradiction follows from ϕ . In other words, my proposal is that “ἀδύνατον,” “impossible,” in our texts can be taken to mean “ἄτοπον,” “absurd,” where “absurd” or “ἄτοπον” is a primitive notion that can be represented formally by “ \perp .” As is usual in modern systems of logic, “ \perp ” can be conceived as a propositional constant that expresses or implies a contradiction or, from a semantic perspective, a proposition that is always evaluated as false. Therefore, \perp is not a modal operator, but a proposition that does not contain modal expressions, and in this sense possibility is defined in terms of nonmodal notions.

I do not claim that text (D) proves this interpretation. First, it is not clear whether the text from which (D) is taken can be attributed to Aristotle. Whereas Bekker printed the version containing (D) as the main text, Ross chose the version containing (C).¹⁰ I would not like to become entangled in this problem, but if we have to choose between the two versions, following Ross would clearly undermine the

reliability of text (D). Moreover, one might question whether the interpretation of “ἄτοπον” in terms of “ \perp ” is compulsory. The word “ἄτοπον” is sometimes used to indicate something strange or unexpected,¹¹ and it does not always have the meaning of “contradictory.” Even in logical contexts it is not always clear that “ἄτοπον” means “contradictory,” and it may simply refer to something absurd because it is implausible or difficult to believe.¹²

However, this proposal is consistent with Aristotle’s terminology. In several passages, he employs “ἄδύνατον” as a synonym for “ἄτοπον.” More interestingly, the formula “ἄδύνατον τι συμβαίνειν” is frequently used to indicate the derivation of something that implies a contradiction,¹³ and “ἄτοπον συμβαίνειν” is employed in the same sense. To be precise, the ἄτοπον or ἄδύνατον that is said to be derived is normally a proposition that yields a contradiction when it is joined to another proposition previously asserted. Moreover, we have to recall Aristotle’s use of “ἄδύνατον” in his terminology for the so-called *reductio ad impossibile*. He normally speaks of συλλογισμός or ἀπόδειξις εἰς τὸ ἄδύνατον,¹⁴ ἀπαγωγή εἰς τὸ ἄδύνατον,¹⁵ or even συλλογισμός διὰ τοῦ ἀδυνάτου,¹⁶ where, of course, “ἄδύνατον” has no specific modal meaning.

As a conclusion of this discussion, we have to admit that there is no decisive evidence for the interpretation of “ἄδύνατον” in terms of logical absurdity. Nonetheless, this interpretation is in my view possible. For one thing, it has no obvious denial in the texts. For another, it is philosophically interesting because it allows us to give an elucidation of the Aristotelian modalities. Therefore, I would like to assume it as a hypothesis in order to see how it works when applied to the relevant texts.

Before engaging in this project, it is safe to present a formal counterpart of our interpretation of (**). An obvious formalization of the Aristotelian definition of possibility is the following:

$$(MD) \quad \Diamond\phi \text{ iff } \phi \not\models \perp.$$

Because (ML) holds, we immediately have a corresponding definition for the necessity operator:

$$(LD^*) \quad \Box\phi \text{ iff } \neg\phi \models \perp$$

which is obviously equivalent to

$$(LD) \quad \Box\phi \text{ iff } \models \phi.$$

In a parallel way, we define an impossible proposition such as $\nabla\phi$ as

$$(ID) \quad \nabla\phi \text{ iff } \phi \models \perp$$

and this is the same as stating

$$(ID^*) \quad \nabla\phi \text{ iff } \models \neg\phi.$$

Needless to say, another way to express the definition of possibility is

$$(MD^*) \quad \Diamond\phi \text{ iff } \not\models \neg\phi^{17}$$

which is equivalent to (MD). Finally, strong possibility can be expressed by the equivalence

$$(ED) \quad \varepsilon\phi \text{ iff } \phi \not\models \perp \text{ and } \neg\phi \not\models \perp$$

or by

$$(ED^*) \quad \varepsilon\phi \text{ iff } \not\models \neg\phi \text{ and } \not\models \phi$$

which is the same. The usual relations between modal propositions are obviously preserved by these definitions.

III

To evaluate this proposal based on such scanty evidence, one should test it against the body of the modal syllogistic in order to show that its adoption brings benefits that other interpretations are unable to give. To my shame, I will not follow this obvious path. I will limit myself to showing how the adoption of a reading of possibility in terms of (MD) is in some cases advantageous in the interpretation of difficult passages and that, in other cases, Aristotle seems to rely on a different characterization of the modal notions.

Paradigmatic for my purpose is a modal thesis that Aristotle proves in two distinct passages and in slightly different ways. One favors my reading of possibility, whereas the other is more naturally framed in terms of a possible-worlds semantic. Let us start with the favorable one, found in the *Metaphysics*. Aristotle says:

(E) (a) At the same time it is also clear that if it is necessary that *B* is if *A* is, then if it is possible that *A* is, it is necessary that it is also possible that *B* is. (b) For if it is not necessary that it is possible that *B* is, there is nothing to prevent its not being possible to be. (c) Take then *A* to be possible. (d) Then when *A* is possible to be, if *A* is supposed to be, nothing impossible to be follows, as we have said. (e) And then it is necessary that *B* is. (f) But *B* was said to be impossible. (g) Suppose then that *B* is impossible. (h) Then if *B* cannot be, it is necessary that also *A* cannot be. (i) But the first was impossible; therefore the second too. (j) Therefore, if *A* is possible, also *B* will be possible, if they are in such a relation that it is necessary that *B* is if *A* is.

(Aristotle, *Metaphysics* 9.4, 1047b14–26)

The passage is complicated but not impossibly difficult. Statement (a) contains the thesis to be proven, and we may express it in the following way:

$$\begin{array}{l}
 \text{(TC)} \quad A \models B \\
 \hline
 \Diamond A \models \Diamond B
 \end{array}$$

where the two operators of necessity are embodied in the two different signs for deduction, “ \models ” and “ $\frac{}{} \vdash$ ” respectively.¹⁸ Clause (b) shows that Aristotle is performing a *reductio ad absurdum*. Suppose that it may happen that B necessarily follows from A and that A is possible and B is impossible. Then, according to this hypothesis, we should have:

- (1) $A \models B$
- (2) $\Diamond A$
- (3) $\neg \Diamond B$.

What comes after (b) proves that such a situation cannot be the case. Clause (c) proposes that we consider $\Diamond A$, namely (2), which is true by hypothesis, and (d) recalls the definition of possibility of text (B). Here “follows” (συμβαίνειν) is used, and this justifies the introduction of “ \models ” in our definitions of the modal operators. The definition of possibility in (d) is supposed to legitimate the move from “ A is possible” to “nothing impossible follows from A .” By taking possibility as noncontradiction, we can replace (2) with

$$(4) \quad A \not\models \perp.$$

On the other hand, B follows necessarily from A , as is said in (e). The implicit conclusion is that B cannot be impossible. In other words, it cannot be the case that

$$(5) \quad B \models \perp.$$

Why? Take B to be impossible, as (g) tells us. If B is impossible, then also A must be impossible. This is what (h) says, and our interpretation offers a nice suggestion for explaining this step of the proof. If B is supposed to be impossible, a contradiction follows from B . But B is also the consequence of A by hypothesis. Therefore, we can state by transitivity that A entails a contradiction, which goes against the claim of its possibility, namely its not having contradictory consequences. We may express all this in a simple deductive line:

$$\begin{array}{l}
 (6) \quad A \models B \qquad B \models \perp \\
 \hline
 A \models \perp.
 \end{array}$$

But the conclusion of (6) contradicts (4); thus, (5) cannot be the case, and we must reject (3), the hypothesis of the *reductio*, namely that B is impossible, that is, (3). We thus have a confirmation of our hypothesis: Aristotle uses (*) in some arguments in a way that shows that (MD) is a reasonable interpretation for (*).

Before the gods punish my hubris, let us consider the second proof of (TC) found in the *Analytics*. Aristotle says:

(F) (a) It must first be said that if it is necessary that *B* is if *A* is, then, if *A* is possible, will be also possible of necessity. (b) For, things being so, let what *A* stands for be possible and what *B* stands for be impossible. (c) If, then, what is possible, when it is possible to be, could come about, and what is impossible, when it is impossible to be, could not come about, (d) and *A* is possible together with *B* being impossible, (e) *A* could come about without *B*, and, if it could come about, it could also before, what is has come about, when it has come about, is.

(Aristotle, *Prior Analytics* 1.15, 34a5–12)

As in text (E), (a) includes the modal thesis to be proven, namely (TC), and (b) introduces the *reductio* by making the hypothesis that (i) *A* implies *B* (οὕτως ἐχόντων, “things being so”: 34a7); (ii) *A* is possible; and (iii) *B* is impossible, namely (1)–(3). The proof starts with (c). This clause has been interpreted in various ways depending on the sense in which its first conditional has been understood, namely: εἰ οὖν τὸ μὲν δυνατόν, ὅτε δυνατόν εἶναι, γένοιτ’ ἄν (34a8–9). A quick survey of its translations may be interesting.

Boethius in the so-called *recensio Florentina* is helpless, because he has “*si ergo possibile quidem, cum possibile esse, fiet*,” leaving all problems of the sentence untouched.¹⁹ But in the *recensio Carnutensis* of the same work he makes a commitment, rendering the Greek as “*si igitur unum quidem sit possibile, quando possibile est, fiet*.”²⁰ Here two fatal decisions are made: “ὅτε” is taken in a temporal sense and “δυνατόν εἶναι” is read as if it were δυνατόν ἐστι. As a consequence, a temporal interpretation of Aristotelian possibility is required.²¹ But “*quando possibile est*” does not correspond to the Greek,²² nor is it easy to understand what a temporal clause means when it is applied to what is impossible: If something is impossible, it is never the case. To avoid this difficulty, two solutions have been adopted. The first consists in amending the *vulgata* at 39a9 by reading ὅτι ἀδύνατον instead of ὅτ’ ἀδύνατον and limiting therefore the temporal qualification to the possibility.²³ Although the variant ὅτι is offered by a single MS, ὅτ’ is attested by all other manuscripts and is confirmed by a quotation of Alexander of Aphrodisias.²⁴ Therefore, there is little chance that ὅτι ἀδύνατον is the right reading. The second, and more reasonable, way of solving our problem is by taking ὅτε not in a temporal but in a causal sense.²⁵ This is the line taken, for instance, by Kirchmann, who renders our sentence as follows: “*da nun das Mögliche, weil es möglich ist, wirklich werden kann*.”²⁶ In this way, no temporal reference is attached to the Aristotelian modalities, but the problem still remains: “*weil es möglich ist*” does not correspond to the Greek.

The majority of English-, Italian-, and French-speaking translators have tried to be more faithful to the Greek. The standard English rendering of our sentence is: “if, then, that which is possible, when it is possible for it to be, might happen.”²⁷

What is the Greek behind this translation? I imagine that whoever accepts this rendering of ὅτε δυνατόν εἶναι takes εἶναι as the verb of an infinitive sentence whose subject is unexpressed and refers to the preceding τὸ δυνατόν. The infinitive sentence should depend on δυνατόν, to which an implicit copula should be supplied, so that the whole expanded sentence should be something like εἰ οὖν τὸ μὲν δυνατόν, ὅτε δυνατόν <ἐστι>, αὐτὸ [= τὸ δυνατόν] εἶναι, γένοιτ' ἄν. If this is the reconstruction of the Greek, I do not understand what it means. Suppose that I take a proposition ϕ , or a *Sachverhalt* if you prefer Seel's terminology,²⁸ and I state that it is possible, $\Diamond\phi$. Imagine then that I say of $\Diamond\phi$ that sometimes it is possible for it to be. What does "to be" mean? "To be possible"? This is rather obscure. One might claim that when Aristotle says of a possible that sometimes it is possible for it to be, what he really means is not that $\Diamond\phi$ may sometimes be the case, but simply that if $\Diamond\phi$, then ϕ can sometimes be the case. This is reasonable but does not correspond to the Greek, because the ὅτε-clause seems to introduce a restriction on the class of the propositions said to be possible and not a mere consequence of their being possible.²⁹

I have a simpler interpretation of our ὅτε-clause that makes its meaning philosophically less attractive and profound but easier to grasp. The idea that Aristotle wants to convey is that if we are dealing with a possible, it may happen that it comes into existence. In his terminology, a δυνατόν (recall that we are here dealing with weak, and not strong, possibility) can be a δυνατόν εἶναι or a δυνατόν μὴ εἶναι.³⁰ Strictly speaking, it is only what is δυνατόν εἶναι that has a chance to sometimes come into existence and therefore to be, because what is δυνατόν μὴ εἶναι has a chance to not be realized sometimes and therefore a chance to not be. I am suggesting that the ὅτε-clause simply contributes to stating that a possible, when it is a possible to be, may sometimes be the case. The reconstruction of the Greek text in this perspective becomes very simple. In this case, εἶναι sticks to δυνατόν and the subject of the whole sentence is the same δυνατόν that plays the role of the subject in the main clause: τὸ μὲν δυνατόν, ὅτε <τὸ δυνατόν ἐστι> δυνατόν εἶναι, γένοιτ' ἄν. This explains in an obvious way the parallel construction with the impossible. The impossible, when it is impossible to be, cannot come to be: τὸ δ' ἀδύνατόν, ὅτ<ε τὸ ἀδύνατόν ἐστι> ἀδύνατον <εἶναι>, οὐκ γένοιτο. This interpretation, although simple and boring, is nicely confirmed by the parallel passage of *Metaphysics* 9.4 that we have already analyzed, which reads, ἔστω δὲ τὸ *A* δυνατόν. οὐκοῦν ὅτε τὸ *A* δυνατόν εἴη εἶναι (1047b17–18). There is no doubt that here εἶναι goes with δυνατόν, which is the predicate of the temporal sentence, its subject being *A*. The conclusion is that we cannot take text (F) as a piece of evidence for a temporal interpretation of the Aristotelian modalities, as some interpreters have done.³¹

The proof contained in (c)–(e) is straightforward. If *A* is possible, there is no reason to deny that it may sometimes be the case. On the other hand, *B* being impossible according to the hypothesis of the *reductio* can never be the case. Therefore, it may be that *A* is the case and *B* is not. It is easy to guess why this conclusion cannot be accepted: It is inconsistent with the assumption that *A*'s being

the case implies B 's being the case. If we try to expand this argument in the way we have done for its previous version, we have to state something like

$$(6^*) \quad \frac{A \neq \perp \quad B \models \perp}{A \wedge \neg B \neq \perp}$$

in order to get the negation of (1), which, for our purposes, we can interpret as $A \wedge \neg B \models \perp$. But (6*) is far from being obvious and requires additional proof to be accepted.

To illustrate the sense of Aristotle's argument, it is much better to put it in a frame that is nearer to its spirit than its syntactic translation. I am thinking of a possible-worlds semantics. Suppose that A is possible; therefore, there is a possible world W in which A is the case. Let us imagine that W_i is such a world. By hypothesis, B is impossible; therefore, there is no possible world in which B is the case. In particular, B is not the case in W_i , the world in which we assumed A to be the case. Thus, there is at least one possible world in which A is the case and B is not the case, against the assumption that A entails B , that is, the assumption according to which in every world in which A is the case B is also the case.³² The conclusion seems unavoidable. In dealing with two different proofs for the same modal thesis, Aristotle appeals to different notions of necessity and possibility. It is difficult to evaluate this fact. My impression is that Aristotle had not sufficiently refined the definitions for the modal notions he was using. He was not able, or maybe did not feel the need, to control the original pre-philosophical intuitions about possibility and necessity with precise and clear-cut definitions, which have the disadvantage of drying the rich complexity of these notions but at the same time make them more manageable and reliable.

IV

I think that we have to resist becoming too frightened by Aristotle's fluctuations in dealing with the modal notions. My suggestion is to proceed with the analysis of the definitions of necessity and possibility we have proposed, because they present interesting problems.

An obvious one is that our definitions of the modal notions are far from being modally free. To be convinced of this, it is sufficient to point out that two elements are present in the definitions of the modal operators according to our interpretation. One is the logical constant for the absurd, " \perp ," which does not involve any modal commitment. The second element is deduction, our " \models ," which expresses what Aristotle calls *συμβαίνειν*; this element does have a modal implication. Aristotle's definition of *συλλογισμός* in the version that appears at the beginning of the *Prior Analytics* provides sufficient evidence for this.³³ Without getting too caught up in the discussion of the many problems this text raises, I shall limit myself to a matter-of-fact observation. The following of a conclusion from its premises is specified by a modal qualification: It must be a *συμβαίνειν ἐξ ἀνάγκης*, a following of necessity. Here a clear modal commitment is implied. Therefore,

the definition of possibility and necessity in terms of deduction or nondeduction of something absurd is circular.

A way to resist this conclusion is to point out that in the definition of συλλογισμός offered in the *Rhetoric* there is no mention of the necessity that qualifies the following of a conclusion.³⁴ But when we notice that in all other versions of the definition a reference to the necessity of the following is made, we begin to suspect that the *Rhetoric* passage does not remove our difficulty.³⁵

One might try to rescue Aristotle's modal definitions from circularity by claiming, against the view held by most scholars, that the definition of syllogism is not a definition of deduction or inference in general, but rather that it refers to a special kind of deduction.³⁶ We cannot consider this difficult problem here.³⁷ However, even if one admits that the definition of syllogism does not represent a definition of deduction in general, in order to defend Aristotle, one should accept that there are inferences in which the following of the conclusion from the premises does not need to be qualified as necessary. But this seems hardly plausible to me.

I do not see any real difficulty in the circularity of the Aristotelian definitions. By examining the way in which Aristotle thought of the modal notions, we cannot expect to find proper definitions of these notions. They are too general to be defined. Even the modern characterization, although precise and in many ways rewarding, is far from being a true definition because it relies on the notion of a possible world, which makes the alleged definitions circular. However, the interpretation of the modal notions via the possible-worlds machinery has the advantage of allowing us to understand better what we may mean by "necessary" and "possible," at least in the sense that it helps correct part of our intuition about these notions by rendering them, to some extent, precise and unambiguous. This, at best, is what we should expect from Aristotle's characterizations of modalities: a sort of elucidation allowing us to understand his own intuition about them.

But there are more pressing problems. To claim that a proposition is necessary if its negation implies a contradiction is the same as saying that every necessary proposition is a theorem. Let us pause to analyze this view. Being a theorem depends upon the theory on which we are working, its logical and linguistic power, and the set of axioms it uses. Without any reference to a theory, we cannot even speak of theoremhood. To solve this problem one might think that the provability and unprovability that make a proposition either necessary or possible are in the frame of logic. A proposition ϕ is necessary if it is a theorem of logic, and it is possible if nothing absurd follows by means of logical axioms only. But such a solution to our problem can easily be dismissed. Even if the borderline between logic and other disciplines may be thought to be vague to some extent, it is clear that a proposition such as

(7) every man is an animal

cannot be a theorem of logic. But Aristotle considers (7) to be a necessary proposition;³⁸ therefore, according to our interpretation, it must be a theorem. The conclusion is that Aristotelian necessary propositions cannot be confined to theorems of logic. With a parallel argument, we can exclude that metaphysics or ontology is a

good candidate for the theory that allows us to distinguish necessary from possible propositions. On the other hand, the theory in which we distinguish theoremhood from non-theoremhood cannot have an inconsistent set of axioms; otherwise, every proposition could be provable in this theory. Therefore, we are looking for a theory, the axioms of which are not chosen at random, but have, at least in the weak sense of a relative comparability, a claim to truth. That rules out dialectic as our theory because there is no requirement of consistency for the premises of dialectical arguments, not even for the ἐνδοξόταται, the more generally accepted of them.³⁹

What remains? It is difficult to say. One might suggest that the best candidate for the role of the theory we are looking for in the Aristotelian context is science. Of course, Aristotle is well aware of the fact that not all scientific theorems can be produced by one and the same science.⁴⁰ We have to face a plurality of independent sciences. Each science has its own appropriate axioms, only some of which are shared by other sciences.⁴¹ In this perspective, we can revise our definitions of modal operators by saying that a proposition ϕ is necessary if and only if there is a science with respect to which ϕ is a theorem. On the other hand, ϕ is possible if and only if in no science ϕ entails something absurd, namely if and only if in no theory $\neg\phi$ is derived. In this way, we are allowed to state

(LD#) $\Box\phi$ iff $S \models \phi$ for some S

and

(MD#) $\Diamond\phi$ iff $S \not\models \neg\phi$ for every S

where “ S ” stands for science.

We solve our question this way, but now must face new difficulties. The most obvious is that Aristotle uses modal notions to define science, as the characterization of science at the beginning of the *Posterior Analytics* clearly shows.⁴² There, we turn to the notion of science to define necessity and to distinguish it from possibility. The definitional circularity becomes heavier and heavier, and I am not sure that, at this time, we should not be worried about it.

I am not at all confident that this problem can have a satisfying solution. However, I would like to propose a further line of approach, which I will try to sketch very briefly.⁴³ In the *Posterior Analytics* Aristotle seems to equate καθ’αυτό and ἀναγκάιον. With a rather controversial argument he proves that scientific premises, being necessary, must also be καθ’αυτά.⁴⁴ The καθ’αυτά at issue here are most probably those propositions in which either the predicate is part of the definition of the subject or the subject is part of the definition of the predicate.⁴⁵ Of them, Aristotle had already claimed, in a very complicated and obscure passage, that they are necessary.⁴⁶ If we take these assertions in a strict way, we face obvious objections.

Consider, for instance, a proposition such as

- (8) every equilateral triangle has the sum of its interior angles equal to the sum of two square angles

which is an application of a standard example of an Aristotelian scientific theorem and therefore a necessary proposition. It is easy to see that it is not a $\kappa\alpha\theta'\alpha\upsilon\tau\acute{o}$ proposition because it is neither the case that its predicate is part of the definition of the subject nor that, vice versa, the subject is part of the definition of the predicate. On the other hand, one could plausibly claim that the necessity of a proposition such as (8) *depends* on the definitions of its subject and its predicate.

We can try to modify our definitions of necessity and possibility in light of these considerations. First, we have to revise the notion of “absurd,” $\alpha\tau\omicron\pi\omicron\nu$, which we have formalized with “ \perp .” In the new perspective, the absurd is no longer a logical contradiction, but a proposition that is or implies the denial of the propositions that express the definitions of the terms involved in the propositions, of which we say that they are necessary or possible. Second, we must interpret the propositions to which modal operators apply as subject–predicate structures, the terms of which can be defined. Therefore, we will say that one of these propositions is, for instance, possible if and only if it entails neither the negation of the definition of its predicate nor the negation of the definition of its subject. Suppose that our proposition is a universal affirmative proposition, which we can represent with “ AaB .”⁴⁷ If we assume that “ $D[A]$ ” and “ $D[B]$ ” stand for the propositions including the definitions of A and B , then we can assume

$$(MD^+) \quad \Diamond AaB \text{ iff } AaB \not\models \neg D[A] \wedge \neg D[B]$$

as a definition of possibility, and

$$(LD^+) \quad \Box AaB \text{ iff } AoB \models \neg D[A] \wedge \neg D[B]$$

or, if you prefer definitions in the style of (LD#) and (MD#)

$$(LD^{++}) \quad \Box AaB \text{ iff } D[A] \vee D[B] \models AaB$$

and

$$(MD^{++}) \quad \Diamond AaB \text{ iff } D[A] \vee D[B] \not\models AoB.$$

The other Aristotelian standard types of propositions can be defined accordingly.

That the definitions of the modal operators are limited in this way to standard Aristotelian propositions, namely *a*- or *i*- or *e*- or *o*-predications, should not be a terrible disadvantage, because Aristotelian logic is a logic for those kinds of propositions only. The true problem is to see what happens if we try to apply these definitions to the modal syllogistic. We cannot face this very complicated question now. My impression is that we have to modify definitions (LD+) and (MD+) to get some results. But this is a different story and cannot be told here.⁴⁸

Notes

- 1 To mention only books, four volumes have been published in the last five years on this specific subject, namely Buddensiek 1994; Patterson 1995; Nortmann 1996; Thom 1996.
- 2 Hintikka 1973a. For an extensive discussion of Hintikka's views, see Waterlow 1982; Seel 1982: 251ff, and Rijen 1989: 59ff. For Nortmann's position see Nortmann 1996: 23 and 232–234.
- 3 The word “ἐνδεχόμενον” as expressing a \diamond -possibility: *Posterior Analytics*, 1.3, 23a38–39; the word “δυνατόν” as referring to ε -possibility: *On Interpretation* 13, 22b29 and 35. The words “δυνατόν” and “ἐνδεχόμενον” as hendiadys: *Posterior Analytics* 1.6, 74b37–38; finally, “δυνατόν” and “ἐνδεχόμενον” as synonyms: *On Interpretation* 13, 22b29 and 35. Cf. Buddensiek 1994: 25–26.
- 4 At *Prior Analytics* 1.14, 33a24–25 Aristotle refers to this passage as a “definition of possibility” (ὁρισμὸς τοῦ ἐνδεχομένου).
- 5 See Becker 1933: 7–11; Bochenski 1947: 68–69; Ross 1949: 327; Łukasiewicz 1957²: 191; McCall 1963: 66–70; Hintikka 1973a: 27–31; Granger 1976: 179–182; Wolf 1979: 102; Seel 1982: 158–160; Waterlow 1982: 16–17; Rijen 1989: 18 ff.; Buddensiek 1994: 19–25; Patterson 1995: 124ff.; Nortmann 1996: 161ff; Thom 1996: 36–37.
- 6 For instance, Aristotle, *Metaphysics* 9.4, 1047b19; *Prior Analytics* 1.15, 34b2; *Physics* 8.5, 256b11.
- 7 This is more or less the analysis of the definition contained in text (A) that Becker (Becker 1933: 7) offers.
- 8 For instance, Aristotle, *Metaphysics* 5.5, 1015a33–35; 12.7, 1072b11–13.
- 9 For instance, Aristotle, *Metaphysics* 5.12, 1019b30–32.
- 10 On this question, see Ross 1966⁴: 11–19.
- 11 For instance, Aristotle, *Nicomachean Ethics* 10.9, 1179a15.
- 12 Aristotle, *Posterior Analytics* 2.7, 92b26–34.
- 13 See, for instance, Aristotle, *Posterior Analytics* 1.23, 41a25–26; *Sophistical Refutations* 29, 181a31–32; *Physics* 4.8, 216a4–5; *On the Heavens* 3.1, 299a11–15; *Generation and Corruption* 1.2, 315b20–21; *On the Soul* 1.4, 408b33–34; *Generation of Animals* 1.18, 722a2–3; *Metaphysics* 3.1, 998a9–10; 3.6, 1002b30–32; 10.6, 1056b4–5.
- 14 Aristotle, *Prior Analytics* 1.23, 41b22–23; 2.17, 65a39–40; *Sophistical Refutations* 5, 167b22–23; *Prior Analytics* 2.12, 62b29–30; *Posterior Analytics* 1.11, 77a22–23; 1.24, 85a16–17.
- 15 Aristotle, *Prior Analytics* 1.7, 29b5–6, 1.44, 50a30–31.
- 16 Aristotle, *Prior Analytics* 2.11, 61a18–19, *passim*.
- 17 By taking “necessity” in the sense of (LD), Aristotle's statement in *Metaphysics* 5.12, 1019b23–24 according to which “that of which the contrary is necessarily true is impossible” can easily be interpreted by means of (ID), if “the contrary” is supposed to mean “the negation of.” In the same way, his characterization of possibility as something that applies to “that which is not necessarily false” at 1019b31–32 can be explained by means of (MD). It remains, however, that his previous definition of possibility at 1019b26–30 is wrong. See also Kirwan 1993: 158–160.
- 18 An alternative reading could be $\Box(A \rightarrow B) \rightarrow (\Diamond A \rightarrow \Diamond B)$ or also $\Box(A \rightarrow B) \rightarrow \Box(\Diamond A \rightarrow \Diamond B)$. For this question, see Nortmann 1996: 24–26. There is no doubt that here weak, and not strong, possibility is at issue.
- 19 Minio-Paluello 1962a: 31. Equally useless is the so-called *Translatio anonyma* (Minio-Paluello 1962c: 214).
- 20 Minio-Paluello 1962b: 164.
- 21 See, for instance, Hintikka 1973a: 190n26.
- 22 Nonetheless this translation has been embraced by Pacius 1623: 188 and Rolfes 1975: 31.

- 23 This line of interpretation has been taken by Pacius (Pacius 1623: 188) and Seel (Seel 1982: 202n77). Seel's rendering of the Greek looks more like a paraphrase than a translation (202): "*Wenn nun der mögliche Sachverhalt zu dem Zeitpunkt, zu dem sein der-Fall-Sein möglich ist, entstehen würde, und andererseits der unmögliche Sachverhalt (zu demselben Zeitpunkt, Zs. d. Vf), da sein der-Fall-Sein unmöglich ist, nicht entstehen würde.*" The corresponding Greek would be something like this: εἰ οὖν τὸ μὲν δυνατόν, ὅτε <τὸ> εἶναι <αὐτοῦ> δυνατόν <ἐστι>.
- 24 Alexander of Aphrodisias, *On Aristotle's Prior Analytics* 177.5–6.
- 25 See *LSJ*, s.v. B: 1265.
- 26 Kirchmann 1887: 32. The same translation is offered by Gohlke (1953: 56).
- 27 This is the translation offered in Jenkinson 1955. Similar translations in Tredennick 1962; Smith 1989: 22; Barnes 1984: I 54; similarly Tricot 1947: 72; Colli 1955: 128; Zanatta 1996: I 287. The same rendering can be found in Ross's paraphrase of the passage (Ross 1949: 333).
- 28 See endnote 23.
- 29 It may be that I have misunderstood the English translation and that it simply conveys the idea that in some cases possibility is possibility to be. If so, I withdraw my criticism.
- 30 To become convinced of this, it is sufficient to recall *On Interpretation* 12–13, where the modal oppositions are at issue.
- 31 I am especially thinking of Nortmann 1996: 233. His interpretation of text (F) is particularly ingenious and sharp but, I believe, based on a false starting point. It is obvious that in my interpretation the ὅτε-clause conveys a temporal reference that does not affect possibility, being equivalent to "in those cases in which . . ."
- 32 For the idea of using a possible-worlds semantics in the interpretation of text (F), I depend on Nortmann 1996: 32–33. However, I do not share his view that we can use a temporal interpretation as an alternative to the possible-worlds semantics. It is true that in order to illustrate the critical step of Aristotle's argument, we can proceed as follows: Suppose that *A* is possible; then, there is a time, maybe t_1 , in which *A* is the case; therefore, at t_1 *A* is the case and *B* is not, the latter being such that in no time is it the case (and in particular at t_1). But there is no hint in the text that a semantic based on instants is behind Aristotle's assertions. The reference to time involved by the notion of coming to be is a reference not to the modal operators but to the propositions or events to which the modal operators apply.
- 33 Aristotle, *Prior Analytics* 1.1, 24b18–22.
- 34 Aristotle, *Rhetoric*, 1.2, 1356b15–16.
- 35 See Aristotle, *Topics* 1.1, 100a25–27 and *Sophistical Refutations* 1, 165a1–2.
- 36 As far as I know, this view was first proposed by Jonathan Barnes in Barnes 1981. It has been reasserted by him in Barnes 1997. The same claim has been made by Smith (1989: xv–xvi) and Striker 1996: 203n2.
- 37 I deal with this problem in Chapter 1.
- 38 See, for instance, Aristotle, *Prior Analytics* 1.9, 30a30–31.
- 39 This clearly results from what Aristotle says about incisive arguments in *Sophistical Refutations* 33, 182b32ff.
- 40 See, for instance, Aristotle, *Posterior Analytics* 1.7, 75a38ff.
- 41 On the distinction between proper principles (which belong to only one science) and common principles (which are shared by more than one science), see Aristotle, *Posterior Analytics* 1.10, 76a37ff.
- 42 Aristotle, *Posterior Analytics* 1.2, 71b9ff.
- 43 I am grateful to Richard Patterson for having suggested this further interpretation during the discussion of an earlier version of this chapter read at Oriel College, Oxford in July 1995.
- 44 Aristotle, *Posterior Analytics* 1.6, 74a5–12. The argument is repeated at 75a28ff. On the problem of Aristotle's proof, see Barnes 1993: 126.

45 Aristotle, *Posterior Analytics* 1.4, 73a34ff.

46 Aristotle, *Posterior Analytics* 1.4, 73b16–24.

47 AaB is the standard way to represent an Aristotelian proposition that is universal and affirmative, with the predicate A and the subject B . AeB stands for a universal negative proposition, AiB for a particular affirmative, and AoB for a particular negative.

48 By the way, if we adopt definitions (LD+) and (MD+) we can solve a problem that has no solution with definitions (LD#) and (MD#). The reduction of necessity to provability has as an immediate consequence that a modal logic adequate for this claim must be shaped in the form of an S5 system. In this system,

$$(\dagger) \quad \Diamond\phi \rightarrow \Box\Diamond\phi$$

holds as a theorem. Actually, (\dagger) is the axiom characteristic for S5 (see Hughes and Cresswell 1996: 49). If we use (LD#) and (MD#) as definitions of the modal operators, (\dagger) amounts to stating that for every possible proposition we can prove in some science that it is possible, namely that its negation does not produce a contradiction in any science, and this claim looks very strange. With (LD+) and (MD+), (\dagger) becomes more manageable. For instance, to say of AaB that it is possible means to assert that from AaB we derive neither the negation of the definition of A nor the negation of the definition of B . To say that this fact is necessary is to claim that it is from the definitions of A or B that we can deduce this underivability. This view does not look completely implausible.

9 Logic and omniscience

Alexander of Aphrodisias and Proclus

I

Let us consider a proposition such as “Socrates is sleeping” or, if you prefer something more delightful:

- (1) Luciano Pavarotti is singing.

The predicate “is singing” – or “is sleeping” in “Socrates is sleeping” – contains an implicit reference to time. Let us call singular propositions in which there is such an implicit temporal reference “indefinitely tensed statements.” An obvious question arises about these sentences: Are they really propositions? If a Fregean semantics is adopted, an affirmative answer to this question is not straightforward. Gottlob Frege distinguishes the sense of a proposition, its propositional content, from its meaning or denotation, which is its truth value.¹ If the truth value of a proposition constitutes its extension, it cannot be the case that two utterances of one and the same proposition have different truth values. If P uttered at t_1 has a truth value different from the truth value of P uttered at t_2 , P at t_1 could not be the same proposition as P at t_2 , because they do not have the same extension. Now (1) is true only if the event A to which (1) refers occurs. But A sometimes does not occur: Luciano Pavarotti is not always singing. Therefore, the truth value of (1) depends on the time at which (1) is uttered, and consequently in one case it can be true and in another case it can be false. The conclusion is that (1) is not a proposition. But in the everyday language we use phrases such as (1) with reference to truth and falsity. It is therefore convenient to find some device in order to reduce these expressions to propositions. The simplest way of doing this is to consider (1) an incomplete proposition, which can be restated as a complete proposition when an explicit temporal reference is introduced, for instance, a date. Thus, (1) can be reduced to the proposition:

- (2) Luciano Pavarotti is singing at t_k

where “ t_k ” is a temporal constant. In (2), “is singing” no longer contains a temporal reference, and (2) does not change its truth value according to the time at which it is uttered. (2) becomes something like:

- (3) Luciano Pavarotti’s singing occurs at t_k

where there is no implicit temporal reference any more than there is in a mathematical statement. Willard Van Orman Quine and Nelson Goodman have notoriously defended this treatment of the indefinitely tensed statements by which they could be eliminated.² There are, of course, other ways of considering them, in which no reduction to atemporal statements is worked out.³ But I will not examine these approaches here, because they can hardly be reconciled with the assumption that a changeless entity that is all-knowing exists in so far as they are based on the idea that tensed statements cannot be detensed.⁴ I will consider later some doctrines in which the existence of such an all-knowing entity is presupposed.

II

I have insisted on describing the modern way of treating indefinitely tensed statements because I wish to emphasize its difference from the way that most of the ancient authors approached this subject. Aristotle, for instance, in *Categories* 5, 4a21–b2 says, rather crudely, that one and the same *logos*, that is, one and the same proposition, can be sometimes true and sometimes false, and he repeats the same point in *Metaphysics* 9.10, 1051b9–17. This latter passage allows us to determine the class of propositions that, according to Aristotle, can change their truth value. In any proposition, the verb “additionally signifies time” (*On Interpretation* 3, 16b6–7). For instance, “recovers” additionally signifies that now someone is getting better (16b8). It apparently follows from this definition that any proposition in which its verb is not specified by an explicit temporal reference is an indefinitely tensed statement, even if any other temporal token reflexive is taken away from the proposition. But one could ask whether a mathematical statement really has an implicit temporal reference according to Aristotle. Does

- (4) every triangle has the sum of its inside angles equal to two right angles
(hereafter *2R*-property)

additionally signify time? If one answers this question in the affirmative, then (4) means that every triangle always has the *2R*-property; otherwise, (4) simply means that every triangle has this property atemporally. I do not wish to discuss this difficult and controversial problem here.⁵ My point is simply that indefinitely tensed statements that are said to change their truth value are not mathematical statements such as (4). These statements must fulfill at least two requirements. First, they must contain an implicit temporal reference. Second, they must be contingent statements, that is, propositions that (unlike mathematical propositions) can be true and can be false. These two conditions are satisfied by the majority of

the statements whose subjects denote (sublunary) individuals and that do not contain an explicit temporal reference. Note that it is not only propositions in which a phasal property (or relation) is attributed to an individual that are to be counted among the indefinitely tensed statements but also many propositions in which a permanent property is truly attributed to an individual. According to Aristotle, a statement like

- (5) Luciano Pavarotti is a man

becomes false after Luciano Pavarotti's death (*On Interpretation* 11, 21a22–23). Therefore, even propositions that express nonphasal and essential properties of individuals are not always true. This probably depends on the fact that individuals of the sublunary world, after they have disappeared, cannot maintain the features they possessed throughout their existence. However, it has to be pointed out that, according to Aristotle, it is not always the case that the death of an individual implies that no property can truly be attributed to him. We can truly say that Homer is a poet after his death (*On Interpretation* 11, 21a25–28). At any rate, the majority of the propositions referring to individuals of the sublunary world are in the precarious situation of our “Luciano Pavarotti is singing,” independently of whether or not the property that is said to be true of the subject is phasal or not.

A similar view can be ascribed to the Stoics. The best evidence comes from Alexander of Aphrodisias. In a passage reported by Simplicius (*On Aristotle's Physics* 1299.36–1300.10 = *SVF* II 206), Alexander considers the conditional statement

- (6) if Dion is alive, then Dion will be alive.

Alexander attributes to the Stoics the view that (6) is not always true, because in the instant immediately preceding the death of Dion, its antecedent is true and its consequent is false. Therefore, one and the same proposition after having been true for a certain time becomes false because the event to which it refers changes.⁶ A particular case of propositions that change their truth values according to time is given by statements that the Stoics called “corruptible propositions,” that is, statements that, after having received a truth value for a given time, are destroyed in the sense that they no longer possess a truth value and therefore cease to be propositions. The example discussed by Alexander of Aphrodisias (*On Aristotle's Prior Analytics* 177. 25ff = *SVF* II 202a) against Chrysippus is

- (7) if Dion is dead, this (man) is dead

where “Dion” admits an anaphoric reference but “this (man)” does not. Before Dion dies, (7) is true because its antecedent is false. When Dion dies, its antecedent becomes true. Against all expectations, the same does not happen to its consequent because the latter loses any truth value, nothing being the reference of “this (man)” in “this (man) is dead.” Apart from the acceptability of such a doctrine,

what interests us is to emphasize that, according to Chrysippus, one and the same proposition can become false and even lose its truth value from being true.

The semantics that seem to be naturally associated with this view about indefinitely tensed statements are one that makes truth values properties of propositions that do not contribute to identifying them. If (1) can change its truth value without losing its identity, its being true and its being false cannot be taken as distinctive features of it. From this point of view, true and false must be considered on the same footing as phasal properties of individuals. The man who is now bald is the same man who had long hair twenty years ago. He is the same man even if twenty years ago he had a property that he no longer possesses. The property of having long hair (or being bald) does not affect the identity of the man at issue any more than the property of being true (or false) affects (1). Using the Aristotelian terminology, one could say that true and false are accidental properties of indefinitely tensed statements. Let us call "thesis (T)" or more simply "(T)" precisely the view that admits that one and the same statement can change its truth value and that true and false are accidental properties of propositions in the sense described. It is sensible to conclude that Aristotle and the Stoics adopted thesis (T). Practically, almost all knowledge we have of the individuals of the sublunary world falls under (T), in as much as it is concerned with individuals and events that change continually, come into being, and pass away.

III

That the ancient philosophers had a view about indefinitely tensed statements different from the modern standard one has been repeatedly observed. First noticed by Peter Geach, this point has been discussed and developed by Arthur Prior, and Jaakko Hintikka has recently tried to show how it is linked with some deep epistemological patterns of ancient Greek culture.⁷ The consequences of the admission of thesis (T) for the development of philosophical ideas in areas different from logic have not been widely studied yet as far as I know. I would like here to examine some aspects of the role that thesis (T) plays in the difficult and controversial discussion on the nature and limits of divine knowledge in post-Aristotelian philosophy. That there is a link between thesis (T) and the way of conceiving divine knowledge can easily be seen. In *Metaphysics* 12.9, Aristotle attributes to the first unmoved mover a very poor knowledge of our world. According to some interpreters, this unmoved mover has no knowledge at all of that which is different from him, whereas others think that this mover knows at least the general principles that constitute all things.⁸ However, the splendid isolation of Aristotle's god was bound to fail even among his followers.⁹ The idea that the world is ruled by provident gods became more and more familiar to a large number of Hellenistic philosophers. The Stoics, for instance, had made divine providence one of the relevant points of their system. According to them, the natural idea that every man has of god is an idea of a provident divinity, and the best way of justifying divination is by admitting that the gods exist and are provident toward man.¹⁰ The existence of a provident divinity was therefore founded on very common and

uncontroversial cultural patterns that no philosophical school could easily disregard. But if god is provident, he must know the things toward which he exerts his benevolent influence. In particular, god must know the individuals and the events that constitute the elements and the history of the world. Hence the problem: God is normally conceived as immutable; but god knows the individuals and the events of this world. By thesis (T) such a knowledge is a changeable knowledge, being sometimes true and sometimes false. Therefore, if the gods know the world, they must change, because they must modify their knowledge in order that it remain true. The immutability of the gods, their knowledge of changing individuals, and thesis (T) are inconsistent. Post-Aristotelian theology had to face this problem, and I will try to illustrate two of its solutions: one from Alexander of Aphrodisias, the other from Proclus.

IV

Full light has not yet been shed on the works traditionally attributed to Alexander of Aphrodisias. Unfortunately, some of the works that are relevant for our inquiry may not have been authored by him, but rather by his pupils.¹¹ It would be safer to speak here of an Alexandrinist doctrine of providence. Hereafter when I use “Alexander,” I mean “Alexander and his pupils.”

Alexander develops a rather original position. On the one hand, he is ready to distance himself from Stoicism. Let us quote an interesting example:

(A) It seemed right to me to be against the view held by the Stoics according to which the intellect, being divine, is even in the most ordinary things and there is an intellect and a purposive providence in the things of this world.

(Alexander of Aphrodisias, *Mantissa* 113.12–15)

Alexander does not deny that there is providence. His criticism affects the way in which the Stoics thought that god was provident. On the other hand, he maintains that only things of the sublunary world are subject to fate, because eternal things and regularly recurrent events of the upper world are exempt from it (*Mantissa* 181.6–22). If what Alexander says here about fate can be extended to providence,¹² one could see a polemical allusion to the view held by some former Peripatetics echoed in *De mundo* 6, 397b24ff and Diogenes Laertius 5.32. Besides, even if it is not true that things of our world are the intended result of the providential activity of the gods (*Questions* 2.21, 69.1–31), it cannot be inferred that the gods are provident toward things of the lower world in an accidental way only (*Questions* 2.21, 66.25–67.2), because an accidental providence, being an inconsistent notion, is no providence at all (*Questions* 2.21, 65.32–66.2).

More specifically, Alexander holds that the movements of the stars lead and master the things that are bound to generation and corruption (*Questions* 2.19, 63.22–28). This regulation is thought to be the result of the activity of a force (*dunamis*)¹³ that constitutes and preserves the different kinds of things (*Questions* 2.3, 48.15–22) and, in particular, makes the forms of the individuals persist

(*Questions* 1.25, 40. 30–41; 2.19, 63. 22–26). This point is confirmed by a fragment of the unfortunately lost *On Providence* quoted by Cyrillus of Alexandria, where it is said that god's providence preserves the forms of individuals.¹⁴ These passages have led scholars to think that according to Alexander, although divine providence reaches the sublunary world, it does not directly concern individuals.¹⁵ This interpretation is confirmed also from the Arabic translation of the lost treatise *On Providence* edited some years ago by Hans-Joachim Ruland.¹⁶ When criticizing the Stoic view, Alexander puts forward several arguments (14.7 ff Ruland) against the thesis according to which the gods are provident toward individuals of this world. At the same time, the Arabic translation allows us to solve some doubts that the works extant in Greek are not able to eliminate. The conception of providence as a force that gives order to the cosmos prevails in the surviving Greek works. The naturalism implicit in this view engenders the suspicion that providence is not coupled with any knowledge or, better, that it does not correspond to the fulfillment of an intended project planned by the gods. As far as I know, there is no evidence in the extant works in Greek that is able to disconfirm this hypothesis.¹⁷ But a plain declaration against a mechanistic conception of providence can be found in the Arabic version of the treatise *On Providence*. Alexander says:

(B) At any rate the view according to which god does not know what is generated through him is absurd. Thus it is necessary that the gods know better than anyone else that their specific nature makes good things. But if they know that, they must also know what depends on such a nature.

(Alexander of Aphrodisias, 66.9–13 Ruland)

It is therefore clear that a kind of knowledge is presupposed by divine providence. The gods know themselves and the things depending on them. This passage does not clarify the precise nature of the relationship between the knowledge that the gods have of themselves and their knowledge of this world. However, their being provident is not the result of a blind force deprived of knowledge.

Luckily enough, another fragment is preserved in the Arabic tradition that gives us some new information on the way in which the link between the gods and our world was conceived by Alexander. Averroes quotes a fragment of Alexander's commentary on Aristotle's *Metaphysics* 12 (*Lambda*):¹⁸

(C) Alexander says that the worst mistake is made by those who maintain that providence concerns all individuals, as the men of the tent do. Providence could flow from the celestial powers only if they possessed knowledge, as has been said before. But how is it possible that they have a knowledge of individuals which is always renewed and besides which is infinite?

(Alexander of Aphrodisias, fr. 36 Freudenthal)

Freudenthal points out that the use of the expression "men of the tent" to refer to the Stoics, combined with the reference to a previous discussion, raises the suspicion that Averroes is not quoting Alexander literally.¹⁹ Nevertheless, he thinks

that the passage corresponds to Alexander's doctrine, as is confirmed by the fact that what is stated in it does not match Averroes's views.²⁰ If this fragment can be trusted, a fundamental step toward the understanding of Alexander's position can be made. Alexander denies that the gods may be concerned with all individuals, and his argument is *ex impossibili*. Suppose that the gods were provident toward all kinds of individuals. Therefore, they must know all individuals. But such a knowledge is impossible, because it should be renewed again and again and should be infinite. Let us leave out the problem that a divine mind cannot know an infinity of things and consider the other reason given by Alexander: If the gods knew all individuals, then their knowledge would have to be renewed again and again. I assume that the renewing that is in question here has something to do with the changing of individuals and events of our world. In our world it happens that what is now a bearer of a property did not exist before and will not exist in the future. The knowledge about such individuals must change correspondingly, if we do not admit that it becomes false. If a is F at t_1 it is true to say at t_1 that a is F . But suppose that at t_2 a is not F anymore; then at t_2 it is not true to say that a is F , being true that a was F . The knowledge about a has therefore changed from t_1 to t_2 . According to this interpretation, the function of thesis (T) in our text is fundamental. Individuals of our world, being liable to change, cannot be known by an immutable mind such as god's mind.

To be cautious, one might say that Averroes's passage could be interpreted in such a way that there is no need of thesis (T). Suppose that one assumes that every statement concerning the future does not have a truth value before the event (or events) it denotes happens (happen). When one posits that

- (i) there will be a sea battle

what one says can be neither true nor false. Therefore, (i) is not a proposition, and there is nothing to know when one utters it. On the other hand, the past is fixed and immutable. Once a sea battle has occurred, the proposition

- (ii) there was a sea battle

is definitely true. Thus, one could think that god's knowledge increases in the sense that new propositions are always being known by him, namely the propositions that become true or false according to the flux of events. In this way there is a change in the knowledge of the individuals without propositions about them changing their truth values. No recourse to thesis (T) is demanded.

This interpretation is not very convincing. First of all, it is doubtful that Alexander maintained that propositions such as (i) have no truth value at all. He does not say it in the treatise *On Fate*,²¹ and the text that comes closest to this position simply states that (i) is not definitely true (or false), and not that it has no truth value (*Questions* 1.4, 12.13–17). Besides, it is by no means clear that a proposition concerning the past cannot change its truth value. Suppose that no sea battle has taken place near Salamis before 480 BC and that someone says in 481 BC:

(iii) there was a sea battle at Salamis.

This proposition is about the past. Therefore, it has a truth value and it is false by hypothesis. But it is difficult to deny that if it were uttered in 479 BC it would be true. Therefore, (iii) changes its truth value according to (T).

One could reply that, according to Alexander (see, for instance, *On Fate* 197.11–15) and the majority of ancient philosophers,²² propositions about the past do not change their truth values. Therefore, one could conclude that the argument, even if it is sound in principle, cannot be applied to Alexander. But what is meant by ancient logicians in general, and by Alexander in particular, is not that every proposition about the past cannot change its truth value, but that a proposition that truly asserts that a past event has happened is necessarily true, because the past cannot be changed. That is the same as saying that every true affirmative proposition about the past does not change its truth value. This claim is plausible: Once (iii) becomes true, it remains always true. But this point does not imply that a proposition that truly denies that an event has occurred always keeps the same truth value. (iii) is false before 480 BC and true after, and

(iv) there was not a sea battle at Salamis

is true before 480 BC and becomes false after this date. Even if Alexander did not explicitly consider this difference (as far as I know), from the mere fact that he says that a proposition such as (iii), if it is true, is always true, it cannot be concluded that he asserts that every proposition about the past cannot change its truth value. However, the same point can be made about statements concerning the present, such as (1). Even if it is admitted that propositions about the future have no truth value at all, (1), being about the present, has a truth value and this truth value changes according to whether or not the event it refers to occurs at the moment in which (1) is uttered. If Luciano Pavarotti is singing at the moment in which (1) is uttered, (1) is true, and it is false if Luciano Pavarotti is not singing at that time. Two different utterances of (1) can be imagined: one in which (1) is true and the other in which (1) is false. It follows that (1) changes its truth value. The moral of the story is that even if one is led to think that every statement about the future is undefined with respect to its truth value, there are propositions about the present and the past that change their truth values. If this interpretation were accepted, one would have to conclude that the knowledge of individuals that is mentioned in the passage reported by Averroes refers only to future individual events. It may be that Alexander also considered this kind of knowledge, but it seems to me to be arbitrary to maintain that he had only this one in mind. If he refers to any kind of knowledge of individuals, then his claim that knowledge of individuals is always renewed can reasonably be explained by appeal to thesis (T).

On Fate 30, a chapter that Alexander dedicates to the problem of the divine foreknowledge of future events, can be interpreted in a way that makes this text consistent with (and even confirm) the point I am trying to defend. I cannot offer here a detailed analysis of this rather controversial and difficult text, nor do I claim

that my interpretation is the only possible one. Things could be put as follows. What Alexander wishes to refute is the claim that one can infer that everything is by necessity from the fact that the gods are supposed to have foreknowledge of any future event (200.12–15). Letting “*A*” stand for any proposition about a future (allegedly) contingent event, the argument attacked by Alexander can take this form:

- (P1) For every *A*: if the gods have foreknowledge of *A* then *A* is necessary
- (P2) For every *A*: the gods have foreknowledge *A*
- (C1) For every *A*: *A* is necessary.

According to some interpreters, Alexander’s refusal of the argument would have led him to affirm that the gods are provident toward everything in the world and that they know everything: what is necessary as necessary and what is contingent as contingent.²³ I do not think that this interpretation, which is claimed to be founded on *On Fate* 201.16–18, is really warranted by this text. Alexander develops a rather complex argument there, and our passage takes on a different meaning if it is considered in its context. A first step of his reasoning is contained in 200.15–201.1, where he intends to assert that (P1) implies

- (P3) For every *A*: if *A* is contingent, the gods cannot have foreknowledge of *A*.

My claim is that Alexander’s next step is directed to refuting (P2). Let us expand it as follows. In order to conclude that there is no contingent proposition, something like (P2) must be assumed or, at least,

- (P2’) For every *A*: the gods can have foreknowledge of *A*.

That means that the gods have the possibility of knowing everything. But this is precisely the point that is disproved by (P3) if one assumes that there are contingent propositions. If *A* is contingent, then the gods cannot know it. According to Alexander (*On Fate* 201.1–6), (P3) simply states that the gods do not possess the power of knowing everything. Putting things in a slightly different way, the argument Alexander tries to counter shows that (P1), (P2), and the negation of (C1) cannot be true together. That simply means that one of them has to be refuted. Therefore, to deny the negation of (C1) does not depend on the argument itself, but on other assumptions that have to be proven. Alexander insists on his point. If (P2’) had to be admitted, one should conclude that the gods can know everything, and therefore, among other things, what is impossible to be known, as, for instance, the measure of the infinite (*On Fate* 201.6–13). Furthermore, if *A* denotes by hypothesis a future contingent event, then *A* must be foreknown as a proposition about a future contingent event, that is, as a contingent proposition, and that is ruled out by (P3) (*On Fate* 201.13–21). Alexander concludes his refutation by proposing a dilemma: *Either* one says that what is impossible becomes possible for the gods; in this case (P2’) becomes true but (P3) cannot be admitted

any more; *or* one maintains that what is impossible is impossible also for the gods; then (P2') has to be proven independently before being put as a premise of the argument (*On Fate* 201. 21–28).

Alexander stops his discussion in *On Fate* 30 with a quite unexpected remark:

(D) We, therefore, do not do away with prophecy or with the gods foreknowledge by saying that they make predictions about things in accordance with the way the things naturally are.

(Alexander, *On Fate* 30, 201.28–30; R. W. Sharples translation)

Whatever the meaning of this rather obscure sentence may be, it is hard to believe that Alexander legitimates here the possibility that the gods have a real foreknowledge of *all* contingent events. Actually the simple admission of thesis (T) does not rule out that some predictions can be true. Suppose, for instance, that someone truly predicts that

(F1) If John has a mole on his cheek, then John will not be a flute player.

It is easy to find a reasonable semantics according to which (F1) is always true.²⁴ One can think that before John's birth and after his death the antecedent of (F1) is false. Therefore, when John is not alive, (F1) is true. But the same holds during his life, because the consequent of (F1) will never be false, because the prediction is, by hypothesis, a true one. Therefore, (F1) never changes its truth value. Thus, (F1) cannot be ruled out from what the gods know by recourse to thesis (T). I do not claim that Alexander had exactly this point in mind when he attributes some kind of foreknowledge to the gods. His attitude toward prophecy is not very clear; he never explains how it could be possible, nor does he even say clearly that he accepts it.²⁵ However, if the statement I have quoted has to be taken to express Alexander's own view and not as a mere dialectical move, it could be reconciled with the use that is made of thesis (T) with reference to divine knowledge by restricting the set of the propositions about sublunary individuals that the gods cannot know. There are propositions about individuals that are predictive and that do not change their truth values. Knowledge of such propositions can safely be attributed to the gods, whereas propositions that change their truth values are out of their reach. At any rate it remains that thesis (T) has an essential function in limiting the domain of divine knowledge: The gods are not omniscient. At this price, the gods can still be admitted to be immutable.

V

We have seen, up to now, how thesis (T) has an influence on theology in the sense that its acceptance contributes to the assigning or denying of certain attributes to the gods and to the precise interpretation of the nature of their knowledge. Late Neoplatonism is perhaps an interesting example of the reverse influence. Proclus, for instance, will modify the traditional semantics of singular

propositions and abandon to some extent thesis (T) in order to keep divine immutability and omniscience. To understand how this goal is reached, we need to describe briefly Plotinus's view. According to him, being provident toward our world is an activity not of the One or the Intellect, but of the Soul. His providence is explained by the fact that our world is produced by the Soul in its contemplation of the ideas that constitute the Intellect (*Enneads* IV 3.11, 8–21). Plotinus is aware that the attribution of providence to the Soul could be thought to imply the attribution to it of a kind of knowledge of the contingent individuals and events of this world. He meets this difficulty when analyzing the problem whether the Soul has memory. According to him neither the Intellect nor even the Soul has memory; otherwise, they would change, because one remembers something that was present before and that is not any longer (*Enneads* IV 3.25, 10–20, 27ff). But if the Soul and the Intellect do not change – Plotinus continues – how can they know changing things? In particular, does the Soul change when registering and knowing the changing things toward which it is provident (*Enneads* IV 3.25, 20; IV 4.9, 1–9)? Even the way in which the problem is stated shows the influence of thesis (T). Plotinus's answer is no less dependent on (T). He denies that (T) does not apply to divine knowledge. He says:

(E) But what prevents such a being <from possessing memory in the sense of> perceiving, without variation in itself, such outside changes as, for example, the cosmic periods? Simply the fact that following the changes of the revolving Cosmos it would have perception of earlier and later; intuition and memory are distinct.

(Plotinus, *Enneads* IV 3.25, 20–24; S. Mackenna translation)

The idea that a divine being can have an immutable knowledge of things that are mutable is ruled out. Positively, Plotinus maintains that Zeus, that is the ordering power of the world – to be identified with the Soul (*Enneads* IV 4.10, 1–4) – is such because he embodies an immutable knowledge that comes to him from the everlasting contemplation of the Intellect (*Enneads* IV 4.13, 17–25). As things of this world reflect and grow out of one and the same nature that permeates them (*Enneads* IV 4.11, 9–11), so the world order depends on the everlasting vision of the intelligible world. This way of thinking is at work for instance in the following passage:

(F) Yes: for what must be stands shaped before the Cosmos, and is ordered without any setting in order: the ordered things are merely the things that come to be and the principle that brings them into being is Order itself; this production is an act of a soul linked with an unchangeable established wisdom whose reflection in that soul is Order. It is an unchanging wisdom, and there can therefore be no changing in the soul which mirrors it, not sometimes turned toward it and sometimes away from it and in doubt because it has turned away but an unremitting soul performing an unvarying task.

(Plotinus, *Enneads* IV 4.10, 9–15; S. MacKenna translation)

The knowledge that the Soul has of the world depends on the knowledge that it has of its archetypes. It is difficult to press Plotinus with more precise questions. For instance: Does Zeus know individuals distinctly? In Plotinus's system the question turns out to be: Is there a distinct knowledge of individuals in the knowledge that Zeus has of the archetypes that form the substance of the Intellect? This issue connects with the debated question about the existence of the forms of individuals.²⁶ I will not discuss these difficult problems here. However we suppose that Plotinus solved these issues, the influence of thesis (T) on his view is evident. If no distinct knowledge of contingent individuals is admitted on the part of god, this happens precisely because of thesis (T). On the other hand, let us assume that Plotinus recognized that god has a distinct knowledge of individuals. Then, the consequences of thesis (T) are exorcized only if one thinks that the knowledge of individuals is given to god, so to speak, a priori in their archetypes. In this way, god's immutability is saved. But one could ask at what price his omniscience is preserved. What the Soul knows is not individuals, but their forms. Now either individuals are the same as their forms or there is some difference between them. If the first case is chosen, divine omniscience is rescued, but the distinction between the material and the intelligible world vanishes and, at the very end, the possibility of attributing a certain degree of reality to our world. On the other hand, if the second option is preferred, the sensible world can be distinguished from the intelligible one but god is not omniscient any more. He does not know individuals of this world, but their immutable and never-changing forms. Individuality, mutability, and temporality are all excluded from god.

VI

Plotinus's approach to the problem of omniscience is, in fact, not very different from Alexander's solution, apart from the metaphysical context in which it is embedded. It is only with the late Neoplatonists that a new position is sketched out. I will consider Proclus as the reference point for describing this change of view. Proclus attributes the job of being provident to the superior gods, the Henads, intermediate entities between the One and the intelligible world.²⁷ What is peculiar to Proclus is that gods take care of individuals and even the most negligible aspects of reality (*Platonic Theology* 1.15 (69. 10–12, 70.22–25, 74.9–16 Saffrey-Westerink)). That implies that the gods must know not only what is universal and eternal but also what is particular and contingent up to the minimal details of individuals (*Platonic Theology* 1.21 (98.5–12 Saffrey-Westerink); *Elements of Theology* 124 (110.10–13 Dodds); *Ten Doubts Concerning Providence* 2.23 Isaac). Proclus insists on this point. When discussing how the gods know this world, he affirms that he is surprised that many Platonists, in order to justify the gods' omniscience, put the intelligible models of individuals and bad things in the divine mind (*Platonic Theology* 1.21 (98.16–20 Saffrey-Westerink)). A criticism of Plotinus's view could be seen in this remark if one is willing to attribute the admission of the existence of intelligible forms of individuals to Plotinus.

To recognize that the gods know individuals and contingent events of this world entails the usual difficulty: Divine knowledge changes according to the way in which the corresponding things vary. Proclus gives a new answer to this question. He does not give up the immutability and omniscience of the gods, but rather he prefers to modify thesis (T). His argument has several steps. A first one consists in an epistemological point. Because knowing is an activity of a knowing subject, the modalities of knowledge depend not on the nature of the object known, but on the nature of the knowing subject. This thesis can be found in its more general form in *Ten Doubts Concerning Providence* (2.7, 1–29 Isaac), and it is ascribed to Iamblichus by Ammonius, a pupil of Proclus (*On Aristotle's On Interpretation* 135. 12 ff). In the major works of Proclus this view is applied to divine knowledge. Let us consider the following passage from the *Elements of Theology*:

(G) For if the gods have all their attributes in a mode consonant with their character as gods, it is surely manifest that their knowledge, being a divine property, will be determined not by the nature of the inferior beings which are its object but by their own transcendent majesty. Accordingly their knowledge of things pluralized and passible will be unitary and impassive: though its object be a thing of parts, yet even of such the divine knowledge will be undivided; though its object be mutable, itself will be immutable, though contingent, necessary, and though undetermined, determinate. For the divine does not get knowledge extraneously, from its inferiors: why then should its knowledge be restricted by the nature of its objects?

(Proclus, *Elements of Theology* 124, 110. 14–23; E. R. Dodds translation)

This text gives us an idea of what, according to Proclus, was a modality of the knowledge depending on the nature of the knower. Knowledge is necessary or contingent, mutable or immutable, undetermined or determined according to the nature of the knowing subject. For instance, the knowledge of a contingent thing is contingent if the knowing subject is contingent, but it is necessary if the knowing subject is necessary. Let us concentrate on the modality of the immutable.²⁸ According to Proclus, then, the gods know immutably the things that are mutable. One writer has judged this position a vain effort to conciliate the two requirements from which Proclus himself had started: to guard the real contingency of the world, on the one hand, and to secure the omniscience of the gods, on the other (*Ten Doubts Concerning Providence* 2.6, 1–20 Isaac). In other words, Proclus would fall into Plotinus's difficulty. From a logical point of view, he should either give up divine omniscience or eliminate contingency and mutability from the world.²⁹ But I think that Proclus is free from this difficulty. First of all, it must be noticed that Proclus does not say that some properties of the objects of knowledge change according to the fact that these objects are known by us and by the gods. What is contingent and mutable does not become necessary and immutable for a divine mind. If it were so, the gods would be mistaken, because, according to Proclus, contingent things are really contingent. What he underlines is that the *knowledge* of what is contingent is not necessarily contingent and that

the *knowledge* of what is mutable is not necessarily mutable. The modality of the objects known is not in question here, but the modality of their knowledge. Second, Proclus insists that the gods, being transcendent, know in a unified way what is manifold and what is stretched out in time. Besides *Elements of Theology* 124, which we have already quoted, the following passage from the *Platonic Theology* is worth keeping in mind:

(H) For, since the gods transcend things which are eternal and things which are in time, they keep gathered in themselves the truth of each and every thing in a single and unified truth.

(Proclus, *Platonic Theology* 1.21, 99. 6–9 Saffrey-Westerink)

In light of these remarks, I propose to interpret Proclus's point as a question of semantics. Let us consider again our initial statement:

(1) Luciano Pavarotti is singing

and let us imagine that (1) could be the content of an act of divine understanding. (1) was traditionally conceived as a proposition that changes its truth value when the event denoted by it changes. Proclus does not deny this fact, but he interprets it in a way that does not affect the immutability of divine knowledge. A possible way of explaining how this is possible is to assume that the semantics for statements like (1) is different from the semantics that is implied by (T). In (T), the truth value of (1) is not embedded in its meaning in opposition to Frege's view. We can guess that Proclus comes back to Frege – if one could say so – but with the difference that the meaning of the proposition is not given by a single truth value, namely the truth value that the proposition takes at the time of its utterance, but is determined by the sequence of the truth values that the proposition takes in time. Using standard mathematical jargon, according to this view, (1) becomes a function that associates truth values to instants. Let us assume that Luciano Pavarotti is really singing at t_j and t_k and that he is not singing any more at t_m (where $t_j < t_k < t_m$). Then (1) associates the truth value True – say “1” – with t_j and t_k and the truth value False – let us call it “0” – with t_m . Therefore, the extensional meaning of a proposition, being a function, is given by a set of ordered pairs whose first member is an instant and whose second member is 1 or 0. In our example, the meaning of (1) is a set of pairs among which there are those containing t_j , t_k , and t_m :

$$(1) = \{ \dots, \langle t_j, 1 \rangle, \langle t_k, 1 \rangle, \dots, \langle t_m, 0 \rangle, \dots \}.$$

Let us call “*a*” the set that defines (1).

It is evident that *a* can be conceived differently according to the ways in which we consider how function (1) is defined. Suppose that the life of Luciano Pavarotti takes place between t_i and t_p . In this interval, (1) will probably be sometimes true and sometimes false. Therefore, *a* will contain pairs, the second members of

which are 1 or 0. But what about the instants before t_i and after t_p ? There are several possibilities that look familiar to people acquainted with modern temporal logic.

- (a) Nothing prevents us from thinking that function (1) is not defined before t_i and after t_p . If Luciano Pavarotti does not exist, it is meaningless to say that he is singing. "Existence is a presupposition for truth values" could be the slogan of this view. Therefore, (1) has to be conceived as a partial function that associates truth values only with a limited set of instants, namely the set of instants comprised between t_i and t_p .
- (b) It is also sensible to think that (1) is defined over all instants. To say something true or false about Luciano Pavarotti does not presuppose that Luciano Pavarotti exists. Of course, when Luciano Pavarotti is not alive, it is false to say that he is singing. Function (1) will have 0 as its value until t_i and the same will happen after t_p . In the interval between t_i and t_p , 1 and 0 are its possible values. Because instants are unlimited, according to this view α will be an unlimited set of pairs.
- (c) Let us consider a third hypothesis. It is also reasonable to assume that function (1) is not defined before t_i and that it is always defined after t_i . Before Luciano Pavarotti was born, it was meaningless to state something about him, but after his birth any proposition concerning him is true or false, even if it is uttered after his death. Of course, saying that Luciano Pavarotti is singing after his death will always be false. In this case α will be an unlimited set in one direction only, and the sequence of the values of (1) will be constant after t_p .

If we limit ourselves to considering the values of (1) in the interval t_i - t_p we can abstain from deciding between the possibilities (a), (b), and (c), because their differences become relevant only if the instants before t_i and after t_p are considered. Let us take this route. It is now clear in what sense Proclus may say that our knowledge of (1) is different from the divine knowledge of it. Because (1) denotes a contingent event, α cannot be conceived as a law-like sequence in the interval t_i - t_p . Whichever partial stretch of the interval may have been covered, it cannot be forecast how the succession will be completed up to t_p . From the fact that Luciano Pavarotti was (or was not) singing in an instant belonging to the interval t_i - t_p it cannot be inferred that he will (or will not) be singing in the following instants up to t_p . Therefore, our knowledge of (1) in the interval t_i - t_p can change significantly. As time passes, our knowledge of α increases step by step by knowing new pairs that are not necessarily the same as the preceding ones, and the development of the resulting sequence cannot be known in advance. On the other hand, let us assume that there is a divine mind that is outside time. Such a mind is able to know every pair of the interval t_i - t_p altogether. Divine knowledge of (1) does not change in time, but is fixed and stable because it is all given in a kind of eternal present. This divine mind does not pass from one element of α to another, and in this sense it has a complete knowledge of what a human mind knows only partially and step by step.

An objection can be raised against this interpretation. There is a difference between the human and the divine way of knowing (1) in the interval t_i - t_p . My knowledge of the fact that Luciano Pavarotti is singing develops and increases along with his life, whereas no such development takes place in the divine knowledge. But is there still a difference when a human mind considers (1) after Luciano Pavarotti's death? When the history of Luciano Pavarotti is accomplished, the knowledge of the fact that he was singing is something stable that a human mind can grasp in a way that is similar to the divine way of knowing the same event. In other words, one could say that, after Luciano Pavarotti's death, (1) is always false or that it does not take a defined truth value. Consequently, the sequence of pairs forming α either becomes constant after or it stops at t_p . Therefore, a human mind that considers (1) after t_p can, at least in principle, have a knowledge of it as complete as the knowledge that the gods have. As Proclus himself says, the indeterminacy of what is contingent depends only on its being put into the future (*Ten Doubts Concerning Providence* 3.14, 225 Isaac).³⁰ It follows that with respect to a past event there is no indeterminacy and a human mind can get the same degree of certainty as the divine mind. This objection can be reinforced by supposing that time is not dense and that the interval t_i - t_p is constituted by a finite set of instants. If hypothesis (a) is preferred, α will be formed by a finite number of pairs that can be grasped by a human mind considering (1) after t_p . But in the case of (b) or (c), too, the infinity of α is not such that it prevents a human mind from knowing (1) completely. According to (c), α is constituted by a sequence of pairs that becomes constant after a certain point. In the case of (b), we have a sequence that is constant up to t_i and that becomes constant again after a finite interval. This kind of infinity does not represent a problem for a finite mind. If I know that α was born at t_i and that α died at t_p , I can say that all the values of $F(\alpha)$ before t_i and after t_p (if any) are 0, without needing to inspect them one by one. Therefore, the objection concludes that the semantics attributed to Proclus are not able to justify the distinction he states between the divine and the human knowledge of this world.

Against this difficulty two lines of defense are possible. First, it is not plain that, according to Proclus, time is discrete.³¹ If it is conceived as dense, then the interval t_i - t_p contains infinitely many instants. Therefore, the sequence of pairs forming α in the interval t_i - t_p is infinite and the human mind cannot grasp them completely, even if it considers (1) after t_p . It has to be remembered that the sequence of the values of (1) in the interval t_i - t_p cannot be determined in its future development from any partial stretch of it, because (1) denotes a contingent event.

This first answer, on the one hand, points to the limits of the human mind, but, on the other hand, it could lead one to think that it attributes too much power to the divine mind. If the gods know (1) completely, they must know infinitely many things, namely the infinitely many pairs that form α . But is it correct to confer such a power to the gods? Alexander of Aphrodisias found in the impossibility of grasping an infinity a reason for denying that the gods know all individuals (*On Aristotle's Metaphysics* 12, fr. 36 Freudenthal). Plotinus, although he maintained that the One and the other hypostases are infinite because they have unlimited powers (*Enneads* VI 9.6, 10–11; IV 3.8, 35 ff.), states that the ideal forms cannot

be infinite (*Enneads* VI 5.8, 39–42). What the Intellect knows is a finite number of forms.³² Does Proclus deviate from this well-established line of thought? If we are inclined to think that he followed Amelius, one of Plotinus's pupils who would not have hesitated to admit that ideal forms are infinite,³³ then we can conclude that the semantics we have sketched out for (1), together with the hypothesis of dense time, are sufficient to establish the radical difference of the divine mind from the human one. If, on the contrary, Proclus has to be put in the Plotinian tradition, we must give up the idea that time is dense and retire to the second line of defense we have spoken about before.³⁴ With inessential modifications, this argument can be coupled with the preceding one, if this latter is supposed to be valid, as a further coordinate answer to the difficulty we are discussing. It can be admitted that a man may attain a knowledge of (1) as perfect as the divine. Nevertheless, our knowledge is always lower than the divine one. The human knowledge of an individual is always *a posteriori*, that is *post factum*, whereas the gods, who are beyond time, know the same individual *ab aeterno*, and therefore even *ante factum*. From the point of view of men, the future is always obscure. It ceases being hidden only when it ceases being future. This implies that although men are in principle able to obtain, step by step, a complete understanding of past events, they cannot hope to know all events; otherwise, they would be at the end of time, and time has no end. For a divine mind, the future is plain as much as the past because there is a future and a past only with respect to things that are in time.

If this interpretation is accepted, Proclus has to be put in a different position not only from Alexander but also from Plotinus. Plotinus as much as Alexander could not really explain divine omniscience, because he was pressed on the one hand by the admission of thesis (T) and, on the other hand, by the idea that the gods are immutable. It is only by Proclus that the immutability and omniscience of the gods can be reconciled to a certain extent with the contingency of the world, because he implicitly adopts a new semantics for indefinitely tensed statements.³⁵

Even if this inquiry is far from being complete and satisfactory, I have to limit myself to two final remarks. First, the interpretation I proposed of Proclus's position contributes (I hope) to explaining the extraordinary success that this view had in Christian culture without casting trivial doubts on the philosophical capacities of its supporters.³⁶ Second, I would like to emphasize a point of method. We have seen that a logical doctrine, namely the semantics of indefinitely tensed statements, is connected with a theological problem affecting the question about the nature of divine knowledge. The thesis that logic, because of its formalism, has nothing to do with ontology is false not only from a conceptual but also from a historical point of view.

Notes

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- 1 Frege 1967: 149–150 (34–35).
- 2 Quine 1981¹²: 191–95; Goodman 1983³: 261–272.
- 3 The distinction between the two treatments of indefinitely tensed statements is clearly sketched out in Rescher and Urquhart 1971: 23–30. See also Sorabji 1980: 91–103.
- 4 Grim 1983: 272–274; Sorabji 1983: 258–260.
- 5 Hintikka 1973a: 82–84 is in favor of the omnitemporality of mathematical statements. See also Sorabji 1983: 125–127.
- 6 Whether this view is consistent with the doctrine of the eternal recurrence held by the Stoics (see, for instance, *SVF* II 625) is a question that cannot be answered here. See Barnes 1978: 3–20.
- 7 Geach 1949: 238–245; Prior 1967; Hintikka 1973a: 62–92.
- 8 David Ross (Ross 1949⁵: 182–183) is the champion of the thesis by which the first unmoved mover knows only himself. *Contra* Norman 1969: 63–74.
- 9 See, for instance, [Aristotle], *De mundo* 6, 397b24 ff.; Diogenes Laertius 5.32. Cf. Moraux 1970: 54. I follow Professor Owens's warning in avoiding capitalization of the word "god" (Owens 1963²: 171n47).
- 10 Cicero, *On Divination* 1.824 = *SVF* II 1192. For an analysis of the Stoic reasons in favor of the existence of divine providence see Dragona-Monachou 1976.
- 11 That is the case particularly for the *Quaestiones*. See, for instance, Todd 1976: 19.
- 12 Robert Sharples (Sharples 1983: 26) is in favor of this extension.
- 13 On this term, see Moraux 1967: 160n2 and Reale 1974: 79 ff.
- 14 Cyrillus of Alexandria, *Against Julian* PG 76, 625B–C.
- 15 Cf. Moraux 1942: 197–200; Happ 1968: 81–83; Hager 1975: 172–175; Todd 1976: 213n13; Donini 1982: 213; Sharples 1983: 25–26.
- 16 Ruland 1976. On the Arabic translation of the *On Providence*, see also Thillet 1960: 313–324. My translations of this work are from Ruland's German version.
- 17 See the discussion of the evidence in Moraux 1942: 200–202.
- 18 The commentary to Aristotle's *Metaphysics* 6–14 that the tradition has attributed to Alexander is not by him (cf. Moraux 1942: 14–19). The true commentary of Alexander is lost except for some fragments preserved by Averroes in his own commentary to the *Metaphysics*. These fragments have been collected in Freudenthal 1885: 1–134. Freudenthal has shown that Averroes was normally accurate in quoting Greek commentators known by him through Syrian versions.
- 19 Freudenthal 1885: 112n7 and 60.
- 20 For instance, in the *Tahafut al Tahafut* (*The Incoherence of the Incoherence*, translation from the Arabic with introduction and notes in S. Van den Bergh, 2 vols. London 1969), Averroes discusses and rejects the theses according to which god would have only a general knowledge of individuals. He says that divine knowledge cannot be qualified either as universal or as particular, being a creative act (462, I.280–281 Van den Berg).
- 21 Cf. Sharples 1983: 11–12.
- 22 The most notable exceptions are supposed to be constituted by Cleanthes and Antipater (*apud* Epictetus 2.19.2 = *SVF* I 489).
- 23 Verbeke 1968: 97. Sharples is much more cautious. Cf. Sharples 1978: "He [*sc.* Alexander] does however suggest – though only hypothetically, his concern being to refute his opponents rather than to establish a position of his own – that the gods have foreknowledge of the contingent in a sense, foreknowledge of the contingent as *contingent*" (260). The same point is made in Sharples 1983: 165.

- 24 This semantics is not inconsistent with the assumptions Alexander makes in his *On Aristotle's Prior Analytics* 177.25 ff. against the Stoics.
- 25 See Sharples 1983: 18–19.
- 26 For *status quaestionis* of this problem, see Armstrong 1977: 49–68.
- 27 See Proclus, *Elements of Theology* [*Institutio Theologica*] 114.120 Dodds. On the origin of the doctrine of the Henads see Dillon 1973: 412–416.
- 28 It is interesting to remark that the pair “determined-undetermined” is used by Proclus (*On Providence* 63–66 Isaac) and by Ammonius (*On Aristotle's On Interpretation* 136.1 ff.) in order to explain divine foreknowledge of contingent future events and to avoid determinism.
- 29 This is the view of Wallis 1981: 227. It would be unfair to underline my disagreement from him without saying how much I owe to his learned article. A similar point is made in Hager 1975: 179.
- 30 Orbetello 1981: 136–137 interprets this passage differently. From *Ten Doubts Concerning Providence* 3.14.22–28 Isaac, Orbetello infers that, according to Proclus, what is undetermined is the same as what is contingent. Because to be present, past, or future depends on us to a certain extent, the same will hold also for what is contingent or necessary. I assume that this point could be proposed by Obertello as a justification of the thesis that the gods have a necessary knowledge of what is contingent. But one could ask whether this way of making the notion of contingency relative entails the impossibility of what Proclus himself admitted to be the starting point of his enquiry, namely that contingency is a real modality of things, “*natura quaedam in entibus*,” as he says (*Ten Doubts Concerning Providence* 2.6.16). Besides, the notion of contingency that Obertello attributes to Proclus, making him an ideal pupil of Diodorus Cronus, cannot be drawn from *Ten Doubts Concerning Providence* 3.14.20–24 Isaac, as he claims. In this passage that Obertello quotes in the translation of Isaac Comnenus, Proclus says: “(a) ὅτι μὲν οὖν πᾶν (b) καὶ τὸ ὅπως οὖν ἀόριστον ἔχει (c) καὶ τοῦτο ὃ φασι ἐνδέχασθαι, [. . .] (d) καὶ οἱ στοχασμοὶ δηλοῦσιν.” Obertello takes τοῦτο in (c) to refer to τὸ ἀόριστον of (b) and implies ἐστὶ after this pronoun. The passage could be read in another way. One could take τοῦτο as proleptic of the relative pronoun and coordinate (c) to τὸ ἀόριστον of (b). In both ways, the sense of the passage will be that what is undetermined takes its indeterminacy and contingency from the fact of not yet being. To be future becomes the same as to be contingent. But neither of these interpretations seems to me to be sensible because of the καὶ in (b) that naturally corresponds to the καὶ introducing (c). The same link can be found in the Latin translation of William of Moerbeke on which the version of Isaac Comnenus depends: “*quod quidem omne qualitercumque indeterminatum in nondum esse indeterminationem habet et hoc quod aiunt contingere*” (22–25). The meaning of the passage becomes the following: What is undetermined *and* what is contingent have their indeterminacy in the fact that they do not yet exist. In other words, being future is the reason, not of an events being contingent, but of its being undetermined, that is, of the fact that the proposition that expresses this event cannot be said to be true or false before the event occurs. What predictions confirm is not that the only contingent thing is the future, but that there is a link between indeterminacy and future.
- 31 Flowing time is said to be continuous by Proclus (e.g., *On Plato's Timaeus* 3.27.2 Diehl). On his conception of time cf. Sambursky and Pines 1971: 17–18 and Sorabji 1983: 184–185.
- 32 Armstrong 1954–1955: 51.
- 33 Syrianus, *On Aristotle's Metaphysics* 147.1–6.
- 34 But it is difficult to refuse to the gods a knowledge of an infinity of things if they are supposed to know individuals of this world with their histories all together.
- 35 Of course, not all problems of divine omniscience are solved by Proclus. Some of them are connected with the so-called essential indexicals. See Kretzmann 1966: 409–421; Grim 1983: 272–274; White 1983: 40–62.

- 36 Proclus's solution was anticipated by Saint Augustine (*The City of God* 11.21, 339. 12 ff; *To Simplicianus* 2.2.2) and restated by Boethius (*On Consolation of Philosophy* 5.3.7, 6.6) through Ammonius (*On Aristotle's On Interpretation* 132.8 ff.) or its Neoplatonic source (on the dependence of Boethius on Ammonius see Orbetello 1974: I, 522–544 (especially 540–542). The path that reaches Thomas Aquinas and other medieval masters has yet to be well explored (cf. Wallis 1981: 227–232. For the whole problem, see also Sorabji 1983: 253–267.

10 Ammonius on future contingent propositions

I

Aristotle's *On Interpretation* 9 is a difficult text, and I would not like to be trapped in a discussion of its complexities.¹ My aim is more modest and commensurate with my capacities: to consider, on its own, Ammonius's solution to the problem of future contingent propositions without feeling committed to evaluating his view as an interpretation of Aristotle.

The problems raised by future contingent propositions are many, and some of them have to do with the question of determinism. If it is now true that a sea battle will take place tomorrow, it cannot be the case that the sea battle does not occur tomorrow; otherwise, it would not be true today that there will be a sea battle tomorrow. That there will be a sea battle tomorrow has forever been fixed and determined. The future in this way appears to be unpreventable and necessary. Therefore, the question can be asked whether it is legitimate, in connection with the events of the world, to speak of contingency in any proper sense.

Aristotle admitted the existence of truly contingent events and corresponding truly contingent propositions. According to many scholars, his answer to the deterministic argument is that future contingent propositions are neither true nor false before the time of the events expressed by them. So the famous Aristotelian proposition

(1) there will be a sea battle tomorrow

cannot be properly called true or false before tomorrow. Unfortunately, this interpretation, which is usually called the traditional interpretation, is not shared by all scholars, and it may be the case that it is not Aristotle's view.²

As I said, I am not concerned with Aristotle. What is relevant to me is that Ammonius is not a follower of the traditional interpretation. This view is not new, because it has been convincingly defended by Richard Sorabji and Bob Sharples,³ and I do not need to spend much time illustrating it.⁴ The core of Ammonius's solution consists in the distinction he proposes between what is definitely and indefinitely true or false. To get an idea of the way in which the distinction is formulated by Ammonius, we may read the following passage:

(A) <Aristotle> simply says that singular propositions concerning the future divide truth and falsity, but not in the same way as propositions concerning

the present or the past. For it is not yet possible to say which of them will be true and which will be false in a definite way (ὀρισμένως), since before its occurring the thing can occur and not occur.

(Ammonius, *On Aristotle's On Interpretation* 130.20–26)

Propositions that do not divide truth and falsity in a definite way are said by Ammonius to be indefinitely (ἀορίστως) true or false:

(B) This is precisely what we are looking for now, i.e., whether every contradiction divides truth and falsity in a definite way, or there is some contradiction which divides them in an indefinite way (ἀορίστως).

(Ammonius, *On Aristotle's On Interpretation* 131.2–4)

The metaphor of dividing truth and falsity is customary among Aristotle's commentators and can easily be explained by reference to the principle of bivalence. The principle of bivalence is usually distinguished from the law of excluded middle. The latter says that a proposition P either is the case or is not the case. We can express it by

(LEM*) $P \vee \neg P$.

If we introduce a truth predicate T into the language, we can restate (EM*) as

(LEM) $T(\ulcorner P \urcorner) \vee \neg T(\ulcorner P \urcorner)$ ⁵

and take (LEM) as the law of excluded middle in the extended language. On the other hand, the principle of bivalence asserts that a proposition is either true or false:

(PB) $T(\ulcorner P \urcorner) \vee F(\ulcorner P \urcorner)$.

We can, then, say that two contradictory propositions, for instance, P and $\neg P$, divide truth and falsity if they are such that one is true and the other false, namely if they satisfy (PB).⁶

Ammonius points out, in the text just quoted and in several other passages, that it is not only propositions concerning the past and the present but also propositions concerning the future that divide truth and falsity. For instance, in trying to elucidate the meaning of Aristotle's οὐχ ὁμοίως at 18a28, he claims that pairs of contradictory singular future contingent propositions

(C) divide at any rate truth and falsity, however not in a definite but an indefinite way. It is necessary that Socrates tomorrow either will or will not bathe and it is not possible that both or neither are true.

(Ammonius, *On Aristotle's On Interpretation* 139.15–17)⁷

It is hard to believe that a follower of the traditional interpretation would have subscribed to such a view, which implies that (PB) holds not only for past and present but also for future contingent propositions. If (1) is supposed sometimes to have no truth value at all, it always satisfies (LEM) and it sometimes does not satisfy (PB): Because it has no truth value now, (1) is not true now, and in this sense (LEM) holds for it even now. However, if (1) is now neither true nor false, (PB) cannot hold for it at any time.⁸

This seems to me the strongest argument against those who place Ammonius among the partisans of the traditional interpretation. I do not want to dwell any longer on this question. Let me simply stress a consequence of Ammonius's admitting that (PB) holds unconditionally. We are allowed to say that both definitely true and indefinitely true propositions are true. This means that a proposition that is indefinitely true cannot be labeled allegedly true or quasi-true.⁹ It is really true no more and no less than any other true proposition. Indefiniteness (or definiteness) qualifies the way in which a proposition is true, just as biped and quadruped determine types of animals. A biped is no less an animal than a quadruped, and a proposition that is qualified as indefinitely true or false is no less true or false than any other proposition.

This interpretation is confirmed by Boethius. It is known that Boethius does not depend directly on Ammonius for his commentary on Aristotle's *On Interpretation*; but the similarity of their treatment of future contingent propositions strongly suggests that they draw their inspiration from a common source.¹⁰ Therefore, we can use Boethius to some extent to understand Ammonius. In a polemical statement against the Stoics, Boethius expressly denies that future contingent propositions can be neither true nor false:

(D) Some people (among them the Stoics) believe that Aristotle said that contingent propositions in the future are neither true nor false. They took his statement that <the contingent> is no more related to being than to not being as a statement that there is no difference in considering <the corresponding propositions> as true or false. For they thought that these propositions are neither true nor false. But falsely. Aristotle does not say this, namely that both <members of a contradiction> are neither true nor false, but rather that one or other of them is true or false, not however in a definite way as in the case of past and present propositions.

(Boethius, *On Aristotle's On Interpretation* 2a, 208.1–11)

Could anything be clearer than this? Boethius rejects the traditional interpretation. If A and $\neg A$ are future contingent propositions, it is not true to say that they are neither true nor false. They are true or false, but not definitely true or definitely false, and in this sense they satisfy not only (LEM) but also (PB).¹¹ We may properly conclude that the same holds for Ammonius.

II

Before making any attempt to understand Ammonius's claim as it emerges from our text (A), we must briefly describe how he proposes to characterize the critical propositions discussed in *On Interpretation* 9. According to Ammonius, they

are temporally qualified with respect to the future, in the sense that they refer to future events.¹² In this he seems merely to repeat Aristotle. What is more interesting is that Ammonius states more clearly than Aristotle does that the propositions in question are not only future but also contingent. From the fact that Aristotle employs the expression “ἐπὶ δὲ τῶν καθ’ ἕκαστα καὶ μελλόντων”¹³ to qualify what is at issue, where “μελλόντων” rather than “ἐσομένων” is used, he infers that the events and propositions in question are contingent events and propositions,¹⁴ or, following Ammonius’s way of putting it, “propositions on contingent matter” (κατὰ τὴν ἐνδεχομένην ὕλην).¹⁵ This means that the propositions in question are not propositions in which modal operators are explicitly stated, but propositions that are said to be contingent because they refer to contingent events.

Finally, Ammonius stresses that the propositions discussed by Aristotle are singular. This is the straightforward and obvious interpretation of Aristotle’s ἐπὶ δὲ τῶν καθ’ ἕκαστα at 18a34. What is strange is that Aristotle’s main example is (1), which is not, on the most natural construction, a singular proposition. In fact, Ammonius never quotes (1);¹⁶ he prefers examples such as

(2) Socrates will bathe tomorrow.

In (2) a pseudo-date is used, but there are also examples where the futurity of the event in question is left open, as in

(3) this sick person will recover.¹⁷

However, most of his examples *do* contain a pseudo-date.¹⁸

It is easy to see that Ammonius assumes that there is a correlation between contingency, futurity, and the way in which a proposition is qualified in its truth value. Past and present propositions in any matter – that is, past and present propositions that can be truly qualified as necessary or contingent – divide truth and falsity in a definite way, and in this sense they are definitely true or false.¹⁹ On the other hand, being indefinite in its truth value is something that can only happen to a proposition concerning the future. However, not every proposition concerning a future event is indefinite in its truth value. If the event referred to is necessary, the proposition expressing it is definitely true or false.²⁰ But the same happens for a contingent event, when all the conditions for its realization are given. The following passage more or less makes this point. Ammonius claims that, in some cases, it is possible to have definite knowledge of future events. He says:

(E) It is clear that even for our knowledge it is sometimes possible to know in a definite way (ὀρισμένως) what is contingent, i.e., when it is no longer contingent in the proper sense but follows by necessity from the causes which precede its generation. A sphere lying on a plane parallel to the horizon can be moved or not by someone, if the plane preserves its position. But if the plane is inclined, it is impossible for the sphere not to move.

(Ammonius, *On Aristotle’s On Interpretation* 137.1–7)

The example of the sphere shows what kind of contingency is at issue with future propositions. A sphere lying on a horizontal plane may be moved or not. It depends on someone's decision. Before the decision is taken, it is open whether the sphere will be moved or not. But after the decision, and when the plane has been inclined, the moving of the sphere cannot be prevented, and in this sense it is no longer open for it to move or not. Because all past and present events cannot be changed, only the future is open, at least for those events for which the causally sufficient conditions for their being or not being are not yet given. This text implies that one and the same proposition can be treated as necessary or contingent according to the different situations to which it is tied. If at a given time before the starting of the battle the decision of the admirals is taken and this means that the event is unpreventable, then at that time (1) is no longer a contingent proposition. On the other hand, before the decision, the future of the battle is still open, and in this sense it is contingent that the battle will take place.

What is in question is definite or indefinite knowledge, a notion that is not the same as having a definite or indefinite truth value. However, one might reasonably assume that we can have definite knowledge of a proposition *P* only if *P* has a definite truth value. On this assumption, the text implies that a future proposition concerning a contingent event may take a definite truth value when all the conditions for the realization of the event are given and it becomes unpreventable. This explains why present and past propositions are said to have a definite truth value. The events that they express are fixed. It is no longer open whether yesterday a sea battle took place. Either it happened or it did not, because the past and the present cannot be changed. To make the point in a different way, when a proposition has a truth value that cannot be different, it is a necessary proposition and it has a definite truth value. On the other hand, a future proposition concerning a contingent event has a truth value that might be different, and for this reason it is true or false in an indefinite way. This analysis suggests a further point about the way in which propositions such as (1) and (2) may be formally represented.²¹ A proposition concerning the past or the present is said to be necessary. The kind of necessity implied by it is not logical necessity, but a sort of historical necessity, the same necessity that is attributed to a contingent event when all the conditions for its realization are given. The historical necessity of a proposition means that it cannot change its truth value: It is either true or false, and this holds forever. Consider now a proposition such as

- (4) yesterday a sea battle took place.

According to Ammonius's account, (4) is a proposition about the past and it cannot change its truth value. But this may not be true if we assign to "yesterday" the proper meaning of a pseudo-date. Suppose that (4) is uttered today and that it is true. This means that yesterday a sea battle took place. Thus, it is still true tomorrow only if today a sea battle takes place, and so on every day. To attribute an unchangeable truth value to (4) we must take "yesterday" as referring to a fixed

date. Suppose that “yesterday” is a way to refer to Sunday, 23 June 1991. Then one might reasonably claim that the proposition

- (5) a sea battle took place on 23 June 1991

is definitely true or false at any time after this date.

The same point is made by Ammonius by discussing the so-called deterministic objection. According to him, this objection can be put as follows:

(F) <Aristotle,> starting the argument again from the beginning, says: “again if something is white now,”²² for instance a baby just born, “it was true to say”²³ the day before that a white baby would be born, and not only the day before, but also at any preceding time. What is the result? It is not possible that that of which we tell the truth by always saying before that it is the case will not be the case, just as it is not possible that that of which we tell the truth by saying that it is the case is not the case. Therefore, it was impossible that a white baby would not be born, because the prediction effected in the indefinitely preceding time is true.

(Ammonius, *On Aristotle's On Interpretation* 144.15–19)

The proposition that is true at any time according to the determinist cannot be

- (4') tomorrow a white baby will be born

because its being true at any time implies that a white baby is born each day.

To overcome this difficulty, we can proceed this way. Let us introduce a temporal constant in the propositions we are going to consider, say k , and take the content of the proposition as a predicate of the temporal constant. For instance, proposition (1) will be transformed into:

- (1*) a sea battle takes place at k

where k is a date and “takes place” has to be taken atemporally. In general, we can express the form of these propositions by “ $P(k)$.” Needless to say, (1*) does not correspond to (1) completely, because (1*) does not involve any reference to the future. To introduce such a reference, we shall consider a certain class of utterances of such propositions, namely those made at n , where $n < k$.²⁴ To be clear, in this analysis of a proposition such as (1), I do not claim that in general propositions containing pseudo-dates can be reduced to propositions in which any relevant reference to time is made by real dates and that the so-called A-series can be reduced to the B-series.²⁵ My point is simpler and weaker. In order to make sense of the way in which Ammonius uses a proposition such as (1), we may read it as (1*).

III

Before offering an interpretation of the distinction between definitely and indefinitely true (or false) propositions, we must reject an easy temptation. Suppose

that $P(k)$, a singular contingent proposition, is the case at k . Therefore, according to Ammonius at some time before k , say j , $P(k)$ is true and indefinitely true. One might try to explain this mysterious reference to an indefinite truth by connecting it to an epistemological situation. Before k , $P(k)$ has an indefinite truth value because we are not able to state it. Indefiniteness does not depend on the objective state of the events and propositions, but on our inability to grasp them adequately.

Some texts may be invoked as evidence for this interpretation. Consider the final part of text (A). In our translation, we have taken “ὀρισμένως” to refer to a proposition’s being true or being false. This interpretation is based on the many passages where “ὀρισμένως” specifies the truth value of propositions. But it would probably be more natural to refer “ὀρισμένως” to “ἔστιν εἰπεῖν.” If so, “ὀρισμένως” is the possibility of saying that a proposition is true or false even when this truth or falsity is not yet determined. In another passage, in order to explain why one of “Socrates will bathe tomorrow” and “Socrates will not bathe tomorrow” is true and the other false but both in an indefinite way, Ammonius says that “it is not possible to know which of them is true before the event occurs.”²⁶ Once again, the fact of having an indefinite truth value is explained by reference to an epistemological situation. In a parallel way, with reference to a pair of contradictory propositions concerning the past or the present, their having a definite truth value is explained by saying that “insofar as the event which is at issue has occurred, which of the two singular propositions is true and which is false is clear.”²⁷ One might take this statement to be asserting that a present or past proposition is definitely true or false because its truth value is clear – namely, it can be grasped.

Although this interpretation is attractive for its simplicity, it must be rejected. First of all, the distinction between definitely and indefinitely true (or false) propositions is appealed to in order to avoid determinism. But a purely epistemic undecidability cannot do the job. In this perspective, although I cannot decide about the truth or falsity of $P(k)$ at, say, j ($j < k$), $P(k)$ nevertheless has a fixed truth value, and this is sufficient to trigger the deterministic argument. In order to escape determinism, we need to interpret the distinction between definitely and indefinitely true or false propositions as an ontological distinction. Again, Ammonius more than once points out that contingent things have an indefinite nature,²⁸ and it is easy to suppose that the indefinite truth value assigned to propositions depends on the indefinite nature of the events expressed by them. This interpretation is confirmed by some statements made by Boethius, where the epistemological interpretation is explicitly rejected.²⁹

The conclusion is that we must look for a different interpretation of Ammonius’s distinction. The lack of knowledge or unclarity we have about the truth value of future contingent propositions is a consequence of their not having a definite status with respect to truth and falsity. Contingent propositions about the future are indefinitely true or false not because the future is hidden or unknown to our mind, but because the ontological status of the facts they refer to is not yet settled. What is uncertain is not the possibility of knowing before tomorrow that Socrates bathes tomorrow, but the event itself, because it is set in the future and is contingent.

IV

In order to construct a positive interpretation of Ammonius’s distinction, it may be useful to summarize the results of our inquiry so far. We have seen that the distinction between past and future contingent propositions depends on the way in which they divide truth and falsity. Past propositions are definitely true or definitely false, whereas future propositions are indefinitely true or indefinitely false. If (1) is interpreted as (1*), one can say that (1) is indefinitely true today and becomes definitely true tomorrow. This means that characterizing the truth value of a proposition as definite or indefinite depends essentially on the time at which the proposition is uttered or evaluated. Again, we must remember that a truth value’s being definite or indefinite depends on the causal milieu in which we consider the event denoted by the proposition. The reason is that an indefinite truth value applies to contingent propositions, and being contingent for a proposition entails that the event denoted by it is not yet causally determined.

In light of these remarks, let us try to specify the context in which a formal characterization of the notions of definite and indefinite truth can be made. Because propositions are definitely or indefinitely true or false with respect to the time at which they are uttered or considered and the situation of the world in which they are stated, we may express the predicates “definitely true” and “indefinitely true” of a proposition A as three-place predicates, the parameters of which are (i) the time in which a proposition A is uttered or considered; (ii) the state of the world in which A is evaluated; and, of course, (iii) A itself. Therefore, we write, for instance, “ $T_D(\ulcorner A \urcorner, S_i, t)$ ” to say that A is definitely true at time t and with respect to the situation of the world S_i . Similarly, “ $T_I(\ulcorner A \urcorner, S_i, t)$ ” is a way of saying that A is indefinitely true at t and with respect to S_i .

Moreover, because contingency must be taken as the open part of our future and linked to time, we can picture the world as a branching structure or a tree with nodes that represent possible states and paths that represent possible histories. Time can be interpreted as an ordered relation on the different stages of possible states of the world.³⁰ To make things easier we may think of the history of a single proposition and represent it by means of an ordinary binary tree as shown in Figure 10.1.

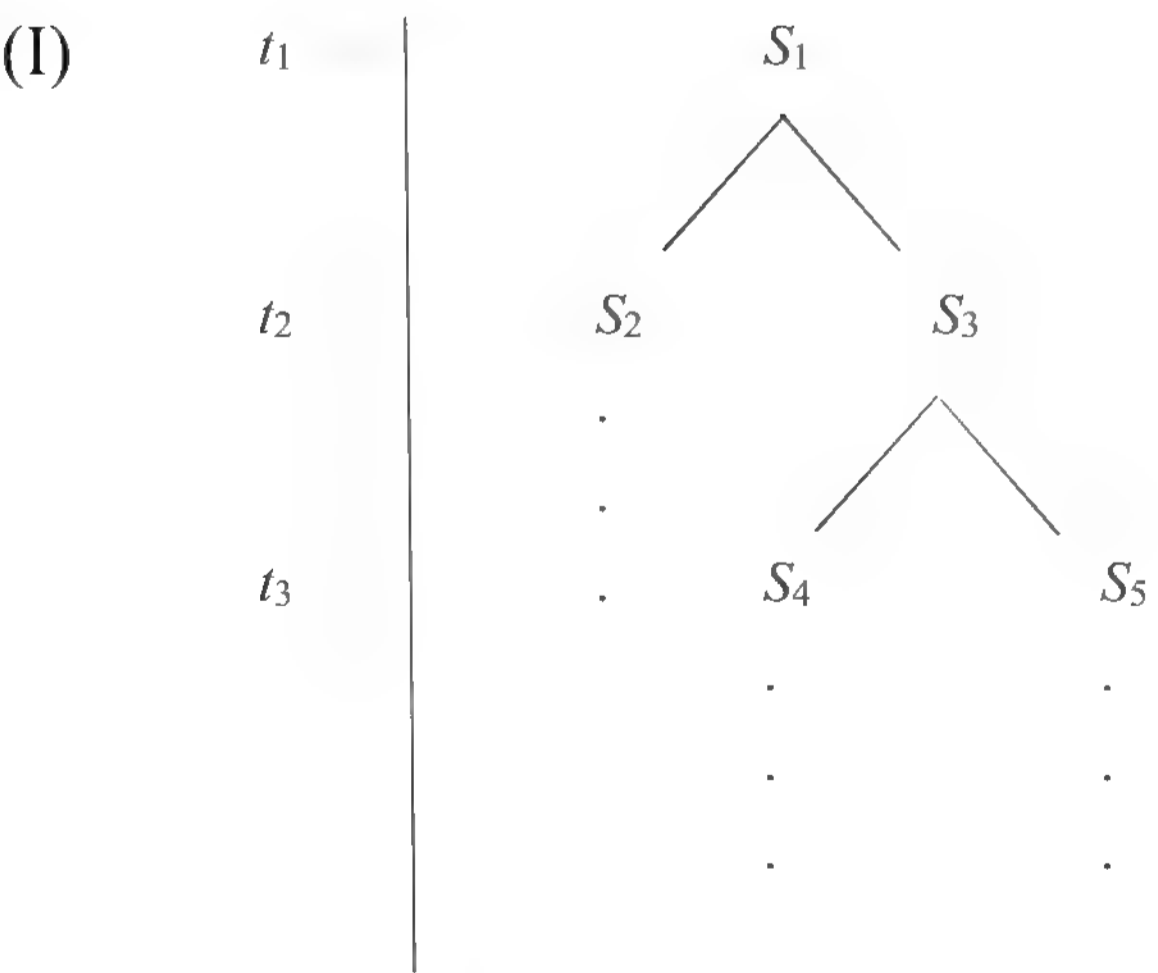


Figure 10.1

S_1 S_5 are nodes, and the lines going through them are paths of the tree.

We can express the relation between time and the nodes of a tree by introducing the notion of level and by saying that an instant t_i is the level of a node S_j . With reference to (I) we can state, for instance, that

$$(6) \quad t_2 = Lv(S_3).$$

It is easy to define in a rigorous way what “being a possible development of S_i ” means for a state S_j in such a structure. This notion is the intuitive counterpart of the relation of accessibility, which is well known to modal logicians. We represent it by “ $Acc(S_i, S_j)$,” and we take it to be reflexive and transitive.³¹ For instance, with respect to (I) we can say that S_5 is a development of S_1 because there is a path going back from S_5 to S_1 through S_3 , whereas S_4 is not accessible to S_2 , because there is no such backward path. Therefore, it is with respect to structures such as (I) that we have to define what it is for a proposition to be definitely or indefinitely true or false.

V

We are still far away from a definition of T_D - and T_I -predicates. To achieve this aim we first have to try to define what it is for a proposition $P(k)$, which is supposed to denote a contingent event, to be true with respect to a given situation of the world S_i . Ammonius has a general correspondence theory of truth: A is true if and only if the event denoted by A is the case.³² Therefore, $P(k)$ has a clear-cut truth value at k . If the event denoted by $P(k)$ is the case at k , then $P(k)$ is true; otherwise, $P(k)$ is false. Therefore, if we consider $P(k)$ in S_i and S_i is on a level equal to or greater than k , we must assign to $P(k)$ either the truth value True or the truth value False according to the way in which we conceive the situation of the world we are referring to. Thus, $P(k)$ in S_i , with $Lv(S_i) \geq k$, has the assigned truth value True or the assigned truth value False. Let us write “ $Assign(P(k), S_i) = 1$ ” for “The truth value True is assigned to $P(k)$ in the node S_i .” In a similar way, we state “ $Assign(P(k), S_i) = 0$ ” for “The truth value False is assigned to $P(k)$ in the node S_i ” or, what is the same, “The truth value True is assigned to $\neg P(k)$ in S_i .”³³ Formally, we have

$$(T^*) \quad T^*(\ulcorner P(k) \urcorner, S_i) \text{ iff } Assign(P(k), S_i) = 1$$

and

$$(F^*) \quad F^*(\ulcorner P(k) \urcorner, S_i) \text{ iff } Assign(P(k), S_i) = 0.$$

What it is important to underline is that a truth value can be assigned to a proposition if and only if the conditions for assigning such a truth value to it are given, namely, if the events referred to by the propositions take place or are at least somehow implied by the due course of events. If we are dealing with a contingent proposition, namely a proposition in which the event denoted by it is not settled before the happening of the event itself, it is reasonable to state that no assignment to $P(k)$ can be made before k , that is, in a node that is on a level preceding k . If it

is now contingent that a sea battle will take place at k (where k is a future instant), in no node on a level preceding k can we assign a truth value to “A sea battle will take place at k .”³⁴ Therefore, we may state

$$(PA) \quad T^*(\ulcorner P(k) \urcorner, S_i) \vee F^*(\ulcorner P(k) \urcorner, S_i) \rightarrow Lv(S_i) \geq k.$$

In other words, if a truth value is assigned to a proposition with respect to a situation S_i , then the level of S_i must be either the same or greater than k .

Moreover, it is easy to understand that if a truth value is assigned to $P(k)$ in S_i , this truth value remains constant in any node accessible to S_i . This corresponds to the intuition that what has happened or is happening cannot be changed so that it is irrevocable in every possible development of the world. Ammonius does not say this in so many words, but it may be implied by his claiming that what is stated about the present or the past is necessarily true or necessarily false:

(G) If it happens that Socrates does not bathe or did not bathe yesterday, it is clear that it is necessary that the negation taken according to the present or the past tense is true and the affirmation saying that Socrates bathes or bathed is false.

(Ammonius, *On Aristotle's On Interpretation* 130.17–20)

If, with respect to the situation S_i , it is the case that Socrates bathes at k , then, however the world might develop, it remains true that Socrates bathes at k . Past and present events are such that the corresponding propositions are eternally preserved in their truth value. The same must be said with respect to falsity. If $P(k)$ takes 0 as an assigned truth value in S_i , then it takes 0 in any node accessible to S_i . We can express formally the point we have tried to make informally by stating

$$(AT) \quad T^*(\ulcorner P(k) \urcorner, S_i) \rightarrow \forall S_j (Acc(S_i, S_j) \rightarrow T^*(\ulcorner P(k) \urcorner, S_j))$$

$$(AF) \quad F^*(\ulcorner P(k) \urcorner, S_i) \rightarrow \forall S_j (Acc(S_i, S_j) \rightarrow F^*(\ulcorner P(k) \urcorner, S_j)).$$

(AT) and (AF) express the condition according to which, when a truth value is assigned to a proposition with respect to a node S_i , it remains the same in any node accessible to S_i .

It is reasonable to think that when a truth value is assigned to $P(k)$ in S_i , where $Lv(S_i) \geq k$, $P(k)$ receives an assigned truth value in every node that is either on the same level as S_i or after S_i independently of its accessibility to S_i . The conditions that allow us to attribute an assigned truth value to $P(k)$ are, at any rate, given at k . Whatever the situation or the history of the world may be, from k onward $P(k)$ is assignedly true or false. If the proposition in question is “There is a sea battle at k ,” we must admit that at k , and after, the conditions are given to assign a truth value to the proposition in any possible history of the world. We can therefore state

$$(AP) \quad Lv(S_i) \geq k \rightarrow T^*(\ulcorner P(k) \urcorner, S_i) \vee F^*(\ulcorner P(k) \urcorner, S_i)$$

If we suppose that the bold line in (II) represents the real path \mathfrak{R} , we can say that $P(k)$ is simply true because there is at least one node on \mathfrak{R} , namely S_3 , where $P(k)$ takes the assigned truth value 1. In a similar way we can characterize an atomic proposition that is simply false by assuming:

$$(F) \quad F(\ulcorner P(k) \urcorner) \text{ iff } \exists S_i (S_i \in \mathfrak{R} \wedge F^*(\ulcorner P(k) \urcorner, S_i)).$$

The idea involved in (T) and (F) is plain and corresponds quite well to our intuition. If conditions for the truth or falsity of $P(k)$ will be given in the future, then $P(k)$ is at any time true or false, in the sense that $P(k)$ will take either the assigned truth value 1 or the assigned truth value 0 in the real future of the world.

It is easy to see that the notions of truth and falsity involved by T - and F -predicates are to some extent atemporal, in the sense that they do not depend on the time of utterance or evaluation of the propositions at issue. To qualify a proposition A as simply true or false, it is sufficient to be sure that A takes an assigned truth value in the real history of the world. If a sea battle actually happens at k , the proposition “There is a sea battle at k ” receives the assigned truth value 1 at k , and before k it has no assigned truth value. But its being true, namely its expressing a conformity to an event, is something that does not depend on the time of utterance. To ensure the possibility of such a correspondence, we need only admit that we are allowed to refer to the series of events that will take place in the *real* development of the world. We do not need to wait until the conditions that give an assigned truth value to a proposition are settled in order to attach a simple truth value to it. In this sense, plain truth and plain falsity are not intrinsically related to time.³⁶ Needless to say, it is with respect to simple truth and falsity that Ammonius can maintain that the principle of bivalence holds in every case. Because $P(k)$ or $\neg P(k)$ will be the case at k , either $P(k)$ or $\neg P(k)$ is at any time (simply) true.

We are now in a position to give a definition of indefinitely and definitely true or false propositions. As we have seen, what contributes in an essential way to the characterization of a definitely true proposition is that it denotes a fixed event that occurs in an inescapable way. We can express these features by saying that a definitely true proposition is a simply true proposition to which the truth value 1 is assigned. By positing that it is a simply true proposition, we say that the event denoted by the proposition is an event of the *real* history of the world; by giving the assigned truth value 1 to it, we state that it refers to an unalterable event. We write this in a condensed way as

$$(T_D) \quad T_D(\ulcorner P(k) \urcorner, S_i, t) \text{ iff } Lv(S_i) = t \wedge t = k \wedge T^*(\ulcorner P(k) \urcorner, S_i) \wedge S_i \in \mathfrak{R}$$

where, as usual, “ \mathfrak{R} ” denotes the *real* history of the world.³⁷ A parallel definition of a definitely false proposition can be given in an obvious way.

A characterization of an indefinitely true proposition can easily be worked out from equivalence (T_D). An indefinitely true proposition is a contingent proposition, namely a proposition that denotes an event, the outcome of which is not yet settled and at the same time is a simply true proposition. Then we can state:

$$(T_1) \quad T_I(\ulcorner P(k) \urcorner, S_i, t) \text{ iff } Lv(S_i) = t \wedge t < k \wedge T(\ulcorner P(k) \urcorner) \wedge \\ \wedge \exists S_j (Acc(S_i, S_j) \wedge F^*(\ulcorner P(k) \urcorner, S_j)).$$

A definition of what it is for a proposition to be indefinitely false can easily be given. To give an example of our definitions, we can say with reference to (II) that $P(k)$ is indefinitely true at t_1 in the node S_1 . This is simply true, because it receives the truth value 1 at S_3 , which is on path \mathfrak{R} representing the *real* history of the world; and it is indefinitely true, because in the node S_2 , which is accessible to S_1 , the truth value 0 is assigned to $P(k)$. On the other hand, $P(k)$ is definitely true in S_3 , because in this node it has the assigned truth value 1, and S_3 is on the real path \mathfrak{R} .

It is easy to see that our characterization of definitely and indefinitely true propositions is able to explain, in a simple way, Ammonius's claim that present or past propositions are definitely true or false, whereas future contingent ones are indefinitely true or false. Take, as usual, $P(k)$. A situation S_i in the real history of the world corresponds to k , and taking (II) as an example, suppose S_i to be S_3 . A truth value is assigned to $P(k)$ in S_3 , the level of S_3 being by hypothesis k .³⁸ Because its assigned truth value in S_3 is 1, $P(k)$ has the assigned truth value 1 in any node accessible to S_3 .³⁹ Therefore, $P(k)$ is definitely true from k onward. On the other hand, suppose that we consider $P(k)$ before k . A situation in the *real* history of the world corresponds to k . With respect to this situation, call it S_n , we can decide whether $P(k)$ is simply true or false. Suppose it to be simply true. Because by hypothesis $P(k)$ refers to a contingent event, there must be a node accessible to S_n and not on the *real* path in which $P(k)$ takes the truth value 0. Therefore, $P(k)$ is indefinitely true in S_n . If we use the example represented by (II), we can identify S_n with S_1 .

VI

What we have to do now is check our interpretation of Ammonius's view against his effort to make the deterministic argument ineffective. Let us return to text (F) where the deterministic objection is summarized by the commentator. The argument can be generalized and divided into the following steps:

- (a) Suppose that $P(k)$ is true at k .
- (b) If $P(k)$ is true at k , it is true at j , which is before k .
- (c) If $P(k)$ is true at j , it is true at any time before k .
- (d) If $P(k)$ is true at any time before k , it is necessary.
- (e) Therefore, $P(k)$ is necessary.

According to the traditional interpretation, step (b) must be denied. From the fact that $P(k)$ is true at k it does not follow that it is true at j and at any time before k . If the event denoted by $P(k)$ is not causally determined before k , $P(k)$ has no truth value before k . But if (b) does not hold, then the deterministic conclusion can be avoided. We have already seen one consequence of this position, namely the invalidation of (PB), and we know that Ammonius did not want to abandon (PB).

There is another consequence of the traditional interpretation, which Ammonius could hardly have swallowed. According to the traditional interpretation, prediction makes no sense if by “prediction” we mean a statement concerning a future contingent event that is true before the time at which the event referred to happens. If $P(k)$ cannot be either true or false before k , and this holds for any future contingent proposition, then any prediction about such events is pointless. If the proposition concerning the event of there being a sea battle at k has no truth value when it is uttered before k , then it is useless to predict today that there will be a sea battle tomorrow. A prediction entails an unavoidable commitment to truth.

It is obvious that the kind of predictions considered by Ammonius have nothing to do with scientific predictions, which are not about contingent events, but about events governed by laws of some sort. Propositions about such events are definitely true or definitely false even before the time at which the events take place. In Ammonius’s account, the predictions in question concern events that, by definition, are not governed by any law. One might think that there is no great harm if such predictions are given up. We might even feel relieved if, in an ideal city ruled by logicians, fortune tellers, soothsayers, and other people of this sort had no admission. But this was not Ammonius’s view. The ancient world paid a great deal of attention to, and regarded as important, oracles, divinations, prophecies, and predictions in general. Philosophers were accordingly interested in these phenomena. The general inclination was to search for a justification for predictions and oracles rather than to deny that they are reliable. Moreover, Ammonius and the late Neoplatonic philosophers had a strong theological reason for admitting predictions. The gods are provident, and they must know the world on which they exert their beneficial influence in such detail that nothing escapes their attention. Therefore, even future contingent events must be known to them, and Ammonius does not waver in maintaining that the gods know future contingent events and that predictions are possible.⁴⁰

If predictions are possible and step (b) must be preserved, how can we escape the conclusion of the deterministic argument? Ammonius’s answer consists in denying step (d). What he says is this:

(H) Against this argument⁴¹ it must be said that with respect to what has already occurred and has already been produced it is not true to say before its occurring that it will be in any case (πάντως) white.⁴² It does not follow from the fact that time has brought this event into being that we have to believe that the event has arrived in virtue of an antecedent necessary condition. Therefore, of people who predict this event the truth is told not by one who says that the baby will be white by necessity, but rather by one who says that this will occur contingently. If so, it is clear that it was possible that the event did not occur. For what occurs in a contingent way cannot be true in any other way. Then, people who say these things⁴³ should not judge what is future from what has occurred, but by keeping it as not yet occurred they should examine whether it occurs necessarily.

(Ammonius, *On Aristotle’s On Interpretation* 145.9–18)

Ammonius's point seems to be that predictions are possible and that they do not rule out contingency. Take a contingent proposition $P(k)$ that is true at k so that (a) is satisfied. According to Ammonius, nothing prevents us from admitting that $P(k)$ is true at any time before k , in accordance with premises (b) and (c) of the deterministic argument. The question is: In what sense is $P(k)$ true before k , being a contingent proposition? If we look at our analysis of the notions of definite and indefinite truth, the answer is clear: It is *indefinitely* true. Because of (a), $P(k)$ takes an assigned truth value at k . Because what is denoted by $P(k)$ is the case in the real history of the world, it is true at the node of the real history of the world, the level of which is k . But, for the same reason, $P(k)$ is also simply true, that is, *T*-true. It is precisely this situation that allows the possibility of $P(k)$ being truly predicted. On the other hand, $P(k)$ is contingent before k . This means that the possibility of its being false is not ruled out, or, if you prefer, that before k the conditions which determine the event denoted by it are not yet given. We must expect, therefore, that in one of the possible histories of the world different from the real one, $P(k)$ takes the assigned truth value. If so, $P(k)$ is indefinitely true according to our definition (T_1).

If this interpretation is correct, the whole point of Ammonius's refutation of determinism lies in the distinction between definite and indefinite truth. If every proposition were definitely true or false before the time at which the event denoted by the proposition occurs, no contingency would be allowed in the world. As he says with reference to a pair of contradictory contingent propositions concerning the future:

(J) If one of them will be definitely true and the rejection of possibility is a consequence of one of the two propositions of a contradiction being definitely true, it is clear that possibility will be expelled from the things which are.

(Ammonius, *On Aristotle's On Interpretation* 143.17–20)

The claim is clear. Suppose that $P(k)$ is not only plainly true, but definitely true before k , for instance, in a node S_j , the level of which is $j < k$. In no node accessible to S_j can $P(k)$ take the assigned truth value 0, and in this sense the future is by no means open. Its truth value state is settled, and in this way $P(k)$ is no different from a present or past proposition. But once again the fact that $P(k)$ is definitely true or false does not depend on the possibility of its being predicted. It depends on the nature of the event it denotes. In this way, the deterministic argument is made ineffective.

Against this view, one might object by repeating the deterministic argument. If $P(k)$ is indefinitely true at S_j , it is simply true. This means that $P(k)$ will, in due course, take the assigned truth value 1 in the real history of the world. Therefore, the future of $P(k)$ at S_j is not at all open, because it is already decided that $P(k)$ will be true in the real world. In other words, the contingent state of $P(k)$ before k is only apparent, because it does not play any role in the development of the real world. This difficulty, I believe, can be met by stressing the difference between being definitely and indefinitely true. What makes $P(k)$ indefinitely true before k

is that the real development of the world at the stage in which $P(k)$ is evaluated is not yet fixed. We may refer to the future real history of the world, but how the world will evolve is still completely open. Therefore, what we actually say when we claim that $P(k)$ is indefinitely true at S_j is that $P(k)$ is true on the condition that the world develops in a certain way. And this condition is a real condition, because at S_j the future of $P(k)$ is still open. But, again, one might urge that, at S_j , $P(k)$ is either true or false. Then, because it is, say, true, then the real development of the world will be such and such and in this sense is already determined. This statement would be true if $P(k)$ at S_j had an assigned truth value. But this is not the case. Attributing to $P(k)$ a simple truth value does not depend on the fact that the course of the events is fixed in the future, but rather is the consequence of admitting that there will be a future and a real history of the world, whatever it may be. In other words, what is simply true, by itself, is not part of the furniture of the world in the sense that it refers to events that are, in some sense, already there and that can be causally related to other events. That there is a sea battle at k is not a fact before k , nor is it causally implied by other facts that are already given, although the proposition that expresses this fact is true or false even before the actual obtaining of the fact.

To admit such a possibility we must concede that the relation between propositions and facts is not a temporal relation. This point is important because it marks the difference between the traditional interpretation and Ammonius's view. Let us pause a little to consider this question. As we have seen, the traditional interpretation is based on the idea that a proposition can only be said to be true or false when the extralinguistic conditions for this attribution are given. In the case of future contingent propositions, these conditions do not obtain. Therefore, no truth value can be assigned to them. What emerges from this conclusion is that truth is a totally temporal notion, that is, a notion that can only be applied when appropriate extralinguistic conditions are the case. The immediate consequence of this view is that the principle of bivalence cannot be considered a logical law and "not being true" for a proposition does not mean "being false."

One might find these implications rather unpalatable. From the fact that now it is not true that tomorrow there will be a sea battle, it cannot be inferred, according to the traditional interpretation, that a sea battle will not take place tomorrow. This looks not at all obvious, and one might prefer to think of truth as something that is not completely given in time. Of course, the conditions that make a proposition true or false are given in time. It is in time that a sea battle takes place, and it is in time that the truth value 1 is assigned to the corresponding proposition. But it does not follow from this that the predicate True can be applied to a proposition only when the corresponding extralinguistic conditions occur. If at some time a proposition becomes true in a proper sense, we may refer to this fact even before it happens. If "There is a sea battle at k " is T^* -true, or true in an assigned way, at k so that the conditions for its truth are given at k , then we can refer to the truth of the proposition at any time whatsoever. This does not mean that the conditions that make the proposition true are given at any time. By hypothesis they are not given before k . Nonetheless, if a sea battle happens at k , the proposition "There is

a sea battle at k ” is “always” or “simply” true, in the sense that, once and for all, it takes the truth value 1 at k . Its receiving this truth value at k does not depend on the time we consider the proposition: It holds atemporally.

I cannot pursue this inquiry, which has deep and controversial philosophical implications, any longer. Rather, I would like to conclude by pointing to one restriction, entailed by the traditional interpretation, that seems to be implicitly overcome by Ammonius’s position. Take, for instance, a proposition such as

(7) it is always raining.

Its formal counterpart may be represented as follows

(8) $\forall t P(t)$.

It seems natural to claim that (8) is true if and only if it rains at every instant; that is, if and only if $\dots, P(t_i), P(t_j), \dots, P(t_m), \dots$. Suppose that (8) is true. If the domain over which the variable t ranges is infinite, then at whatever time we evaluate (8) we cannot say that (8) is true. For instance, at t_m we cannot conclude that (8) is true. Of course, if it happens that $\neg P(t_{m+1})$, we can infer at this stage that (8) is false. But if $P(t_{m+1})$, we have still to postpone an assignment of a truth value to (8) and so there is no time at which we can say that it is true.

Because we cannot state that (8) is true, we cannot assert that a particular indefinite proposition such as

(9) $\exists t P(t)$

is false, because the falsity of (9) is the same as the truth of $\forall t \neg P(t)$, and we have seen that we can assign to (9) the truth value 1 at no time.

If we move from the traditional interpretation to a conception in which truth is no longer meant as a temporal predicate, we are immediately able to treat propositions such as (8) and (9). It is sufficient to have the capacity of referring to the totality of the truth values assigned to propositions $P(t_1), P(t_2), \dots, P(t_m), \dots$ in the nodes of the *real* path. If these assigned values are all 1, then (8) is true; if they are all 0, then (9) is false. It does not matter how we can go along an infinite path. The important point is that we can understand what it means for a proposition such as (8) to be true. This is possible exactly because it is not implied that truth and falsity are temporal notions, namely predicates that can only be attributed to propositions at given times. From this point of view Ammonius’s position seems to offer the possibility of analyzing a greater variety of propositions than the traditional interpretation.⁴⁴

VII

Let us turn from these difficult philosophical questions to more familiar historical problems. It will readily be guessed that Ammonius was not the man who invented

the theory we have tried to present. He did not possess the capacity for such a creative and difficult task.

The problem arises: Who was the common source of Ammonius and Boethius? The question has been studied with reference to Boethius, and for the Latin commentator the answer seems to be Porphyry.⁴⁵ However, there is no clear evidence for Ammonius. He quotes Porphyry, who seems to be his main source for the discussion of alternative readings of Aristotle's text, more than once.⁴⁶ Sometimes he mentions some of his views with approval,⁴⁷ and in one case he says that in his exposition he will follow Porphyry's theory, trying to make it clearer.⁴⁸ From this evidence we cannot infer even that Ammonius had direct access to the works of Porphyry, because it may be that his quotations of Porphyry were taken from a later source. In fact, his main source seems to be Proclus, who is mentioned at the beginning of the commentary in a rather solemn way as the divine teacher whose research on Aristotle had made Ammonius's work possible.⁴⁹ In the course of Ammonius's interpretation of chapter 9, Iamblichus is quoted for the decisive step concerning the solution of the problem of how the gods can know future contingent events.⁵⁰ Here the distinction between having definite and indefinite knowledge of future contingents plays an important role, and this distinction has something to do with the distinction between definitely and indefinitely true or false propositions.⁵¹ But once more Iamblichus's point was well known to Proclus,⁵² and we can once more suppose that Proclus was the direct source of Ammonius. A prudent conclusion may be that Ammonius refers to a doctrine whose existence can be traced back to Porphyry.

However, we could try to push our inquiry a step further by asking whether Porphyry was the creator of this doctrine. There are some texts that make the answer controversial. We must take into account a passage of Simplicius to which Richard Sorabji first drew attention.⁵³ A certain Nicostratus is mentioned in it, who is probably to be identified as the Nicostratus who received an honorific inscription at Delphi and was a Platonic philosopher who flourished in the middle of the second century AD.⁵⁴ Simplicius reports that Nicostratus denied any truth value to future contingent propositions, making him a partisan of the traditional interpretation.⁵⁵ If we may trust Cicero's evidence, Nicostratus was not the only ancient follower of the traditional interpretation, because Epicurus was also among its supporters.⁵⁶ After Nicostratus, Simplicius considers the position of the Peripatetics. He says:

(K) But the Peripatetics say that the contradiction regarding the future is true or false, while it is by nature ungraspable and uncertain which part of it⁵⁷ is true and which part is false. For nothing prevents us from saying the contradiction with respect to any time, as for instance "it will be or it will not be," and each of the two parts contained in it, as for instance "it will be" or "it will not be," is already (ἤδη) true or false in a definite way (ἀπορισμένως) with respect to the present or past time. But those parts of a contradiction which are said with respect to the future are not yet (ἤδη) true or false, but they will be true or false. Let these things be sufficient against (πρός) Nicostratus.

(Simplicius, *On Aristotle's Categories* 407.6–14)

One might think that the view of the Peripatetics is not clear. On the one hand, the adverb ἀφορισμένως (407.10–11), which is to be connected with ἡ ἀληθῆ ἡ ψευδῆ, suggests that Ammonius's doctrine is hinted at here. On the other hand, contingent propositions concerning past or present events are contrasted not with future propositions that have an indefinite truth value, as one might expect, but rather with propositions that are not yet true or false. That future contingent propositions are not yet true or false would not be admitted by Ammonius, and this statement reminds us rather of the traditional interpretation. Were the Peripatetics referred to by Simplicius followers of the traditional interpretation or rather partisans of the position that Ammonius holds?

A little reflection shows that the second alternative is the right one. Suppose that the Peripatetics embraced the traditional interpretation. If *A* is a future contingent proposition, the principle of bivalence cannot hold for *A*. But Simplicius at the beginning of our passage says that according to the Peripatetics “the contradiction regarding the future is true or false,” and this statement can only mean that (PB) applies also to future contingent propositions. Moreover, if the Peripatetics had adopted the traditional interpretation, they would have held the same view as Nicostratus. But the position of the latter is clearly opposed by Simplicius to the view of the Peripatetics.⁵⁸ The conclusion is that the Peripatetics did not embrace the traditional interpretation. How can we then explain the view that Simplicius attributes to them with respect to future contingent propositions? In what sense are these propositions not yet true or false? The question is easily answered if we admit that “not yet true or false” means “not yet definitely true or false,” namely if we understand ἀφορισμένως to be connected to ἡδὲ μὲν οὐκ ἔστι ἡ ἀληθῆ ἡ ψευδῆ at 407.13. In this way, the Peripatetics must be taken as representatives of the view defended by Ammonius.

Unfortunately, Simplicius does not tell us who the Peripatetics holding the same view as Ammonius are. Nor does he give us any hint toward identifying them. One might think that the Peripatetics were led to formulate their doctrine as a reaction to the position put forward by Nicostratus. If so, we would have a *terminus post quem* for the origin of Ammonius's view, and we might suppose that it was created before Porphyry in a Peripatetic milieu after the middle of the second century AD. The name of Alexander of Aphrodisias comes at once to mind. But Simplicius's words establish neither the starting point nor the consequences of this interpretation. He exploits the Peripatetic view against Nicostratus to show that his position is not the only possible one. But this does not mean that the Peripatetics themselves elaborated their conception to avoid Nicostratus's view. To make things worse, the position of Alexander about future contingents, which is known to us from his remaining works, is far from being clear. We cannot examine this question here. It is sufficient to remember that some scholars who have studied this problem at length are inclined to think that Alexander was rather near to the traditional interpretation, although he is never explicit on this point.⁵⁹ Therefore, no relevant clue can be extracted from Simplicius's passage to find an answer to our question.

There is, however, another text that has led some scholars to seek the origin of Ammonius's doctrine among the Peripatetics near Alexander of Aphrodisias.⁶⁰

I refer to a passage in the *Questions* traditionally attributed to Alexander but in fact made up of rather heterogeneous materials.⁶¹ This is true especially for the text that interests us: *Question* 1.4.⁶² In the last part of it, a hint is made at a doctrine that is *prima facie* similar to Ammonius's position. There are two passages where ἀφορισμένως is used in connection with the truth and falsity of a contradictory pair of future contingent propositions. The first of them runs as follows:

(L) And further, if that is possible from which, if it is supposed that it is the case, nothing impossible results; and if, from everything of which the opposite is truly predicted, there results, if it is supposed that it is the case, something impossible, i.e. that the same thing both is and is not at the same time; then none of those things of which one part of the contradiction referring to the future is true definitely (ἀφορισμένως ἀληθές ἐστι) would be the case contingently. But they say that in all cases one part of the contradiction is true definitely (ἀφορισμένως ἀληθές εἶναι).

(Alexander of Aphrodisias, *Question* 1.4, 12.13–18)⁶³

The Greek is in a rather poor condition, and it is not easy to follow the development of the argument in favor of determinism outlined here. The main idea seems to be that if a contingent proposition $P(k)$ is definitely true before k , then it is necessary, because the hypothesis that $\neg P(k)$ is true entails a contradiction. It is important to stress that in the last lines of the passage a sort of principle of bivalence is laid down. It can be represented in the following way:

$$(PB^*) \quad T_{D^*}(\ulcorner A \urcorner) \vee T_{D^*}(\ulcorner \neg A \urcorner)$$

where, of course, $T_{D^*}(\ulcorner A \urcorner)$ stands for “ A is definitely true.” The crucial point is to see whether the predicate “definitely true” which is mentioned here is the same as the predicate used by Ammonius, namely whether “ T_{D^*} ” can be identified with “ T_D .” The simple fact that the expression “definitely true” is used is not a sufficient reason to give an affirmative answer to our question. It might be that “ἀφορισμένως ἀληθές” has the same meaning as Ammonius's expression, but that is neither necessary by itself nor imposed by the context. The expression “ἀφορισμένως ἀληθές” might simply refer to what is already true in contrast to what is *not yet* true. From this point of view, the deterministic argument would have its main point in the premise that even future contingent propositions always have a truth value. But if $P(k)$ is true even before k , then it is always true and therefore necessarily true.⁶⁴

The answer of the author of this *Question* to the deterministic argument contains the other occurrence of “ἀφορισμένως ἀληθές.” He says:

(M) But if it is alike possible for the same thing to come to be and not to come to be, how is it not absurd to say, in the case of these things, that one part of the contradiction uttered beforehand is true definitely (ἀφορισμένως ἀληθές), and the other false, when the thing in question is alike capable of both?

(Alexander of Aphrodisias, *Question* 1.4, 13.2–6)⁶⁵

Unfortunately, even here it is not clear what “ἀφορισμένως ἀληθές” means. The core of the answer to the deterministic argument is that it is inconsistent to maintain that every proposition is definitely true or definitely false and that there are contingent events. If $P(k)$ is always true in a definite way, then there is no possibility that the event denoted by it does not obtain. It is absurd to reject the existence of contingent events. Therefore, it cannot be admitted that every proposition always has a definite truth value. But the question here is: Shall we infer that future contingent propositions have an indefinite truth value, or must the conclusion rather be that these propositions have no truth value at all? If we give the first answer, we have Ammonius’s view and we are entitled to say that the doctrine was born among the pupils of Alexander. On the other hand, if we prefer the second answer, we have to reckon Alexander’s school among the supporters of the traditional interpretation, and the problem of the origin of Ammonius’s theory is left in the dark. Needless to say, we would like to embrace the first answer, because it gives a nice solution to our problem. But it would be unfair to adopt it simply because it explains what we are looking for. I do not see any reason to prefer the first interpretation to the second. In our passage, it is not said to what ἀφορισμένως ἀληθές is opposed, and it might be contrasted either to what is indefinitely true or to what is not yet true. Consequently, the author of the *Question* might equally be a forerunner of Ammonius or a follower of Nicostratus.

Although we do not know where his view ultimately comes from, Ammonius’s doctrine is far from uninteresting in both a historical and philosophical perspective. Its commitment to an atemporal theory of truth, on the one hand, and its exploiting of the notions of necessity and possibility, on the other, clearly show how ample the range of the problems involved is and how modern they are.

Notes

It is impossible to express my gratitude to all colleagues and friends who have contributed with their observations and criticisms to give the final form to this chapter. However, I would like to thank at least Jonathan Barnes, who has not only given me useful suggestions but has also tried to make my English less Italian.

- 1 The bibliography up to 1973 can be found in Celluprica 1977. Further bibliographical references are available in Frede 1985: 8–47 and Talanga 1986: 169–185. Add to them Kirwan 1986: 167–187; Eck 1988: 38; Donini 1989; Rijen 1989.
- 2 One of the best presentations of the traditional interpretation is found in Sorabji 1980: 91 ff. Different views have recently been proposed in Rijen 1989; Eck 1988.
- 3 See Sorabji 1980: 92; Sharples 1978: 263. Jan Łukasiewicz has briefly expressed the same view in Łukasiewicz 1930: 51–77.
- 4 Although Dorothea Frede in a recent article published after Sorabji’s and Sharples’s work still attributes the traditional interpretation to Ammonius. See Frede 1985: 43. She repeats the interpretation already defended in Frede 1970: 24. On the same line is Talanga 1986: 144–145.
- 5 By $\ulcorner P \urcorner$ I express the so-called nominalization of P , which is the argument of the T -predicate in $T(\ulcorner P \urcorner)$.
- 6 As is obvious, an equivalent formulation of (PB) is
(PB’) $T(\ulcorner P \urcorner) \vee T(\ulcorner \neg P \urcorner)$.
- 7 See also *On Aristotle’s On Interpretation* 139.32–140.4; 140.11–13.

- 8 Günther Patzig (Patzig 1973: 921–929) has clearly underlined the fact that the traditional interpretation is inconsistent with the admission of (PB).
- 9 Pace Frede 1985: 43 (and Frede 1970: 25), who thinks that Ammonius' speaking of indefinitely true or false propositions is only a diplomatic way of saying that (PB) admits of exceptions.
- 10 Courcelle's thesis that Ammonius was Boethius's main source is now rejected by all scholars (Courcelle 1948: 264). The view that both commentators depend on a common source was proposed by Shiel 1958: 228–234; it is shared by Orbetello 1974: I, 522–544; Zimmermann 1981: LXXXVIII; and Kretzmann 1987: 66–67.
- 11 See also Kretzmann 1987: 65–66.
- 12 Although Ammonius does not say so explicitly, I assume that he would not have counted as a proposition concerning the future a sentence such as "It will be true tomorrow that three years ago Philip had a car accident."
- 13 Aristotle, *On Interpretation* 9, 18a33.
- 14 Ammonius, *On Aristotle's On Interpretation* 138.34 ff.
- 15 See, for instance, Ammonius, *On Aristotle's On Interpretation* 139. 10.
- 16 As far as I remember, the only exception is at *On Aristotle's On Interpretation* 154.32. However, Ammonius considers a proposition such as "a white baby will be born tomorrow" (*On Aristotle's On Interpretation* 144.16), which seems to be of the same type as (1).
- 17 Ammonius, *On Aristotle's On Interpretation* 140.15–16.
- 18 I borrow the terminology of "pseudo-dates" from Rescher and Urquhart 1971: 27.
- 19 Ammonius, *On Aristotle's On Interpretation* 130.1–20.
- 20 Ammonius, *On Aristotle's On Interpretation* 130.1–5.
- 21 I leave aside (3), where no pseudo-date is expressed.
- 22 These are Aristotle's words: *On Interpretation* 9, 18b9–10.
- 23 Again, Aristotle's words: *On Interpretation* 9, 18a10.
- 24 I would not like to get involved in the difficult problems concerning whether time ought to be conceived as a continuum, a succession of discrete instants, or a succession of intervals.
- 25 There is a well-known debate among philosophers on this subject. A useful discussion of the question can be found in Sorabji 1980: 97–102.
- 26 Ammonius, *On Aristotle's On Interpretation* 139.17–18.
- 27 Ammonius, *On Aristotle's On Interpretation* 130.11–14.
- 28 See, for instance, Ammonius, *On Aristotle's On Interpretation* 136.12–13.
- 29 Boethius, *On Aristotle's On Interpretation* 2a, 208.11–18; 245.19–28.
- 30 See Rescher and Urquhart 1971: 125 ff.
- 31 For the notion of accessibility, see Hughes and Cresswell 1968: 75–80. Because the accessibility relation is supposed to be reflexive and transitive, the modal system involved is at least as strong as S_4 .
- 32 See Ammonius, *On Aristotle's On Interpretation* 139.26 ff. 32 ff.: 154.16–20.
- 33 To be precise, we have to distinguish between atomic and nonatomic propositions. A truth value can be assigned just to atomic propositions and their negations, for instance, to $P(k)$ or $\neg P(k)$. From the notion of assigned truth value we can work out the more general notion of being assignedly true or false in a node S_i , which can be applied to compound propositions. Having defined what it is for an atomic proposition $P(k)$ to have the assigned truth value 1, we can construe the definition of "assignedly true (false) in a node S_i " which we can represent by " $T^+(\ulcorner A \urcorner, S_i)$ " and " $F^+(\ulcorner A \urcorner, S_i)$." If A is an atomic formula, then $T^+(\ulcorner A \urcorner, S_i)$ iff $T^*(\ulcorner A \urcorner, S_i)$ and $F^+(\ulcorner A \urcorner, S_i)$ iff $F^*(\ulcorner A \urcorner, S_i)$. If A is $\neg B$, then $T^+(\ulcorner A \urcorner, S_i)$ iff $F^+(\ulcorner B \urcorner, S_i)$. If A is $B \wedge C$, then $T^+(\ulcorner A \urcorner, S_i)$ iff $T^+(\ulcorner B \urcorner, S_i)$ and $T^+(\ulcorner C \urcorner, S_i)$. The other propositional operators can be defined derivatively. The definition can be easily extended to first-order formulas. But for our purposes the unextended definition is enough, because Ammonius only considers atomic propositions and their negations.

- 34 In principle, “A sea battle will take place at k ” might have an assigned truth value even before k if the conditions that unequivocally determine the happening or not happening of this event are given at some time before k (see text (E)). But to avoid complications we may suppose that these conditions are not given before k .
- 35 In speaking of “simple truth (or falsity),” I have been inspired by the terminology of Georg Henrik von Wright, who uses the expression “plain truth” (Wright 1984: 5).
- 36 This view has recently been developed by Wright 1984: 6.
- 37 An alternative characterization of definite truth would be:

$$(T_D') \quad T_D(\ulcorner P(k) \urcorner, S_i, t) \text{ iff } Lv(S_i) = t \wedge t = k \wedge T(\ulcorner P(k) \urcorner) \wedge T^*(\ulcorner P(k) \urcorner, S_i).$$

Remember that the *Assign*-function that defines T^* -truth is monotonic. Therefore, $Assign(\ulcorner P(k) \urcorner, S_i) = 1$ implies $\forall S_j (Acc(S_i, S_j) \rightarrow Assign(\ulcorner P(k) \urcorner, S_j) = 1$.

- 38 Cf. (AP).
- 39 Cf. (AT).
- 40 See, for instance, Ammonius, *On Aristotle's On Interpretation* 132.17 ff. On this attitude toward the gods in late Neoplatonism, see e.g., Proclus, *Elements of Theology* 124 (110.10–11, Dodds); *Platonic Theology* 1.15 (69.10–12; 70.22–25; 74.9–16 Saffrey and Westerink) and 1.21 (98.5–12); *Ten Doubts Concerning Providence* 2.6–23 Isaac; Ammonius, *On Aristotle's On Interpretation* 132.13 ff. (These texts are discussed in Chapter 9.)
- 41 The deterministic argument developed in text (F).
- 42 Ammonius is hinting at the example of the white baby who is just born mentioned in text (F).
- 43 The Greek οἱ ταῦτα λέγοντος is probably a misprint because the sense requires λέγοντες.
- 44 It must, however, be pointed out that if propositions such as (8) and (9) are systematically considered, the definitions of definite and indefinite truth and falsity must be modified accordingly. These have been given with reference to time, and no such reference can be applied to propositions that range over the whole of time. Nontemporally committed modalities would have to be introduced to explain the indefinite or definite truth of (8) and (9). But this is another story.
- 45 See Mignucci 1987: 38–41. What is still in dispute is whether Boethius had direct access to Porphyry's commentary on the *On Interpretation* or rather translated a Greek codex with marginalia mostly taken from Porphyry. On this question, which does not affect our problem very greatly, see Shiel 1958; Ebbesen 1990: 375 ff.; Barnes 1983: 80.
- 46 For instance, a different reading of *On Interpretation* 16b9–10 is attributed by Ammonius to Porphyry (*On Aristotle's On Interpretation* 50.12), and the same happens with reference to *On Interpretation* 16b22 (*On Aristotle's On Interpretation* 56. 18). Again, Porphyry's discussion of *On Interpretation* 17b16 ff. is considered with his reading ἀποφαντικῶς instead of ἀντιφαντικῶς at *On Interpretation* 17b17 (*On Aristotle's On Interpretation* 190. 24 ff.), and a variant at *On Interpretation* 19b24–5 is discussed by quoting Porphyry (*On Aristotle's On Interpretation* 171.1–6).
- 47 For instance, Ammonius, *On Aristotle's On Interpretation* 32.3; 70.3 ff. 99.8 ff.
- 48 Ammonius, *On Aristotle's On Interpretation* 94.25–28.
- 49 Ammonius, *On Aristotle's On Interpretation* 1.6–11. Strangely enough, Proclus is quoted in only one other passage, at *On Aristotle's On Interpretation* 181.30 ff. (cf. Stephanus, *On Aristotle's On Interpretation* 46.25–6).
- 50 Ammonius, *On Aristotle's On Interpretation* 135.14.
- 51 Ammonius, *On Aristotle's On Interpretation* 135.12 ff.
- 52 See Proclus, *Elements of Theology* 124 (110.10–13 Dodds); *Platonic Theology* 1.15 (69.10–12, 70.22–25, 74.9–16 Saffrey and Westerink); *Ten Doubts Concerning Providence* 2.6–23 Isaac.

- 53 See Sorabji 1980: 92.
- 54 See Praechter 1973: 101–113; Dillon 1977: 233–236.
- 55 Simplicius, *On Aristotle's Categories* 406.13–16.
- 56 Cicero, *On Fate* 9.18; 10.21; 16.37; *Prior Academics* 97.
- 57 Adopting Kalbfleisch's suggestion, I read at 407.7 πότερον δὲ ἔσται μόριον αὐτῆς ἀληθές, instead of αὐτῶν ἀληθές. Cf. 407.9–10.
- 58 This remark has also been made by Sharples 1978: 263.
- 59 See Sharples 1978: 264; Sharples 1983: 11–12; and also Sorabji 1980: 92–93 and especially 93n.5; Frede 1984: 286.
- 60 See Frede 1970: 26; Sharples 1978: 264; Sharples 1982: 38; Sorabji 1980: 93n10.
- 61 On the *Questions*, see Sharples 1990: 83–111. For *Quaestion* 1.4, which will be at issue here, see Bruns 1889: 619–630; Sharples 1990: 23–38.
- 62 See Sharples 1982: 24–25.
- 63 Following Ivo Bruns, I delete μή and add συμβήσεται at 12. For an analysis of this passage, see also Bruns, “Studien zu Alexander von Aphrodisias:” 627. The translation is taken from Sharples 1982: 36, with a small change.
- 64 Kretzmann 1987: 67–68 expresses similar worries about the interpretation of the passage.
- 65 Here again I follow Sharples's translation (Sharples 1982: 37).

11 Truth and modality in late antiquity

Boethius and future contingents

I

Aristotle's analysis of future contingents in *On Interpretation* 9 has generated a great deal of discussion, ranging from the interpretation of his own words to the philosophical meaning and adequacy of the solution proposed by him. Unfortunately, the former question is entailed by the latter, and there is no agreement about the kind of answer that Aristotle gives to the question of determinism despite the astonishing quantity of works dedicated to it.¹ I do not want to involve myself in the problem of Aristotle's interpretation. My task here is to illustrate the meaning and relevance of Boethius's analysis of future contingents, and I will consider his commentary on the *On Interpretation* for its own sake. In other words, I will not try to evaluate the adequacy of Boethius's proposal with respect to Aristotle, even if, of course, he believed that his interpretation was faithful to the text of the *On Interpretation*. Nor will I try to compare Boethius's solution with other solutions that have been proposed by ancient and modern interpreters alike who have tried to explain Aristotle's text. I will consider only one view other than that of Boethius because Boethius himself discusses it and his discussion is relevant to the understanding of his position.

II

Boethius begins his analysis by distinguishing conditionals from predicative statements – namely compounds from simple propositions where a predicate is attributed to a subject. He says that, in the latter case, when we leave out the case of general nonquantified statements,² propositions concerning present or past events divide truth and falsity in an appropriate way. Indeed, they are such that for any contradictory pair of them, it not only always happens that one is true and the other is false, but also they have a definite truth value (*On Aristotle's On Interpretation* 2a, 199.1–200.1). To use Boethius's words, if we consider an affirmation and its negation concerning present or past events, we can say that each of them has

(A) a certain and definite truth (*certam et definitam veritatem*) or a certain and definite falsity (*certam et definitam falsitatem*).

(Boethius, *On Aristotle's On Interpretation* 2a, 199.28–200.1)

According to Boethius the same can be said of predicative propositions referring to necessary future events, provided that they are not both general and nonquantified. His example is

(B) this year the sun will be in the Ram in winter.

(Boethius, *On Aristotle's On Interpretation* 2a, 200. 1–9)³

However, it is not with propositions regarding such future events that we have to concern ourselves, but rather with future contingent propositions (200.9–11). Boethius gives a first characterization of this kind of proposition:

(C) As we have already said,⁴ contingent are those things that are in the same relation with being and not being, and as they have an indefinite being and not-being, so their affirmations <and negations> have an indefinite truth or falsity, since one is always true and the other always false, but it is not yet known which one is true and which one is false with respect to contingent things. For as with respect to things of which it is necessary that their being is definite, and with respect to things of which it is impossible that their not-being is definite, so with respect to things that can be and can not be neither being nor not-being is definite, but truth and falsity are taken from the way in which the being of the thing is and from the way in which the not-being of the thing is. For if what is said to be is, then it is true, and if what is said to be is not, then it is false. Therefore, with respect to contingent and future things, as being and not-being themselves are unstable, although it is necessary to be or not to be, so also in affirmations <and negations> that express contingent things truth and falsity are uncertain (for it is unknown from the nature of propositions themselves which of them is true and which is false), but one must be true and the other false.⁵

Boethius's main claim seems to be that singular contingent propositions referring to future events are always true or false, but they are not definitely true or definitely false. To stress the point, Boethius contrasts singular contingent propositions with necessary ones, which are not only always true or false but also definitely true or definitely false. The reason that the latter fulfill the condition of being definitely true or definitely false is seen by Boethius in their referring to stable states of affairs. If it is necessarily true that Socrates will die, it cannot happen that this event does not occur. The event will take place, and in this sense Socrates's future is not open. Therefore, the truth value of the corresponding proposition in which it is said that Socrates will die is already determined. Consider, by contrast, a contingent future proposition. Because it is future, the event to which it refers has not yet occurred; because it is contingent, the event is not causally determined. With respect to the event denoted by the proposition

- (i) Socrates will be debating in the gymnasium tomorrow (*On Aristotle's On Interpretation* 2a, 202.20)

which is both future and contingent, it may happen that Socrates changes his mind and never goes to the gymnasium; or it may happen that he will go and take part in the discussion. The event is not yet settled, and in this sense the proposition is not yet definitely true or definitely false. From this point of view, propositions concerning the past and the present, even if they do not refer to necessary events, are more on the side of necessary than on the side of future contingent propositions. Because what has happened and happens cannot change – if Socrates is speaking now it is no longer possible that he is not speaking now – propositions concerning past and present events are definitely true or definitely false for the same reason that necessary and impossible propositions are so.

III

Up to this point, I have tried to rephrase what Boethius says in the passage just quoted. But what do his words mean? Is there a meaning in the claim that there are propositions that are true but not definitely true and propositions that are true and definitely true? In answering this question we have first to face an interpretation that, despite the efforts made by Richard Sorabji and Bob Sharples, has many supporters. According to a well-established usage, we can refer to this interpretation as the traditional interpretation.⁶ The main idea that characterizes the traditional interpretation is that future contingent propositions are neither true nor false before the time at which the events denoted by these propositions are said to occur. So, for instance, (i) is neither true nor false before tomorrow. It is only from tomorrow onward that it becomes true or false. The point is clear, and modern gap semantics have made us familiar with such a view. The consequences of this way of presenting things are also clear. The most relevant of them for us is that in this perspective we have to distinguish the Law of Excluded Middle (LEM) from the so-called Principle of Bivalence (PB). Roughly speaking, the first says that every proposition is either true or not true, whereas the second claims that every proposition is either true or false. Formally we have:

$$(LEM) \quad T(p) \vee \neg T(p)$$

and

$$(PB) \quad T(p) \vee F(p).^7$$

Now if there are propositions that do not always possess a truth value, then (LEM) does have the status of a logical law, whereas (PB) does not. Even if (i) is neither true nor false, it is either true or not true – in fact, it is not true. However, we cannot say that it is true or false because it is by hypothesis neither true nor false. It is not important to decide whether, in the case of Aristotle, we have to do with a proper or a temporary restriction of (PB).⁸ Whatever the case, the point is that (PB) does not apply to every proposition and in particular to future contingent ones.

This remark should be sufficient to conclude that Boethius cannot be placed among the supporters of the traditional interpretation. In the long passage quoted earlier, it is said twice that future contingent propositions are true or false, and this seems to imply that (PB) holds even for these sentences. Boethius's view is that future contingent propositions are always true or always false but not definitely true or false.

In spite of this (obvious) situation, Dorothea Frede has maintained that Boethius must be treated as a follower of the traditional interpretation. In reaction to Richard Sorabji and Bob Sharples, she repeats and expands on the view that she has already defended in her book on Aristotle's *On Interpretation* 9.⁹ By her reasoning, Boethius (along with Ammonius), maintains that future contingent propositions do not split the true and the false in a definite way, whereas present and past propositions do.¹⁰ In Frede's view, this would be only "a diplomatic way of expressing oneself,"¹¹ which Boethius and Ammonius would have used to say that (PB) undergoes a suspension.¹² Unfortunately, that is not what Ammonius and Boethius mean. The indefinite distribution of the truth values among p and $\neg p$ does not amount to an unaffected distribution of them, as Frede seems to believe.¹³ Truth and falsity are distributed, and they are distributed indefinitely if p and $\neg p$ receive indefinite truth values. In other words, "dividing indefinitely the true and the false" is not the same as "not dividing the true and the false." If it were so, we could not explain why Boethius maintains that future contingent propositions are true or false and why he can speak of indefinite truth values.¹⁴ To have an indefinite truth value implies having a truth value and is not the same as not having a truth value.¹⁵

However, there is additional evidence against the claim that Boethius was a supporter of the traditional interpretation. In a polemical statement, Boethius seems to deny that future contingent propositions can be neither true nor false:

(D) Some people (among them the Stoics) believed that Aristotle says that contingent propositions in the future are neither true nor false. They took his statement that <the contingent> is no more related to being than to not-being as a statement that there is no difference in considering <the corresponding propositions> as true or false. For they thought that these propositions are neither true nor false. But falsely. Aristotle does not say this, namely that both <members of a contradiction> are neither true nor false, but that one whatsoever of them is true or false, not however in a definite way as in the case of past and present propositions.¹⁶

Later we will consider the reasons that may have led Boethius to reject the traditional interpretation. For the time being, it is sufficient to point out that he does reject the traditional interpretation. If p and $\neg p$ are future contingent propositions, it is not true to say that they are neither true nor false. They are true or false but not definitely true or definitely false, and in this sense they satisfy not only (LEM) but also (PB). I do not need to insist on this point any longer, which has already been clarified by Richard Sorabji and Bob Sharples.¹⁷

IV

The provisional conclusion of our inquiry is that Boethius, if we trust his way of speaking and his statement against the Stoics, was not a supporter of the traditional interpretation. That makes it urgent to solve the problem of what it means for a proposition to be said to be indefinitely true or indefinitely false. As a preliminary step to this aim, we have to get free from two possible misunderstandings.

First of all, one might be led to think that Boethius considers future contingent propositions indefinite with respect to truth and falsity because of an epistemological indeterminacy. Future contingent propositions, one might think, are by themselves definitely true or definitely false, but because they are contingent, we cannot state or know their truth value until the events denoted by them occur. It is now definitely true that Socrates will be debating in the gymnasium tomorrow; nevertheless, nobody can be certain that it is true before tomorrow, because the event in question is not causally determined by antecedent facts. So being indefinitely true or indefinitely false for a proposition does not depend on truth and falsity as such, but rather on our knowledge of facts. This interpretation might be supported by Boethius's asserting that, with respect to a contradictory pair of future contingent propositions

(E) it is not yet known which of them is true and which is false (*sed quae vera quaeve falsa sit, nondum in contingentibus notum est*).

(Boethius, *On Aristotle's On Interpretation* 2a, 200.17–18)

Or again:

(F) it is unknown according to the nature of the propositions themselves which of them is true and which is false (*quae enim vera sit, quae falsa secundum ipsarum propositionum naturam ignoratur*).

(Boethius, *On Aristotle's On Interpretation*. 2a, 200.30–201.1)

But the presence of the qualification “according to the nature of the propositions themselves” (*secundum ipsarum propositionum naturam*) in the second passage should warn us against too hasty a conclusion. Indeed, the lack of knowledge we have about the truth value of future contingent propositions is a consequence of their not having a definite status with respect to truth and falsity. Boethius is very clear on this point a few pages later, when he states:

(G) the nature of sentences is in a certain sense of two kinds. Some of them are such that not only one of them is true and the other false, but also one of them is definitely true and the other definitely false; others are such that one of them is true and the other false, but indefinitely and in a changeable way, and this happens because of their nature, not with respect to our ignorance and knowledge.¹⁸

From this passage it is clear that the indefiniteness and instability that qualify the truth or falsity of some propositions depend on the nature of the things referred to

and not on the status of our knowledge of them. In the same vein, Boethius points out that if the distinction between definite truth (definite falsehood) and indefinite truth (indefinite falsehood) were just a matter of knowledge, it would be impossible to refute determinism (*On Aristotle's On Interpretation* 2a, 245.19–28). Because he claims that the distinction is able to defeat determinism, it must be concluded that it is not just the difference between epistemological indeterminacy and epistemological determinacy that is in question here. Contingent propositions about the future are indefinitely true or indefinitely false not because the future is hidden or unknown to the mind, but because the ontological status of the facts they refer to is not yet settled.

A second preliminary remark concerns the way in which future contingent propositions may be treated and formalized. Let us consider Boethius's examples of tensed propositions. Besides the case of (i), which has already been mentioned, and the famous Aristotelian example

- (ii) there will be a sea battle tomorrow (*On Aristotle's On Interpretation* 2a, 246.6–7),

Boethius uses propositions such as

- (iii) Alexander will have breakfast today (*On Aristotle's On Interpretation* 2a, 106.2–3), and
 (iv) I will read Theocritus' *Bucolics* again today (*On Aristotle's On Interpretation* 2a, 234.12–13).¹⁹

These propositions are not only expressed in a future tense, but they also refer to future events.²⁰ From this point of view, a proposition such as

- (v) it will be true that a sea battle occurred yesterday

cannot be counted as a future proposition in Boethius's sense, even if the tense of the main verb is in the future. Similarly, the proposition

- (vi) it has been true that there will be a sea battle tomorrow

is not a past proposition in spite of the tense of its main verb. Propositions (v) and (vi) do not refer to future or past events, and for this reason they cannot be put among the propositions with which Boethius is concerned.

Propositions (i)–(vi) express their reference to time by means of the tense of the verbs and the adverbs “tomorrow” or “today.” We are therefore dealing with pseudo-dates²¹ and what is called the “A-series” of temporal events, in which a reference to the time at which a proposition is uttered, considered, or evaluated is essential. This does not happen when temporal events are ordered by means of the so-called “B-series.” The battle of Cannae is after the battle of Marathon independently of the moment one considers these two events.²² A dispute is ongoing among logicians and philosophers whether the A-series can be reduced to the B-series.²³ In putting

forward my proposal, I would not like to get involved in this difficult discussion. My aim is to present Boethius's position with reasonably formal simplicity. With this task in mind, take k to stand for a real date (in the B-series) and suppose that k is in the future (after now; henceforth " N " for "*nunc*.") Then, (ii) can be read as

(iia) there is a sea battle at k

where "there is" has to be taken atemporally (all temporal reference being committed to the date k), and futurity is expressed by the metalinguistic condition that N , the time in which (iia) is considered, evaluated, or uttered is before k , namely that $N < k$. From a formal point of view, we will take (iia), and consequently (ii), as an instance of $p(k)$, where $N < k$. The present and the past of a proposition can be represented accordingly. It is sufficient to put as a condition in the metalanguage that the date to which the event expressed by the proposition refers is after or the same as *now*, namely to state that either $k < N$ or $k = N$.

It should be noted that Boethius not only mentions propositions (i)–(vi) but also uses examples such as

(vii) Socrates will have dinner

where no adverb such as "tomorrow" or "today" are found.²⁴ It would be natural to treat (vii) as

(viia) $\exists t(\text{Socrates will have dinner at } t)$

and to consider (viia) as an instance of

(1) $\exists t(N < t \wedge p(t))$.

I will omit consideration of these propositions, because I am inclined to think that they can be treated in a way that is parallel to propositions with an explicit reference to time. However, we will return briefly to this point later on.

Finally, propositions (i)–(vi) are modally characterized, although no modal operator affects them. They are said to be contingent because they denote or refer to contingent events. The following passage can be taken as an implicit definition of a contingent event:

(H) Contingent is, according to the Aristotelian doctrine, everything that is either brought about by chance, or occurs by anyone's free choice and voluntary act, or it is possible to bring into both parts – namely that it happen and that it not happen – in virtue of a readiness of nature.²⁵

We will come back to this point later. For the present, it is sufficient to say that an event is presently contingent if it is not included in a causally determined chain in such a way that its outcome is already established and inescapable. To take advantage of an example that Boethius makes a few lines later (*On Aristotle's On*

Interpretation 2a, 190.24 ff.) hinting at Aristotle (*On Interpretation* 9, 193.12–13), it is contingent that my tunic will be cut, in the sense that it may happen that it will be cut, but it may also happen that it will not be cut. Of course, it cannot happen that my tunic will be both cut and not cut. Just one of the two events will occur, but it is not yet settled or determined which of the two will occur. From this point of view, we can say that the future is open.

V

After these preliminary remarks, we have to try to explain Boethius's view without any further delay. As we have seen, the major and most difficult point is constituted by his distinction between propositions that are definitely true or definitely false and those that are indefinitely true or indefinitely false. It is by distinguishing these two kinds of propositions that Boethius claims to avoid determinism. Therefore, we must first try to elucidate what "definitely true" and "indefinitely true" mean, and subsequently examine if (and possibly how) these notions enable him to avoid deterministic conclusions.

In order to accomplish the first task, we begin by making a rather obvious remark. Both definite truth (falsehood) and indefinite truth (falsehood) are time-relative notions. Boethius claims that past and present propositions are definitely true or definitely false, whereas some future propositions are only indefinitely true or indefinitely false.²⁶ That makes the qualification of these propositions doubly dependent on time or, more precisely, on the relation between two different temporal indices. On the one hand, we have to consider the time at which the event denoted occurs, which we have decided to express by a constant, for instance, k . On the other hand, the time at which the propositions are uttered, evaluated, or considered must be taken into account. If, for instance, $p(k)$ is a contingent proposition (is a proposition that predicts a contingent event) and $N < k$, then $p(k)$ is now indefinitely true or indefinitely false. It becomes definitely true or definitely false when $k \leq N$, that is, if the present time n , at which we evaluate $p(k)$, is later than k . This means that, for a proposition, being definitely or indefinitely true depends essentially on the time at which we consider it with respect to the time referred to by the event denoted by it.

Another obvious remark is that futurity is not the only criterion for deciding whether a proposition is definitely or indefinitely true or false. Boethius is very clear on this point. A proposition such as

(viii) this year in winter the sun will be in the Ram

is necessary and therefore has a definite truth value (*On Aristotle's On Interpretation* 2a, 200.1–9), although it is a future one. In order to be indefinitely true or indefinitely false a proposition must be not only future but also contingent, or, more exactly, it must refer to a contingent event. This relation between contingency and being indefinitely true or false is stressed more than once by Boethius. Let me quote one of the many passages concerning this point:

(I) As we have already said, contingent events are those that are equally related to being and not-being, and as they have an indefinite being and not-being, so affirmations <and negations> about them have an indefinite truth or falsity.²⁷

As is pointed out in this text, being contingent is a necessary condition that contributes to an event being indefinite. An event is indefinite only if it is contingent. If indefinite, a proposition about it is indefinitely true or indefinitely false. Therefore, we have to examine what to be indefinite means for an event. An event is indefinite only if it is not yet settled whether the event itself or its opposite will take place in the history of the world. Its being related to events that are definite can help clarify the point. Present and past events are definite, even if they are contingent, because their happening in the world is given and cannot be changed. The following passage illustrates Boethius's view:

(J) It is, therefore, clear that for present and past things, even among those that are contingent, the result is definite and settled. On the other hand, in the case of future events, either one of the two results may take place, but one of them is not definite. Rather, it is open to both results and one or the other occurs of necessity, but it cannot be that the one or the other is in a definite way. For things that are contingent can happen both ways.²⁸

What Boethius hints at when he says that in the case of future contingent events “*unum quidem quodlibet duorum fieri posse*” or “*in utramque partem vergere*” is pretty clear. I take it to mean that the event or its opposite can take place. The following passage makes the point clearly:

(K) The result of such things is undivided: it can happen so and so. And this occurs because the taking place of these things does not depend on some preceding causes, in a way that there is a sort of necessary chain, but these things depend rather on our decision and free will, where there is no necessity.²⁹

When an event is indefinite, the situation of the world is such that its occurrence is not yet settled, determined, or fixed. Only its possibility is given, and this possibility is an open one in the sense that the event itself or its opposite can be the result of the process. In other words, an event is definite or indefinite with respect to the situation or the state of the world at a given time. That explains why the way of being of an event can change and become definite when before it was indefinite. If the world develops in an appropriate way, it may happen that conditions are given that make the event not merely possible but actual, so that its being is no longer indefinite.

If propositions are definitely or indefinitely true or definitely or indefinitely false with respect both to the time at which they are uttered or considered and to the situation of the world in which they are evaluated, the following conclusion can be drawn. A proposition *p* is definitely or indefinitely true or definitely or indefinitely false with reference to two parameters: the time in which *p* is uttered

or considered and the status of the world in which p is evaluated. Let “ $T^*(. . .)$ ” stand for “... is indefinitely true” and “ $T^{**}(. . .)$ ” for “... is definitely true.” Consequently, we may write “ $T^{**}(p, S_i, t_j)$ ” to say that p is definitely true at time t_j and with respect to the situation of the world S_i . In a similar way, “ $T^*(p, S_i, t_j)$ ” is a way to express that p is indefinitely true at t_j and with respect to S_i .

In considering a proposition we have to consider time and, by the same token, contingency; that is, we have to represent propositions denoting opposite outcomes. A possible way to combine these different features is to think of a tree, the nodes of which represent possible states of the world and the paths of which are possible histories. Time can be interpreted as an ordered relation of the different stages of the possible states of the world. Of course, if we want to represent the development of the world from a certain state onward, we must consider a tree, the vertex of which is constituted by the state we take as the starting point of the process and the paths of which represent all the possible future developments of the initial state of the world. Figure 11.1 illustrates perhaps better what I have in mind.

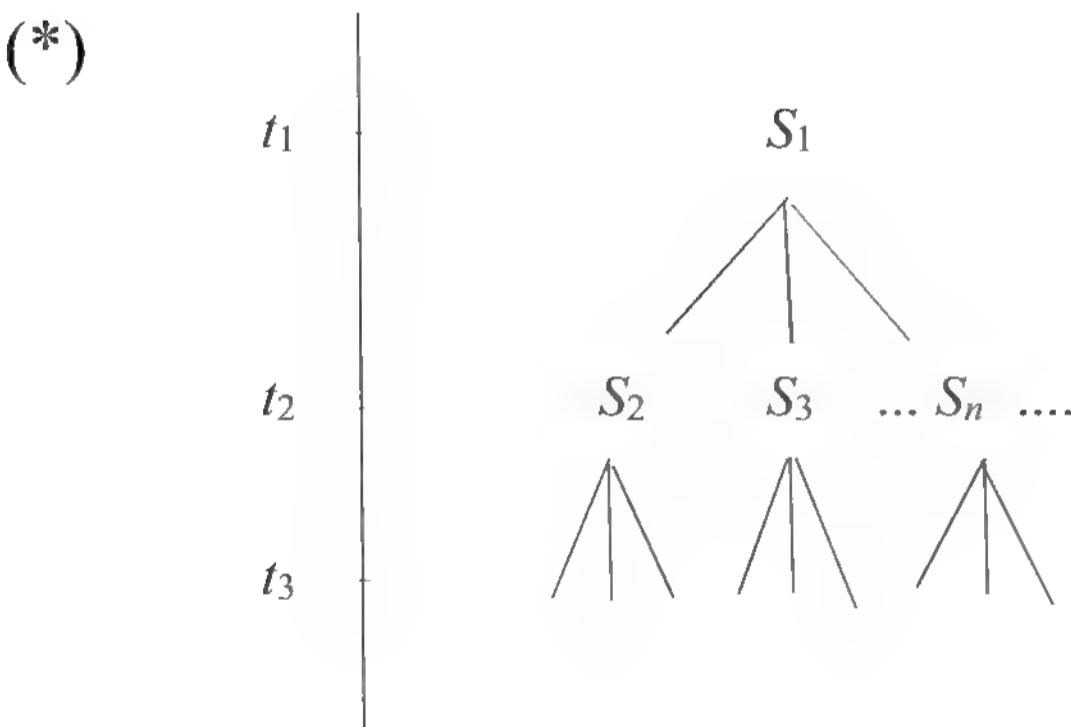


Figure 11.1

We may simplify this rather complicated figure by thinking of a tree where the history of just one proposition, and not the whole history of the world, is depicted. Let us represent this easier situation as shown in Figure 11.2.

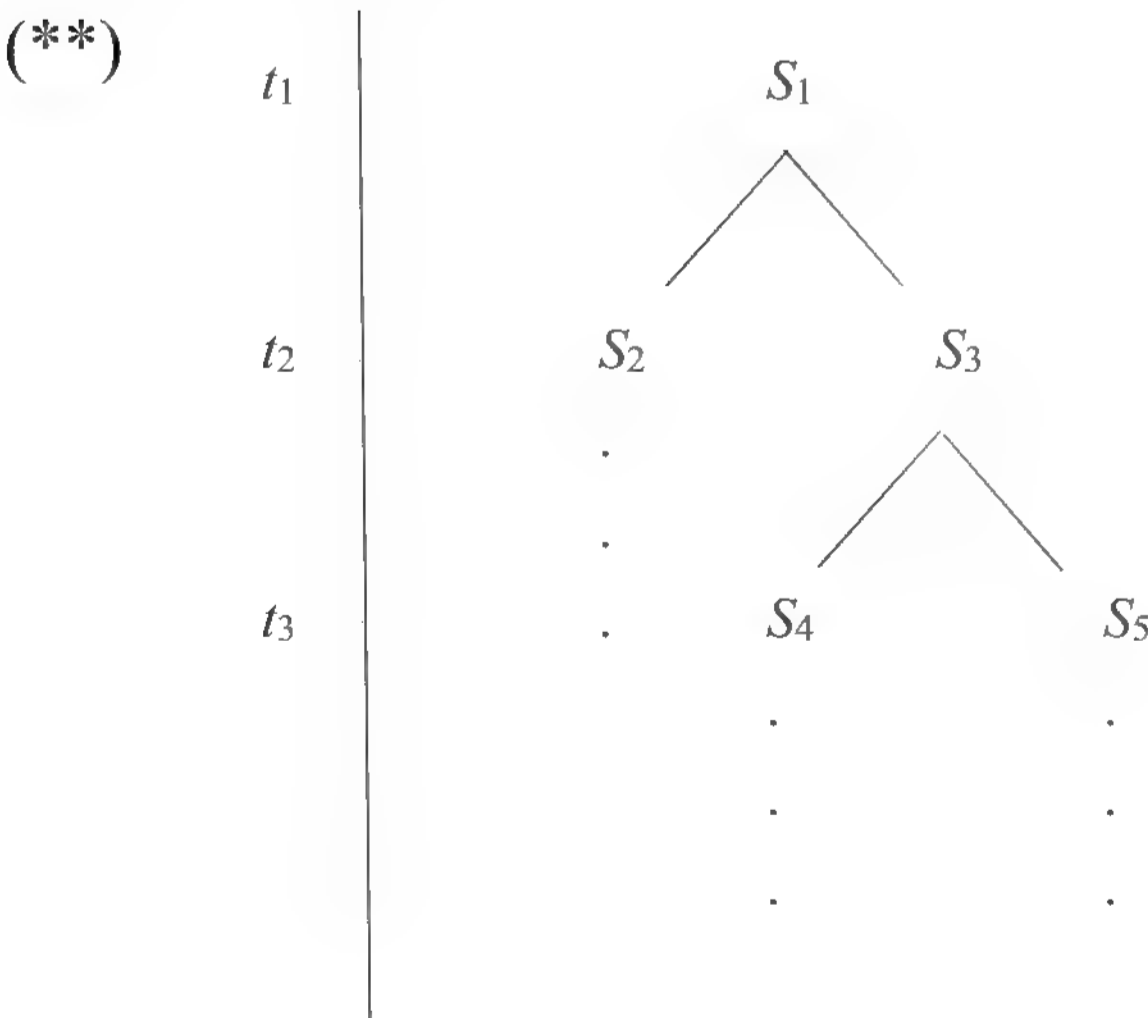


Figure 11.2

For the sake of simplicity we will refer to some instance of (**) as the standard case under the assumption that what we are going to say about (**) can easily be transferred to (*) with a few minor changes. We can express the relation between time and nodes by introducing the notion of level and by saying that an instant t is the level of a node S . With reference to (**) we can state, for instance, that

$$(2) \quad t_2 = Lv(S_2) = Lv(S_3).$$

It is easy to define what it means for a state S_j to be a possible development of S_i in such a structure. This notion is the intuitive counterpart of the relation of accessibility, which is well known to modal logicians. We represent it by “ $Acc(S_i, S_j)$,” and we take it to be reflexive and transitive.³⁰ For instance, with respect to (**) we can say that S_5 is a development of S_1 , because there is a path going back from S_5 to S_1 through S_3 , whereas S_4 is not accessible to S_2 for no such path exists. Therefore, it is with respect to structures such as (**) that we have to define what it is for a proposition to be definitely or indefinitely true and to be definitely or indefinitely false.

VI

We are still far from a definition of T^* - and T^{**} -predicates. With this aim in mind, we first have to try to define what it is for a proposition $p(k)$ to be true with respect to a given situation of the world S_i . It is clear that Boethius has a general correspondence theory of truth: p is true if and only if the event denoted by p takes place.³¹ Therefore, $p(k)$ has a clear-cut truth value at k . If the event denoted by $p(k)$ takes place at k , then $p(k)$ is true; if the event denoted by $p(k)$ does not take place at k , then $p(k)$ is false. If we consider $p(k)$ in S_i and S_i is on a level that is equal to or greater than k , we must assign to $p(k)$ either the assigned truth value True or the assigned truth value False according to the situation of the world to which we are referring. So, for instance, $p(k)$ can have the assigned truth value True in S_i or the assigned truth value False in S_i . Let us write “ $Assign(p(k), S_i) = 1$ ” for “the truth value True is assigned to $p(k)$ in the node S_i .” In a similar way, we state “ $Assign(p(k), S_i) = 0$ ” for “the truth value False is assigned to $p(k)$ in the node S_i ” or, what is the same, “the truth value True is assigned to $\neg p(k)$ in S_i .” To be precise, we have to distinguish between atomic and nonatomic propositions or, to use Boethius’s words, between categorical (or predicative) and conditional propositions.³² A truth value can be assigned just to atomic propositions and their negations, for instance, to $p(k)$ or $\neg p(k)$. From the notion of assigned truth value, we can work out the more general notion of being assignedly true or false in a node S_i , which can be applied to compound propositions. But for our purposes the notion of truth value assignment is enough, because Boethius only considers atomic propositions and their negations.³³ Therefore, when we speak of propositions, we normally mean atomic propositions or their negations.

What is important is that a truth value can be assigned to a proposition if and only if the conditions for assigning such a truth value to it are given; that is, if the events referred to by the propositions take place or are at least somewhat implied by the due course of the events. If we are dealing with a future contingent proposition, namely a proposition in which the event denoted is not settled, it is reasonable to state that no truth value can be assigned to $p(k)$ before k . If it is now contingent that a sea battle will take place at k (where $N < k$), we cannot assign a truth value to “a sea battle will take place at k ” before k .³⁴

Besides, it is easy to see that if a truth value is assigned to $p(k)$ in S_i it remains constant in any node accessible to (in the future of) S_i . That corresponds to the intuition that what has happened or is happening cannot be changed so that it is irrevocable in every possible future development of the world. If with respect to the situation S_i it is the case that there is a sea battle at k (with $k \leq Lv(S_i)$), however the world might develop, it remains true that there is a sea battle at k . Past and present events are eternally preserved in time. The same must be said with respect to falsity: If $p(k)$ takes 0 as its assigned truth value in S_i then it takes 0 in any node accessible to S_i . We can formally express the points we have tried to make informally by stating

$$\begin{aligned}
 (\text{AT}) \quad & Assign(p(k), S_i) = 1 \Rightarrow Lv(S_i) \geq k \wedge \forall S_j (Acc(S_i, S_j) \Rightarrow \\
 & \hspace{15em} \Rightarrow Assign(p(k), S_j) = 1) \\
 (\text{AF}) \quad & Assign(p(k), S_i) = 0 \Rightarrow Lv(S_i) \geq k \wedge \forall S_j (Acc(S_i, S_j) \Rightarrow \\
 & \hspace{15em} \Rightarrow Assign(p(k), S_j) = 0)
 \end{aligned}$$

where $p(k)$ is, as usual, a future contingent proposition. The first member of the conjunctions that constitute the right side of implications (AT) and (AF) meets the requirement that a truth value cannot be assigned to $p(k)$ before k . The second member expresses the condition according to which, when a truth value is assigned to a proposition with respect to a node S_i , it remains the same in any node accessible to S_i .

It is reasonable to think that if a truth value is assigned to $p(k)$ in S_i where $Lv(S_i) = k$, then $p(k)$ receives an assigned truth value in every node that is either on the same level as S_i or after S_i . The conditions that allow us to attribute an assigned truth value to $p(k)$ are at any rate given at k . Whatever the situation or the history of the world may be, from k onward $p(k)$ is assignedly true or assignedly false. If the proposition in question is “there is a sea battle at k ,” we must admit that at k and after the conditions are given to assign a truth value to the proposition. We can therefore state:

$$\begin{aligned}
 (\text{AV}) \quad & Assign(p(k), S_i) = 1 \vee Assign(p(k), S_i) = 0 \Rightarrow \\
 & \Rightarrow \forall S_j (Lv(S_i) \leq Lv(S_j) \Rightarrow Assign(p(k), S_j) = 1 \vee Assign(p(k), S_j) = 0).
 \end{aligned}$$

An example of what we are saying is the following. Consider (***) and suppose that we have the following assignation of truth values to $p(k)$ (see Figure 11.3).

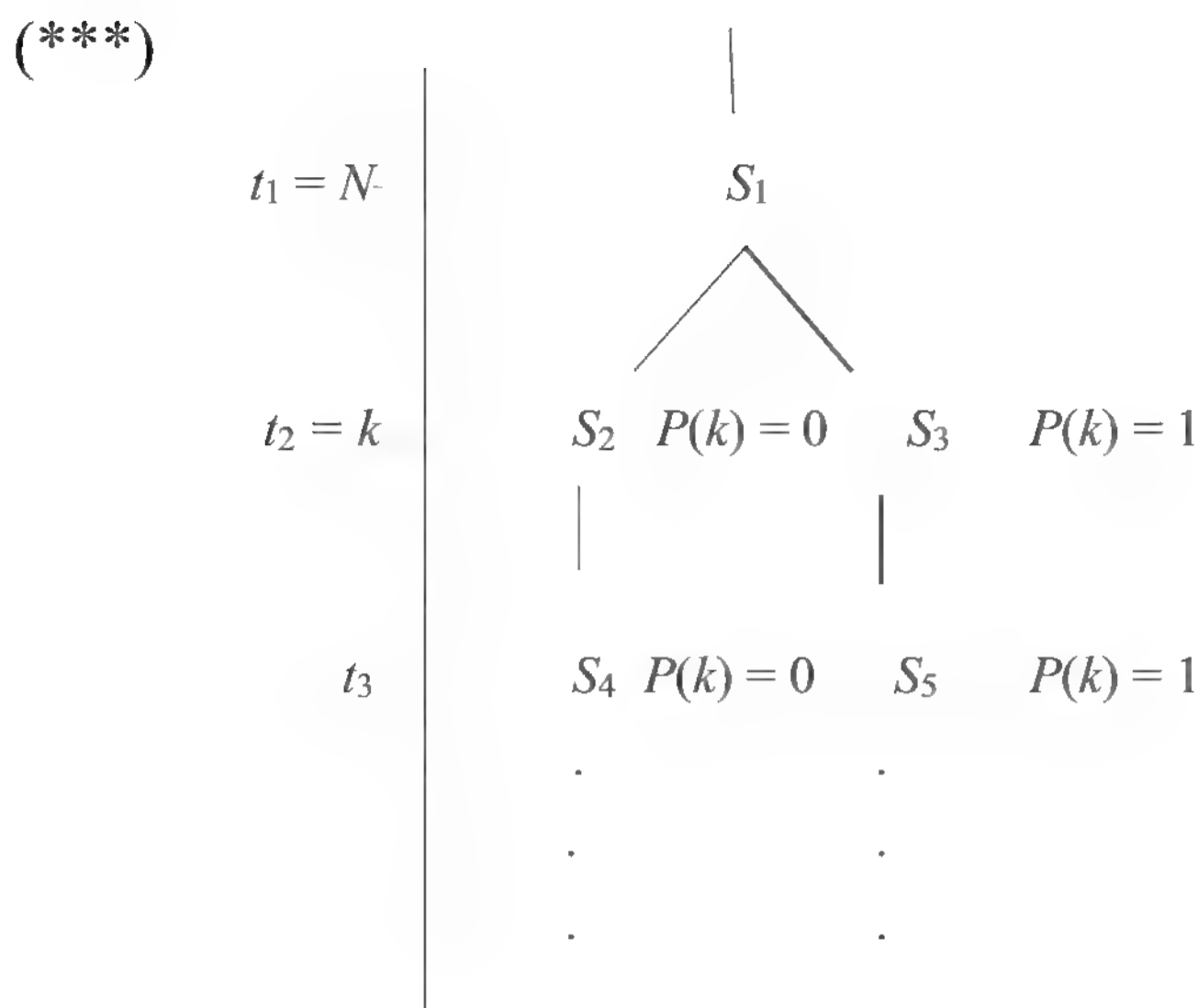


Figure 11.3

Because by hypothesis $t_2 = k$, we can assign a truth value to $p(k)$ with respect to S_2 and S_3 . Suppose that $p(k)$ takes the assigned truth value 1 in S_3 and the assigned truth value 0 in S_2 . Then, at any level after k , $p(k)$ will have an assigned truth value, and this value is 1 at the nodes on the same path as S_3 and is 0 at the nodes on the same path as S_2 , because we have assumed that the *Assign*-function is monotonic.

On the other hand, no truth value can be assigned to $p(k)$ in S_1 , because its level is t_1 , which is, by hypothesis, before k . Shall we conclude that $p(k)$ is neither true nor false in S_1 , because no truth value can be assigned to it? Boethius's answer is, of course, negative. His view is that even before k , $p(k)$ is either true or false, but not in a definite way, namely, in a way that makes the happening of the event denoted by it inescapable. We can try to make his point clear as follows. Suppose that we are able to refer to what is happening in the "real" future, namely to what will in fact happen, whatever that may be. Then if $p(k)$ will really take the assigned truth value 1 at k , it is in some sense always true that $p(k)$ will take such a value at k . What I mean is that if the truth value 1 is assigned to $p(k)$ in a node of the path that is supposed to be part of the "real" history of the world, then we are entitled to say that $p(k)$ is factually or plainly or simply true. Take P^* to be the path representing a part of the "real" history of the world, and suppose that S_n , which is at a level that is by hypothesis equal or greater than k , belongs to P^* , namely that $S_n \in P^*$. If the truth value 1 is assigned to $p(k)$ in S_n , $p(k)$ is factually or plainly or simply true. If " $T(p(k))$ " stands for " $p(k)$ is simply true"³⁵ we can write:

$$(ST) \quad T(p(k)) \text{ iff } \exists S_i (S_i \in P^* \wedge \text{Assign}(p(k), S_i) = 1).$$

If we suppose that the bold line in (***) represents the "real" path P^* , we can say that $p(k)$ is simply true because there is one node on P^* , namely S_3 , where $p(k)$ takes the assigned truth value 1. In a similar way, we can characterize an atomic proposition that is simply false by assuming:

(SF) $F(p(k))$ iff $\exists S_i (S_i \in P^* \wedge \text{Assign}(p(k), S_i) = 0)$.

The idea found in (ST) and (SF) is plain and corresponds quite well to our intuition. If conditions for the truth or falsity of $p(k)$ will be given in the future, then $p(k)$ is at all times true or false, in the sense that $p(k)$ will take either the assigned truth value 1 or the assigned truth value 0 in the real future of the world. It is easy to see that the notions of truth and falsity involved in T - and F -predicates are to some extent atemporal. In order to say that $p(k)$ is simply true, it is sufficient that it is possible to refer to conditions for the assigned truth of a proposition p . If a sea battle takes place at k , the proposition “there is a sea battle at k ” will receive the assigned truth value 1 at the node of level k belonging to the real path. But its being true, that is, its expressing conformity to an event, is something that does not depend on time. To guarantee the possibility of such a correspondence, we have only to admit that we are allowed to refer to the series of events that will take place in the “real” development of the world. We do not need to wait for the conditions that give a proposition an assigned truth value in order to attach a simple truth value to it. In this sense, plain truth and plain falsity are not intrinsically related to time.³⁶

One might think that this view does not completely correspond to Boethius’s doctrine because, in several places, he claims that in a pair of contradictory future contingent propositions one is *always* true and the other *always* false. Therefore, a reference to time seems to be implied by him. I do not think that this is a real objection to what I am trying to say. First of all, the mere use of expressions such as “always true” or “always false” do not commit us to a temporal interpretation of truth. The meaning of “always” is not strictly temporal and could be introduced to convey the idea of generality. Besides, I am inclined to think that Boethius uses “*semper verum*” and “*semper falsum*” simply to stress that simple truth and simple falsity do not depend on the particular time in which the proposition in question is uttered or considered. This would explain why Boethius sometimes omits any reference to time when he speaks of simple truth and falsity.

Needless to say, it is with respect to simple truth and falsity that Boethius can maintain that the principle of bivalence holds in every case. Because $p(k)$ or $\neg p(k)$ takes the assigned truth value 1 at an actual state, the level of which is k , it is simply true that $p(k)$, or alternatively, it is simply true that $\neg p(k)$.

VII

We are now in a position to give a definition of both definite and indefinite truth (falsehood). As we have seen, what contributes in an essential way to characterize a definitely true proposition is that it denotes a fixed event that occurs in an incapable way. We can express these features by saying that a definitely true proposition is a simply true proposition to which the truth value 1 has been assigned. By positing that it is a simply true proposition, we say that the event denoted by the proposition is an event of the “real” history of the world; moreover, we state that the level of the state of evaluation is either equal to or greater than that of the event denoted. Therefore, we write:

$$(T^{**}) \quad T^{**}(p(k), S_i, t_j) \text{ iff } Lv(S_i) = t_j \wedge k \leq t_j \wedge T(p(k)) \wedge \\ \wedge Assign(p(k), S_i) = 1.^{37}$$

A parallel definition of a definitely false proposition can be given in an obvious way.

A characterization of an indefinitely true proposition can easily be worked out from (T^{**}) . An indefinitely true proposition is a future contingent proposition with respect to the level of the state of evaluation, namely a proposition that denotes an event without an outcome that is already settled. Then we can state:

$$(T^*) \quad T^*(p(k), S_i, t_j) \text{ iff } Lv(S_i) = t_j \wedge t_j < k \wedge T(p(k)) \wedge \exists S_j (Acc(S_i, S_j) \wedge \\ \wedge Assign(p(k), S_j) = 0).$$

We can easily define what it is for a proposition to be indefinitely false.

With reference to (T^{**}) we can say that $p(k)$ is indefinitely true at t_1 and S_1 . For it is simply true, because it receives the truth value 1 at S_3 , which is on the path P^* representing the “real” history of the world; and it is contingently true, because in the node S_2 , which is accessible to S_1 , the truth value 0 is assigned to $p(k)$. On the other hand, $p(k)$ is definitely true in S_3 , because in this mode it has the assigned truth value 1, and S_3 is on the “real” path P^* .

It is easy to see that our characterization of definitely and indefinitely true propositions is able to explain in a simple way Boethius’s claim that present or past propositions are definitely true or false, whereas future contingent ones are indefinitely true or false. Take as usual $p(k)$. A situation S_i of the world in the real history of it corresponds to k , and taking (T^{**}) as an example, suppose S_i to be S_3 . A truth value is assigned to $p(k)$ in S_3 , with the level of S_3 being by hypothesis k . Because its assigned truth value in S_3 is 1, $p(k)$ has the assigned truth value 1 in any node accessible to S_3 . Therefore, $p(k)$ is definitely true from k onward. On the other hand, suppose that we consider $p(k)$ before k in a node S_i . As before, a situation in the “real” history of the world corresponds to k . With respect to this situation, say it to be S_j (where $Lv(S_j) = k$), we can decide whether $p(k)$ is simply true or simply false. Suppose it to be simply true. Because by hypothesis $p(k)$ refers to a future contingent event, there must be a node accessible to S_i and not on the “real” path in which $p(k)$ takes the truth value 0. Therefore, $p(k)$ is indefinitely true in S_i . If we use the example represented by (T^{**}) , we can identify S_i with S_1 .

VIII

I do not think that we need to insist on explaining the distinction between T^* - and T^{**} -predicates. It is more interesting to see how this distinction works in Boethius’s attempt to reject the deterministic view.

Boethius makes an important point when he claims that the deterministic view follows from taking every proposition as definitely true or false. Among the many passages, let us recall the following one:

(L) Therefore, with respect to this kind of contingent propositions, if in the future one of them³⁸ is always true and the other always false, and one of

them is definitely true and the other is definitely false, and things correspond to words, everything necessarily is or is not, and everything that takes place takes place necessarily, and nothing can be that can not be, and there will be no free will and no chance in any of the things in which necessity rules.³⁹

The same point is repeated few lines after:

(M) Aristotle, after having assumed this conditional proposition, namely if everything which is said in the future is definitely true or definitely false, everything takes place by necessity and nothing by chance or by decision or by possibility, explains these points in a suitable order.⁴⁰

If we rely on the given interpretation of what it is for a proposition to be definitely true or definitely false, we can take Boethius's words as follows. Determinism, namely the view that "everything necessarily is or is not, and everything that takes place takes place necessarily, and nothing can be that can not be," is a consequence of the view that every proposition has an assigned truth value in any node of the path representing the "real" history of the world. Take, once again, (***). Suppose that the part of the history of the world that interests us starts at t_1 .⁴¹ Imagine that $p(k)$ already had an assigned truth value in S_1 .⁴² For instance, let $p(k)$ have the assigned truth value 1 in S_1 . Therefore, it has the assigned truth value 1 in any node accessible to S_1 , that is, in any node whatsoever of the tree. In this sense $p(k)$ is on the same footing as present and past propositions, and therefore it enjoys the same unalterable condition as those. If the future behaves like the past and the latter is necessary, the former is too.

Boethius's view is strongly against the assumption that every proposition is definitely true. If it is not the case that a truth value is assigned to any proposition at every state, determinism cannot be derived. If a truth value is not assigned to $p(k)$ in S_1 – once again we refer to (***) – we cannot infer that $p(k)$ has the same assigned truth value in all nodes that are accessible to S_1 . There is no reason to rule out that, in a node accessible to S_1 , $p(k)$ takes the assigned truth value 1, whereas in another node, which is also accessible to S_1 , $p(k)$ takes the assigned truth value 0, as in fact happens in (***). This possibility depends on the assumption that the event denoted by $p(k)$ is not yet determined or fixed by the state of the world constituted by S_1 . This means that the development of the world is not implicit in S_1 , so that it is still open whether the world will evolve in a way that allows us to assign either the truth value 1 or 0 to $p(k)$. In this precise sense, $p(k)$ is indefinitely true in S_1 . It is simply true under the hypothesis that the "real" history of the world will turn toward S_3 . At S_1 , however, it is still possible that the world will evolve in such a way that the future event denoted by $p(k)$ does not actually obtain. But to admit such an open future implies that determinism is ruled out.

Boethius stresses this point by underlining that the truth or falsity of an indefinitely true or false proposition is uncertain and open to change. For instance, with reference to future contingent propositions, he says:

(N) If neither is it the case that both propositions <of a contradictory pair> are true or that both are false, nor is it case that one is definitely true and the other definitely false, it remains that one of them is true and the other is false, not definitely, but in an open and unstable way, that it is necessary that this or that occurs, but in a way that it cannot occur that one thing comes out by a sort of necessity or definitely.⁴³

I take the qualification of the truth or falsehood of an indefinitely true or indefinitely false proposition, namely “*non tamen definite, sed utrumlibet et instabili modo*,” not as a statement that what is true can become false. If $p(k)$ is indefinitely true in S_1 ,⁴⁴ then it is simply true and once and for all so qualified. From this point of view, there is no instability or indeterminacy in the simple truth of $p(k)$. What makes its truth unstable and open is that it is consistent with the assignment of the truth value 0 to $p(k)$ in a merely possible future of S_1 .

Similarly, in the passages where Boethius claims that in a pair of contradictory future contingent propositions

(O) it is indefinite and undivided (*indefinitum enim et indiscretum*) which of them is true and which is false.

(*On Aristotle's On Interpretation* 2a, 247.28–248.2)

one has not to see a reference to the curious doctrine according to which a disjunction can be said to be true without its members being true or false. Boethius repeats immediately before the passage we are considering that in such a contradictory pair, one proposition is true and the other is false.⁴⁵ What he wants to stress is, once again, that the indefinite truth of $p(k)$ does not rule out that there is a possible state at which $p(k)$ takes 0 as its assigned truth value.

Reading these passages, one might be tempted to take a weaker characterization of what it is for a proposition to be indefinitely true or indefinitely false. If the reference to simple truth is dropped in (T^*) , the idea of a changing truth might be comparatively reinforced. To make the point clearer, let us suppose that we have, instead of (T^*) ,

($T\#$) $T\#(p(k), S_i, t_j)$ iff $Lv(S_i) = t_j \wedge t_j < k \wedge \exists S_m \exists S_n (Acc(S_i, S_m) \wedge$
 $\wedge Acc(S_i, S_n) \wedge Assign(p(k), S_m) = 1 \wedge Assign(p(k), S_n) = 0)$.

It is easy to see that (T^*) is stronger than $(T\#)$ in the sense that a proposition is $T\#$ -true if it is T^* -true, but not conversely. Because of this, one can understand in what sense Boethius says that an indefinitely true proposition is open to opposite assignments of truth values. But the temptation to interpret indefinite truth as $T\#$ -truth must be resisted. By doing so, we should give up explaining why Boethius claims that an indefinitely true proposition is true (that is, simply true) and an indefinitely false proposition is false (that is, simply false). Therefore, a reference to simple truth must be included in the definition of an indefinitely true

proposition. Consequently, (T^*), but not ($T\#$), must be taken as the correct interpretation of indefinite truth.

There is, however, a passage that seems to contradict this conclusion. By contrasting indefinitely true or false with definitely true or false propositions, Boethius says that in a pair of indefinitely true or false propositions in which one is the negation of the other

(P) *una quidem vera est, altera falsa, sed indefinite et commutabiliter.*
(*On Aristotle's On Interpretation* 2a, 208.16–17)

In this passage, the difficult word is “*commutabiliter*.” Properly speaking, an indefinitely true or false proposition cannot be said to be *commutabiliter* true or false, because it is not sometimes true and sometimes false, but *semper vera* or *semper falsa*, namely simply true or simply false. “*Commutabiliter*” would better fit the interpretation of “indefinitely true” in terms of $T\#$ - than in terms of T^* -predicate, and it has to be ruled out that Boethius was not completely clear about this point.⁴⁶

This impression is reinforced by a rather strange statement that can be found in the *prima editio*, where Boethius stresses the variable character of the truth and falsity of indefinite propositions:

(Q) Things which do not always exist or it is not always the case that they do not exist are the only things for which it is contingent to be or not to be. If they always existed, their state could not change and therefore they would necessarily exist; on the other hand, if they always did not exist, they necessarily would not exist. For as the nature of events varies, so one or the other part of a contradiction has a variable truth (*habet variabilem veritatem*). One of them is always true or false, but not in such a definite way that this or that is determinately true, but one or the other. As the state of the things is changeable, so the truth and falsity of the propositions is doubtful.⁴⁷

The beginning of this passage, where Boethius seems to endorse what is called “the statistical interpretation of the modalities” according to which what is always true must be classified as necessary, does not concern us. I am inclined to believe that Boethius has here blundered in his effort to establish a parallel between the state of contingent events and the state of propositions that express them and that the interpretation is not his standard view about modalities.⁴⁸ What is relevant to us is that he speaks of a variable truth of indefinite propositions and that he puts this statement together with the claim that they are always true or false. Shall we infer that the variable truth of indefinite propositions is nothing more than a way to assess that an indefinitely true proposition does not rule out the possibility that it takes the assigned truth value 0 in the future? His words seem to say more than this. Perhaps Boethius is here thinking not of propositions such as $p(k)$, which contain a definite reference to time, but of temporally indefinite propositions, with the form $p(t)$. If $p(t)$ is contingent, it can be roughly described as a proposition that

can change its truth value in time, and from this point of view it imitates better than $p(k)$ the inconstant nature of facts.⁴⁹

However, it seems to me that Boethius's sometimes careless wording does not compromise the whole of my interpretation. What is relevant to my view is that both aspects of his doctrine find a reasonable explanation: both the aspect according to which future contingent propositions are simply true or false and the aspect according to which they are indefinite. By stressing the latter, Boethius is able to dismiss the deterministic view; by endorsing the former, he is able to justify predictions. We will see in the next section how this happens. For the present, it is sufficient to point out that if predictions of future contingent events are admitted as possible, then a truth value must be assigned to indefinitely true propositions. These propositions cannot be merely such that they *can* be true or false; they *must* be either simply true or simply false. Therefore, we cannot take ($T\#$) as a sufficient elucidation of what it is for a proposition to be indefinitely true.

The same argument makes it impossible to think that what we have called the "simple truth" or "simple falsity" of an indefinitely true or false proposition is merely an arbitrarily given truth value that can be changed according to the way in which the world develops. Of course, such an arbitrary truth value would justify the validity of the principle of bivalence: If the arbitrary truth value True is attributed to p , the truth value False must be attributed to $\neg p$ for logical reasons. Although this view might be reasonable in itself, it is hardly consistent with the possibility of predictions admitted by Boethius. If $p(k)$ is arbitrarily true before k and this is the only truth we can confer on this proposition, it does not follow from its truth that the event denoted by $p(k)$ will occur at k . But it is precisely this that makes a prediction a true prediction.

To avoid misunderstandings, I would like to insist on how attributing simple truth or falsity to indefinitely true or false propositions makes sense. Let us refer once more to (**). A consequence of (T^*) is that although a truth value is not assigned to $p(k)$ in S_1 , because by hypothesis the conditions according to which the proposition is assigned the truth value 1 or 0 are not yet given, $p(k)$ is nonetheless in some sense already true or false in S_1 . As we have seen, $p(k)$ has no assigned truth value in S_1 , but it is simply true. What does this mean? One might maintain that if $p(k)$ is already true in S_1 , that is, before k , its truth is already settled and therefore the event denoted by it has always been determined. But to say this implies confusing two quite different cases, namely the assigned truth value 1 at S_1 and that of simple truth. As we have said, "assigning the truth value 1 at a given state to a proposition" means that the conditions are given that make p true at that state. To say that p is simply true amounts to saying that p takes the truth value 1 if and only if the situation of the world evolves such that the conditions for assigning 1 to p obtain. It is simply true that tomorrow there will be a sea battle if and only if a sea battle takes place tomorrow. The assigned truth of this statement depends uniquely on the way in which the world, or better the "real" history of the world, evolves. Therefore, by saying that $p(k)$ is simply true in S_1 , one is committed to maintaining that the path going from S_1 to S_3 represents the "real" development of the world. But this does not entail that the path going from S_1 to S_3 *must be* the

real one. That would happen only if one could rule out that in a possible future of S_1 , $p(k)$ takes the assigned truth value 0. And because we cannot rule out such a possibility, we must conclude that the development of the world is not necessarily implied by S_1 . Therefore, the simple truth of $p(k)$ does not entail that its taking the truth value 1 in the future is already determined by the present state of the world. The obvious consequence is that the simple truth of future contingent propositions does not entail determinism.⁵⁰

It is now clear what role the distinction between propositions that are definitely and indefinitely true or false plays in Boethius's defense against the deterministic attack. If any true proposition is definitely true, and this happens at any state of the world, then any true proposition is necessary and the event denoted by it is determined. But if there are propositions that are indefinitely true or false, the same conclusion cannot be drawn. From the mere fact that a proposition is simply true or false, it cannot be inferred that it is necessarily true or false. The corresponding event is not yet fixed and the deterministic claim therefore cannot be maintained.

IX

Is Boethius's solution sufficient to reject determinism? The answer to this question is controversial, and I cannot enter such a discussion here. I would rather like to point out that his position is interesting not only because of the question of determinism but also because it entails a theory of truth that is worth considering. A comparison with the traditional interpretation can be helpful.

As we have seen, Boethius knew the traditional interpretation and rejected it. A possible reason for such a negative attitude involves the question of predictions. According to determinists, predictions show that if an event A happens at k , then the proposition $p(k)$ that refers to this event is true not only at k but also at any time before k . This implies that $p(k)$ is necessarily true and the occurrence of A at k is at any time predetermined.⁵¹ The traditional interpretation had a radical answer to this objection. Even if A occurs at k , $p(k)$ cannot be said to be true before k , or at least before the time at which A becomes part of a causal chain of the world. Therefore, it cannot be said that $p(k)$ is always true and therefore necessary. The price to pay for this solution is clear. No prediction is possible, if by "prediction" a statement concerning a future contingent event is meant that is true before the time in which the event referred to happens. If $p(k)$ cannot be true before k and this holds for any future contingent proposition, any prediction about such events is meaningless.

It is obvious that the kind of predictions considered by Boethius have nothing to do with scientific predictions, which are not about future contingent events, but are about events that are submitted to laws of some sort. Propositions about such events are definitely true or definitely false even before the time to which the events refer. In Boethius's account predictions concerning events that by definition are not submitted to any law are in question. One might think that it is not a great harm if such predictions are given up. We might even feel relieved if in an ideal city ruled by logicians fortune tellers, soothsayers, and other people

of this sort had no admission. But, of course, this was not Boethius's view. As is well known, the ancient world paid a great deal of attention and gave a large place to oracles, divinations, prophecies, and predictions in general. Philosophers were accordingly interested in these phenomena. The general attitude was more inclined to search for a justification for predictions and oracles than to deny that they are reliable. Besides, Boethius and in general the late Neoplatonic philosophers who constituted his sources had a theological reason for admitting predictions. God is provident, and he must know the world toward which he exerts his beneficial influence in such detail that nothing escapes attention. Therefore, even future contingent events must be known to him.⁵² Boethius does not waiver in maintaining that god knows future contingent events and that predictions are possible (*On Aristotle's On Interpretation* 2a, 224.27–226.6).

If predictions are admitted, how can one deny that future contingent propositions are definitely true even before the time to which the events denoted refer? And how can the conclusion that everything is necessary or impossible be avoided? Boethius's answer is contained in the following passage:

(R) If an event occurs necessarily, it was true to say that it will be. But if the event does not happen by necessity, but contingently, it was not true to say that it will be, but rather that it happens contingently. For people who say it will be put some necessity in the prediction itself.⁵³ Therefore, it must be understood that if one truly says that something will be and that which has been foreseen cannot not happen, this happens by necessity. Therefore, one who says that some contingent event will be, is perhaps wrong in so far as one says that what happens contingently will be. Even if the thing foreseen happens, the person is nevertheless wrong. For it is not the case that the event is false, but the way in which the foreseeing is made is false. For it should have been said this way: a sea battle will take place tomorrow in a contingent way, that is: if it happens, it happens in a way that it was possible for it not to happen. One who speaks this way, says something true, since one has foreseen an event in a contingent way. On the other hand, one who begins this way: there will be a sea battle tomorrow, as if it was necessary, speaks in this way. If this event happens, one has not spoken the truth because one has foreseen, since one has foreseen that the event which happens contingently will take place necessarily. The falsity is not in the event, but in the way of the foreseeing. For if one says while Socrates is walking: "Socrates is necessarily walking," one is wrong not because Socrates is walking, but because one says that Socrates is necessarily walking; in the same way one who says that something will be is wrong even if the thing happens, not because it happens, but because it does not happen in the way in which one has foreseen that it would be. For if the proposition would definitely be true, the event would necessarily happen [. . .]. It is therefore necessary that with respect to contingent events predictions are made in such a way that, if the proposition is true, one says that something will be, but in a way that it is still possible that it will not be [. . .]. If someone simply foresees that an event will be which will perhaps

take place in a contingent way, he foresees that a contingent thing will necessarily be. Therefore, even if that which is said would happen, nonetheless one is wrong, since on the one hand the event would take place contingently, and on the other hand the prediction was that the event would necessarily be.⁵⁴

Boethius's explanation is on the whole clear. He *does* admit the possibility of predictions about future contingent events. His worry is about the way in which a prediction must be expressed, and he says that it must be done in a way that does not carry the impression that what is foreseen will necessarily happen. If a truth teller says that tomorrow there will be a sea battle, we can say that he is right only if we do not take his forecasting as implying that the event will take place necessarily. Apart from the linguistic question of the way in which a prediction must be expressed, the point remains that $p(k)$ can be true even before k , and this assumption does not imply that the event denoted by $p(k)$ takes place necessarily. The event would take place necessarily if the truth of $p(k)$ were definite even before k . But there is no reason to assume that $p(k)$ has a definite truth value before k . As we know, $p(k)$ has an indefinite truth value before k . Therefore, neither is $p(k)$ a necessarily true proposition nor does the event denoted by it take place necessarily. If we reconsider the formal representation that has been given to the notions of definite and indefinite truth, the point will appear immediately clear. Take (***) once again and suppose that a truth teller X is in S_1 . The possibility that X can predict that the truth value 1 is assigned to $p(k)$ in S_3 is warranted by the fact that $p(k)$ is simply true. Therefore, if X has the capacity to see what the conditions of the world will be at k , he can predict what the future will bring with respect to $p(k)$. But by hypothesis the event denoted by $p(k)$ is not yet settled in S_1 , namely the conditions that make the outcome of that event inescapable are not yet given. Therefore, $p(k)$ is not definitely true in S_1 . The world might develop in a way in which 0 is assigned to $p(k)$. Consequently, even if the prediction about $p(k)$ is simply true, it is not necessary that $p(k)$ is simply true, and the event denoted by it is not unpreventable just because it has been truly predicted.

When Boethius claims that a true prediction must be made in a way to avoid commitment to necessity, he does not maintain that a prediction must be uncertain. The contingency of a prediction has nothing to do with its uncertainty, and a prediction is not contingent because it is uncertain. If it were so, Boethius would not have said that god is able to know the future (*On Aristotle's On Interpretation* 2a, 224.27–226.25) and that he knows it as contingent:

(S) God knows future events not as necessarily, but as contingently, happening in a way that does not ignore that a different event could have taken place.⁵⁵

A passage from Boethius's *On Consolation of Philosophy* can also be invoked in defense of our interpretation. Boethius is considering the way in which god foreknows future contingent events. He denies that god foreknows the future in the way Tiresias does:

(T) How does it⁵⁶ compare with that ridiculous prophecy of Tiresias? – “Whatever I say will happen or not”? (Tester translation.)⁵⁷

As we have seen, the contingency of a prediction merely depends on the contingency of the event predicted, in the sense that the truth of a prediction is a factual truth and it does not rule out the possibility that the predicted proposition takes a different truth value in the future. The same point is clearly made in *On the Consolation of Philosophy*:

(U) Therefore things which, while they are happening, lack any necessity of being so, these same things, before they happen, are future without any necessity. And therefore there are some things going to happen the occurrence of which is free from all necessity. For I do not think that any man would say that those things which are happening now were not going to happen before they happened; therefore of these, even foreknown, the occurrence is free. For just as knowledge of present things introduces no necessity into those things which are happening, so the foreknowledge of things introduces none into those things which are to come (Tester translation, slightly modified).⁵⁸

God is surely able to identify whether the path in the tree of our future that corresponds to the actual history of the world and to know that $p(k)$ is true in any given node of such a path. There is nothing doubtful in his prediction. But still it remains contingent in S_1 that $p(k)$ obtains in S_3 , in the sense that the development of the world in S_3 is not necessarily implied by the stage S_1 . Therefore, the contingency of the events predicted has nothing to do with the certainty of the forecasting.

X

The capacity of offering a justification of predictions is not the only distinguishing mark of Boethius's position with respect to the traditional interpretation. There is a subtler and more important difference that concerns the notion of truth. As we have seen, the traditional interpretation is based on the idea that a proposition can only be said to be true or false when the extralinguistic conditions for this attribution are given. In the case of future contingent propositions, these conditions are not given. Therefore, no truth value can be assigned to them. What comes out from this conclusion is that truth is a totally temporal notion, namely a notion that can only be applied when appropriate extralinguistic conditions are the case. The immediate consequence of this view is that the principle of bivalence cannot be considered a logical law, and for a proposition, not being true does not mean that it is false.

One might find these implications unpalatable. From the fact that, according to the traditional interpretation, it is now not true that tomorrow there will be a sea battle, it cannot be inferred that a sea battle will not take place tomorrow. This is not at all obvious, and one might prefer to think of truth as something that is not completely given in time. Of course, the conditions that make a proposition true

or false are given in time. It is in time that a sea battle takes place, and it is in time that the truth value 1 is assigned to the corresponding proposition. But it does not follow from that the predicate True can only be applied to a proposition when the corresponding extralinguistic conditions occur. If at some time a proposition becomes true in a proper sense, we are allowed to refer to this fact even before it happens. If “there is a sea battle at k ” is true in an assigned way at k , that is, the conditions for its truth are given at k , then we can refer to the truth of the proposition at any time whatsoever. This does not mean that the conditions that make the proposition true are given at any time. By hypothesis they are given only at k . Nonetheless, if a sea battle happens at k , the proposition “there is a sea battle at k ” is “always” or simply true, in the sense that it takes the assigned truth value 1 at the actual state the level of which is k . Its receiving this truth value at k does not depend on the time we consider the proposition, but it holds atemporally.

I cannot pursue this inquiry, which has deep and controversial philosophical implications, any longer. I would like to conclude this section by pointing to a limit of the traditional interpretation that seems to be overcome by Boethius’s position. Take, for instance, a proposition such as

(ix) it will always rain.

Its formal counterpart is

(3) $\forall t(N < t \rightarrow p(t))$.

If we put ourselves in the traditional interpretation, we cannot be sure that there is a time at which (3) becomes either true or false if we take the domain over which the temporal variable t ranges as infinite. Suppose that $p(t_1), p(t_2), \dots, p(t_n), \dots$. At t_n we cannot conclude that (3) is true. Of course, if it happens that $\neg p(t_n)$, we can infer that (3) is false. But if $p(t_n)$ still we have to postpone an assignment of a truth value to (3). In particular, if (3) is true, there is no time in which we can say that it is true. If we cannot state that (3) is true, we cannot assert that (1) is false either, because the falsity of (1) is the truth of $\forall t \neg(N < t \wedge p(t))$.

If we move from the traditional interpretation to a conception in which truth is no longer intended as a temporal predicate, we are immediately able to treat propositions such as (1) and (3). It is sufficient to have the capacity of referring to the totality of the truth values assigned to propositions $p(t_1), p(t_2), \dots, p(t_n), \dots$, in the nodes of the “real” path. If these assigned values are 1, then (3) is true; if they are all 0, then (1) is false. It does not matter how we can go along an infinite path. The important point is that we can understand what it means for a proposition such as (3) to be true. This is possible exactly because simple truth and simple falsity are not temporal notions, namely predicates that can only be attributed to propositions at a given time. From this point of view Boethius’s position seems to offer the possibility to analyze a greater variety of propositions than the traditional interpretation.

It can, however, be pointed out that if propositions such as (1) and (3) are systematically considered, the definitions of definite and indefinite truth and falsity must be modified accordingly. These have been given with reference to time, and no such reference can be applied to propositions that range over the whole of time. Nontemporally committed modalities should be introduced to explain the indefinite or definite truth of (1) and (3). But this is another story. In fact, there is no need to modify the theory to account for propositions such as (vii) if the assumption is made that contingent propositions, the subjects of which are spatiotemporal individuals, become and remain false when their subjects cease to exist. This is the doctrine that Boethius attributes to Aristotle in his commentary on the *Categories*.⁵⁹ By exploiting this view it can be said that to decide whether (vii) has the assigned truth value 0 one is not compelled to consider the whole series of temporal instants, but only the interval of Socrates's life. If in no instant of this interval Socrates has dinner, then, after this interval, (vii) takes the assigned value 0, because it cannot be true that Socrates has dinner after his death. Therefore, in assigning a truth value to (vii), no reference to the whole series of time is demanded, and thus we can think that (vii) can take an assigned truth value at a certain time of the development of the world. The same can be said about the instances of (3), the subjects of which are spatiotemporal individuals. We can conclude that Boethius does not consider propositions such as (ix), although his theory can be adapted to explain also this kind of proposition much better than the traditional interpretation.

These comments show, I hope, that Boethius's doctrine is far from being uninteresting in a historical and philosophical prospective. Its commitment to a well-refined theory of truth, on the one hand, and its exploiting of the notions of necessity and possibility, on the other, clearly reveal how ample the range of the problems involved by it is and how modern they are. (*)

Notes

Many friends and colleagues helped me avoid mistakes and helped me elaborate my interpretation. Among them, I would like to thank Richard Sorabji, Bob Sharples, Simo Knuuttila, Enrico Martino, Paolo Leonardi, and Pierdaniele Giaretta.

- 1 A bibliographical survey of the relevant books and papers until the year 1973 can be found in Celluprica 1977. Further references are in given in Frede 1985: 84–87; Talanga 1986: 169–185; and Kirwan 1986: 167–187.
- 2 Cf. Boethius, *On Aristotle's On Interpretation* 1a, 105.11–18.
- 3 Cf. Boethius, *On Aristotle's On Interpretation* 1a, 105.24–26.
- 4 Boethius, *On Aristotle's On Interpretation* 2a, 191.5–11.
- 5 Boethius, *On Aristotle's On Interpretation* 2a, 200.11–201.2: *contingentia autem sunt (ut supra iam diximus) quaecumque vel ad esse vel ad non esse aequaliter sese habent, et sicut ipsa indefinitum habent esse et non esse, ita quoque de his adfirmationes <et negationes> indefinitam habent veritatem vel falsitatem, cum una semper vera sit, semper altera falsa, sed quae vera quaeve falsa sit, nondum in contingentibus notum est. Nam sicut quae sunt necessaria esse, in his esse definitum est, quae autem sunt impossibilia esse, in his non esse definitum est, ita quae et possunt esse et possunt non esse, in his neque esse neque non esse est definitum, sed veritas et falsitas ex eo quod est esse rei et ex eo quod est non esse rei sumitur. Nam si sit quod dicitur, verum est, si non sit*

quod dicitur, falsum est. Igitur in contingentibus et futuris sicut ipsum esse et non esse instabile est, esse tamen aut non esse necesse est, ita quoque in adfirmationibus <et negationibus> contingentia ipsa prodeuntibus veritas quidem vel falsitas in incerto est (quae enim vera sit, quae falsa secundum ipsarum propositionum naturam ignoratur), necesse est tamen unam veram esse, alteram falsam.

- 6 For this terminology see, for instance, Frede 1970: 24 ff.; Sorabji 1980: 94; Seel 1982: 240.
- 7 If $F(p)$ is defined as $T(\neg p)$, an equivalent way to express (PB) is to say that given a pair of contradictory propositions p and $\neg p$ either p is true or $\neg p$ is true, namely $T(p) \vee T(\neg p)$. Properly speaking, we should write $T(\ulcorner p \urcorner)$ and $F(\ulcorner p \urcorner)$ where “ $\ulcorner p \urcorner$ ” is the so-called “nominalization” of p . But we can omit such refinements.
- 8 The point is made by John Ackrill (Ackrill 1963: 133).
- 9 See Frede 1985: 42–45; Frede 1970: 24–27.
- 10 See, for instance, Boethius, *On Aristotle's On Interpretation* 1a, 111.17–20: *quod si in futurum omnis adfirmatio vel negatio definite semper dividunt veritatem et falsitatem, erit rerum quae praedicuntur necessarius eventus et omnia ex necessitate contingent vel non contingent*. Cf. also Ammonius, *On Aristotle's On Interpretation* 130.20–26. Frede 1985: 43; Frede 1970: 25.
- 11 Frede 1985: 43; Frede 1970: 25.
- 12 Frede's interpretation is shared by Talanga 1986: 144–145.
- 13 See, in particular, Frede 1985: 45n26.
- 14 Frede 1985: 44–45 is compelled to say that Boethius expresses himself in a misleading way, simply because his words do not fit her interpretation. Even worse Talanga 1986: 145 qualifies as “*besonders irreführend*” Boethius's speaking of a proposition as “*indefinite vera*” or “*indefinite falsa*.”
- 15 Rescher 1968: 186n5 puts Boethius and Ammonius among the supporters of the traditional interpretation. As far as Ammonius is concerned, he quotes *On Aristotle's On Interpretation* 154.35–155.5, and his interpretation is based on a wrong translation of the passage. With respect to Boethius, he refers to *On Aristotle's On Interpretation* 2a, 249.5–7, where the Latin commentator says: *manifestum est in futuris et contingentibus propositionibus non esse unam veram, alteram falsam*. But the context makes it clear that “*definite*” must be understood before “*veram*” and “*falsam*.” Cf. Rescher 1963: 43–44.
- 16 Boethius, *On Aristotle's On Interpretation* 2a, 208.1–11: *putaverunt autem quidam, quorum Stoici quoque sunt, Aristotelem dicere in futuro contingentes nec veras esse nec falsas. Quod enim dixit nihil se magis ad esse habere quam ad non esse, hoc putaverunt tamquam nihil eas interesset falsas an veras putari. Neque veras enim neque falsas esse arbitrati sunt, sed falso. Non enim hoc Aristoteles dicit, quod utraeque nec verae nec falsae sunt, sed quod una quidem ipsarum quaelibet aut vera aut falsa est, non tamen quemadmodum in praeteritis definite nec quemadmodum in praesentibus*.
- 17 Sorabji 1980: 93–94; Sharples 1978: 263–264. See also Łukasiewicz 1930: 51–77.
- 18 Boethius, *On Aristotle's On Interpretation* 2a, 208.11–18: *enuntiativarum vocum duplicem quodammodo esse naturam, quarum quaedam essent non modo in quibus verum et falsum inveniretur, sed in quibus una etiam esset definite vera, falsa altera definite, in aliis vero una quidem vera, altera falsa, sed indefinite et commutabiliter et hoc per suam naturam, non ad nostram ignorantiam atque notitiam*. The same point is made in Boethius, *On Aristotle's On Interpretation* 2a, 191.24–192.22.
- 19 Other examples of the same sort can be found in Boethius, *On Aristotle's On Interpretation* 2a, 188.7–8; 192.9–10, 17–18; 202.23–25; 207.21–22; 240. 10–11; 245.12–13; 1a, 116.9–10; 23–24.
- 20 Cf. Boethius, *On Aristotle's On Interpretation* 1a, 106.3–8.
- 21 I take this expression from Rescher and Urquhart 1971: 27.
- 22 On this distinction, see Prior 1967: 1–4.

- 23 A discussion of this question can be found in Sorabji 1980: 97–102.
- 24 Boethius, *On Aristotle's On Interpretation* 1a, 108.29. See also 1a, 106.23; 107.3–4; 123.181–19. As far as I can see, no example of this kind is mentioned in the *secunda editio*. If one looks at the whole of Boethius's examples, then Simo Knuuttila's assertion that "for Boethius the basic form of a singular declarative statement is temporally indefinite" cannot be admitted without qualification (Knuuttila 1981: 171).
- 25 Boethius, *On Aristotle's On Interpretation* 2a, 190.1–6: *contingens autem secundum Aristotelicam sententiam est, quodcumque aut casus fert aut ex libero cuiuslibet arbitrio et propria voluntate venit aut facilitate naturae in utramque partem redire possibile est, ut fiat scilicet et not fiat*. Cf. Boethius, *On Aristotle's On Interpretation* 1a, 106.8–14.
- 26 Boethius, *On Aristotle's On Interpretation* 2a, 188.30–189.19; 1a, 107.20–108.11.
- 27 Boethius, *On Aristotle's On Interpretation* 2a, 200.11–16: *contingentia autem sunt (ut supra iam diximus) quaecumque vel ad esse vel ad non esse aequaliter sese habent, et sicut ipsa indefinitum habent esse et non esse, ita quoque de his adfirmationes <et negationes> indefinitam habent veritatem vel falsitatem*.
- 28 Boethius, *On Aristotle's On Interpretation* 2a, 191.2–11: *perspicuum ergo in praesentibus atque praeteritis vel earundem rerum quae sunt contingentes definitum constitutumque esse eventum. In futuris autem unum quidem quodlibet duorum fieri posse, unum vero definitum non esse, sed in utramque partem vergere et aut hoc quidem aut illud ex necessitate evenire, ut autem hoc quodlibet definite vel quodlibet aliud definite, fieri non posse. quae enim contingentia sunt, in utraque parte contingunt*. Cf. also Boethius, *On Aristotle's On Interpretation* 1a, 112.22–113.7.
- 29 Boethius, *On Aristotle's On Interpretation* 2a, 246.14–19: *eventus autem ipsorum indiscretus est: et illud enim et illud poterit evenire. Hoc autem idcirco est quoniam non est ex antiquioribus quibusdam causis pendens rerum eventus, ut quaedam quodammodo necessitatis catena sit, sed potius haec ex nostro arbitrio et libera voluntate sunt, in quibus est nulla necessitas*.
- 30 For the notion of accessibility, see Hughes and Cresswell 1968: 75–80.
- 31 See, for instance, Boethius, *On Aristotle's On Interpretation* 1a, 110.16–18.
- 32 See, for instance, Boethius, *On Aristotle's On Interpretation* 2a, 199.1–14. In spite of the examples he makes, Boethius's conditional propositions include any kind of compound propositions. Cf. Boethius, *On Aristotle's On Interpretation* 2a, 186.13–22; *On the Hypothetical Syllogism*. 1, 1.5 and 1, 3.1–2 and Barnes 1985: 563.
- 33 Having defined what it is for an atomic proposition $p(k)$ to have the assigned truth value 1, we can construe the definition of "assignedly true (false) in a node S_i ," which we can represent by " $T+(A, S_i)$ " and " $F+(A, S_i)$." If A is an atomic formula, then $T+(A, S_i)$ iff $Assign(A, S_i) = 1$ and $F+(A, S_i)$ iff $Assign(A, S_i) = 0$. If A is $\neg B$, then $T+(A, S_i)$ iff $F+(B, S_i)$. If A is $B \wedge C$, then $T+(A, S_i)$ iff $T+(B, S_i)$ and $T+(C, S_i)$. The other propositional operators can be defined derivatively.
- 34 In principle "a sea battle will take place at k " might have an assigned truth value even before k if the conditions which unequivocally determine the happening or not-happening of this event are given at some time before k . But to avoid complications we do not consider this case, and we suppose that the determining conditions cannot be given before k .
- 35 By speaking of simple truth (falsity), I have been inspired by Georg Henrik von Wright, who uses the expression "plain truth" in a similar sense (Wright 1984: 5).
- 36 This view has recently been developed by Wright 1984: 6.
- 37 Remember that the *Assign*-function is monotonic with respect to truth. Therefore, $Assign(p(k), S_i) = 1$ implies $\forall S_j (Acc(S_i, S_j) \Rightarrow Assign(p(k), S_j) = 1)$. (It is not clear whether the concept of definite truth defined in (T**) fully corresponds to its informal introduction.)
- 38 One of a pair of future contingent propositions such as p and $\neg p$.

- 39 Boethius, *On Aristotle's On Interpretation* 2a, 203.13–20: *ergo in huiusmodi contingentibus si in futurum una semper vera est, altera semper falsa et una definite vera, falsa altera definite et res verbis congruent, omnia necesse est esse vel non esse et quidquid fit ex necessitate fit et nihil neque possibile est esse, quod possibile sit non esse, neque liberum erit arbitrium neque in rebus ullis casus erit in omnibus necessitate dominante.*
- 40 Boethius, *On Aristotle's On Interpretation* 2a, 204.8–12: *Aristoteles vero sumens istam hypotheticam propositionem, si omne quod in futuro dicitur aut verum definite aut falsum definite, omnia ex necessitate fieri et nihil casu nihil iudicio nihil possibilitate, ea convenienti ordine monstrat.* Cf. also Boethius, *On Aristotle's On Interpretation* 1a, 111.22–112.2.
- 41 To assume that the history of the world has a beginning has no relevance for the development of the argument, and it merely depends on taking (***) as our example.
- 42 Of course, in order to obtain this, the conditions for assigning a truth value to a proposition expressed by (AT) and (AF) should be modified.
- 43 Boethius, *On Aristotle's On Interpretation* 2a, 214.5–12: *quod si neque utraeque verae sunt neque utraeque falsae neque una definite vera, falsa altera definite, restat ut una quidem vera sit, altera falsa, non tamen definite, sed utrumlibet et instabili modo, ut hoc quidem aut hoc evenire necesse sit, ut tamen una res quaelibet quasi necessarie et definite proveniat aut non proveniat fieri non possit.* Cf. also Boethius, *On Aristotle's On Interpretation* 1a, 111.22–112.2.
- 44 As always, I refer to the case illustrated by (***) .
- 45 He says: *quamquam in tota contradictione una vera sit, altera falsa* (*On Aristotle's On Interpretation* 2a, 247.27–28).
- 46 However, it is interesting to point out that one of the best manuscripts has the reading “*incommutabiliter*,” which has been corrected into “*commutabiliter*.” In our interpretation, “*incommutabiliter*” might be defended by considering “*incommutabiliter verum (falsum)*” as a synonymous expression for “*semper verum (falsum)*.”
- 47 Boethius, *On Aristotle's On Interpretation* 1a, 124.30–125, 11: *sola enim sunt quae et esse et non esse contingit, quae non semper sunt et non semper non sunt. Si enim semper essent, status eorum mutari non posset atque ideo ex necessitate essent; si autem semper non essent, ea non necesse essent. Etenim sicut ipsa natura rerum evenientium est varia, ita quoque altera pars contradictionis habet variabilem veritatem. Et semper quidem vera vel falsa est, non tamen una definite, ut hoc verum sit determinate aut illud, sed utrumlibet, ut sicut status ipse rerum mutabilis est, ita quoque veritas aut falsitas propositionum dubitabilis sit.*
- 48 The “statistical interpretation” seems to be ruled out by other passages of the *Secunda editio*, as, for instance, *On Aristotle's On Interpretation* 2a, 237.19–23: *multaque nobis perspicua sunt ita sese habentia, ut in utraque parte eventus sine ullo alicuius rei impedimento vertatur, ut vestem quam possibile est quidem secari, sed fortasse ita contingit, ut non ante ferro dividatur, quam eam exerat vetustas.*
- 49 I have been led to formulate this hypothesis by reading Knuuttila 1981: 176–178.
- 50 This view has recently been defended in Wright 1984.
- 51 See Boethius, *On Aristotle's On Interpretation* 2a, 209.29–210.11; 1a, 113.12–114.4.
- 52 On this attitude toward the gods in late Neoplatonism, see Proclus, *Elements of Theology* 124 (110.10–13, Dodds); *Platonic Theology* 1.15 (69.10–12; 70.22–25; 74.9–16 Saffrey and Westerink; *Ten Doubts Concerning Providence* 2.6–23 Isaac; Ammonius, *On Aristotle's On Interpretation* 132.13 ff. (The Neoplatonic position is analyzed in Chapter 9.)
- 53 Following Meiser's suggestion, I read *praedictione* instead of *praedicatione* at 212.5.
- 54 Boethius, *On Aristotle's On Interpretation* 2a, 211.29–213.18: *nam si ex necessitate contingit id quod evenit, verum fuit dicere quoniam erit. Quod si ex necessitate non contingit, sed contingenter, non potius verum fuit dicere quoniam erit, sed magis*

quoniam contingit esse. Nam qui dicit erit, ille quandam necessitatem in ipsa praedictione ponit. Hoc inde intellegitur, quod si vere dicat futurum esse id quod praedicitur non possibile sit non fieri, hoc autem ex necessitate sit fieri. Ergo qui dicit, quoniam erit aliquid eorum quae contingenter eveniunt, in eo quod futurum esse dicit id quod contingenter evenit fortasse mentitur; vel si contingerit res illa quam praedicit, ille tamen mentitus est: non enim eventus falsus est, sed modus praedictionis. Namque ita oportuit dicere: cras bellum navale contingenter eveniet, hoc est dicere: ita evenit, si evenerit, ut potuerit non evenire. Qui ita dicit verum dicit, eventum enim contingenter praedixit. Qui autem ita inquit: cras bellum erit navale, quasi necesse sit, ita pronuntiat. Quod si evenerit, non iam idcirco quia praedixit verum dixerit, quoniam id quod contingenter eventurum erat necessarie futurum praedixit. Non ergo in eventu est falsitas, sed in praedictione modo. Quemadmodum enim si quis ambulante Socrate dicat: Socrates ex necessitate ambulat, ille mentitus est non in eo quod Socrates ambulat, sed in eo quod non ex necessitate ambulat, quod ille eum ex necessitate ambulare praedicavit, ita quoque in hoc qui dicit quoniam erit aliquid, etiam hoc si fiat, ille tamen falsus est, non in eo quod factum est, sed in eo quod non ita factum est, ut ille praedixit esse futurum, quod si verum esset definite, ex necessitate esset futurum [. . .]. Oportet enim in contingentibus ita aliquid praedicere, si vera erit enuntiatio, ut dicat quidem futurum esse aliquid, sed ita, ut rursus relinquat esse possibile, ut futurum non sit [. . .]. Quod si quis simpliciter id quod fortasse contingenter eveniet futurum esse praedixerit, ille rem contingentem necessarie futuram praedicit, atque ideo etiam si evenerit id quod dicitur, tamen ille mentitus est in eo quod hoc quidem contingenter evenit, ille autem ex necessitate futurum esse praedixerit.

- 55 Boethius, *On Aristotle's On Interpretation* 2a, 226.9–12: *novit enim futura deus non ut ex necessitate evenientia, sed ut contingenter, ita ut etiam aliud posse fieri non ignoret.*
- 56 That is, god's foreknowing.
- 57 Boethius, *On the Consolation of Philosophy* 5, 3.73–75: *aut quid hoc refert vaticinio illo ridiculo Tiresiae: Quidquid dicam, aut erit aut non?* Boethius refers here to Horace *Satires* 2.59.
- 58 Boethius, *On the Consolation of Philosophy* 5, 4.53–62: *quae igitur cum fiunt carent existendi necessitate, eadem prius quam fiant sine necessitate futura sunt. Quare sunt quaedam eventura quorum exitus ab omni necessitate sit absolutus. Nam illud quidem nullum arbitror esse dicturum, quod quae nunc fiunt, prius quam fierent, eventura non fuerint. Haec igitur etiam praecognita liberos habent eventus. Nam sicut scientia praesentium rerum nihil his quae fiunt, ita praescientia futurorum nihil his quae ventura sunt necessitatis importat.*
- 59 Boethius, *On Aristotle's Categories* 279 B–C.



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Part IV

Paradoxes



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12 The Stoic analysis of the Sorites

I

Diogenes Laertius attributes the discovery of the Sorites, along with many other famous paradoxes, to Eubulides.¹ It is neither our concern to decide whether this testimony is reliable nor to examine why such a concentration of difficult puzzles was created by a single man or by a small group of dialecticians. There are scholars who are convinced that the real inventor of the Sorites was Zeno of Elea with his argument of the millet seed,² whereas others think that his reasoning in this argument has little to do with the Sorites.³ Even wider is the range of opinions about the target of the Eubulidean paradoxes, and in particular of the Sorites. A widespread view is that Eubulides used the Sorites to attack Aristotle, a sport he was fond of, and in particular his doctrine of the mean.⁴ A piece of evidence for this position may be that both Aspasius and the anonymous commentator on the *Nicomachean Ethics* refer to the Sorites as an objection to Aristotle's position.⁵ But there are no clear hints that Aristotle discussed it, nor is there any clue that he felt the soritical argument as a difficulty for his own views.⁶ So even if Aristotle was aware of the existence of the Sorites, he does not seem to have paid a great deal of attention to it. This would be consistent with his general attitude toward paradoxes, which were classified by him as eristical arguments. They may be worth considering, but on the whole they are nothing more than puzzles that can be easily solved if one is properly trained in dialectics.⁷

II

The way of approaching paradoxes changes with the Stoics, and with Chrysippus in particular. If we examine the list of his writings preserved by Diogenes Laertius, we can immediately notice that a surprisingly high number of his works are dedicated to paradoxes. At least six of his titles mention the Liar paradox,⁸ and he is credited with three books: *On Soritical Arguments Concerning Words*,⁹ possibly a work on the application of the Sorites to grammatical contexts.¹⁰ In the logical section of the catalog, an *On the Argument from Little-by-Little* in two books is attributed to him,¹¹ and we know that ὁ παρὰ μικρὸν λόγος was another name for ὁ σωρίτης λόγος or simply ὁ σωρίτης.¹²

Chrysippus's interest in paradoxes can be explained not only by calling to mind the attention he paid to logical problems but also by the hypothesis that the fiercest adversaries of the Stoics, the Academics, used Megarian logic to attack the major tenets of the school. It is not too daring to suppose that Sextus's criticism of the distinction between apprehensive and nonapprehensive presentations in which soritical arguments are used reflects earlier discussions with the Academics. While presenting his criticism, Sextus quotes Chrysippus's answer to a soritical argument against the distinction.¹³ Therefore, we may suppose that Chrysippus was involved in studying the paradoxes because he needed to defend his philosophical positions.

III

As is easy to guess, “ὁ σωρίτης” comes from “σωρός,” “heap” or “pile,” and according to Galen the argument was called this way after the matter that first gave rise to this question: the heap.¹⁴ It is plausible that the original formulation of the Sorites is reflected by the following passage of Galen:

(A) Wherefore I say: tell me, do you think that a single grain of wheat is a heap? Thereupon you say: No. Then I say: What do you say about 2 grains? For it is my purpose to ask you questions in succession, and if you do not admit that 2 grains are a heap, then I shall ask you about 3 grains. Then I shall proceed to interrogate you further with respect to 4 grains, then 5 and 6 and 7 and 8, and you will assuredly say that none of these makes a heap. Also 9 and 10 and 11 grains are not a heap. For a conception of a heap which is formed in the soul and is conjured up in the imagination is that, beside being single particles in juxtaposition, it has quantity and mass of some considerable size. In my opinion, therefore, you would not be at fault were you to declare that 100 grains even would not be what men may call a heap, despite the fact that it has quantity which may be taken in [one's] hand. I appreciate your caution and foresight there in speaking thus, but I shall interrogate you further all the same: Would you allow 101 grains of wheat to be called a heap? I think you will again say: No. Then I would have you tell me: What do you say with regard to grains whose number has reached 102? I know that here, too, you will not affirm it. And again I would ask you: What do you think of grains whose number is now 103. You will say: No. And concerning the grains whose number has reached 104? And you will say: Not yet. And then I shall ask you: And the grains whose number amounts to 105? You will answer: Assuredly not. I, for my part, shall not cease from continuing to add one to the number in like manner, nor desist from asking you without ceasing if you admit that the quantity of each single one of these numbers constitutes a heap. I shall proceed to explain the cause of this. If you do not say with respect to any of the numbers, as in the case of the 100 grains of wheat for example, that it now constituted a heap, but afterward when a grain is added to it, you say that a heap has now been formed, consequently the quantity of corn became a heap by the addition of the single grain of wheat, and if the grain is taken

away the heap is eliminated. And I know of nothing worse and more absurd than that the being and not-being of a heap is determined by a grain of corn. And to prevent this absurdity from adhering to you, you will not cease from denying, and will never admit at any time that the sum of this is a heap, even if the number of grains of wheat reaches infinity by the constant and gradual addition of more. And by reason of this denial the heap is proven to be non-existent, because of this pretty sophism.

(Galen, *On Medical Experience* 17, 1–3.115–116; R. Walzer translation)

The ancient authors were well aware that the argument had a general form and could be applied to many things. For the same reason that we are forced to conclude that there are no heaps, we are driven to state that there is no wave, no flock of sheep, no herd of cattle, no open sea, no crowd.¹⁵ And for the same reason we must deny that there is boyhood, adolescence, manhood, or seasons.¹⁶ Horace, in one of his *Epistles*, applies the soritical argument in a playful way to the notion of an old writer. Is an old and respected writer one who was born a hundred years ago? And what happens with people born ninety-nine years ago? What follows is easy to guess.¹⁷

Galen gives us the form of the ascending Sorites: Even if we add millions of grains to a given collection of grains that is not a heap, we never reach a heap. The ancients also knew the descending form of the Sorites. The most famous version of it is called the φαλακός, the Bald Man. Consider a man with luxuriant hair; everybody will agree that he is not bald. Take a hair away. He is still not bald. By repeating the operation we reach a stage when our poor man has no hair at all and is not yet bald.¹⁸

IV

Given the variety of ways in which the Sorites were presented, we are entitled to look for the logical structure that is common to all of them. By isolating this form we will be able to determine which ingredients make the arguments paradoxical and what the possible solutions are.

The argument is surprisingly simple. Consider again the case of the heap and take a collection of grains, let us say of wheat, made up of a single grain. Call it a_1 . It is pretty clear that it is not a heap. To keep a general form and avoid complications, we omit the negative way in which the case of the heap is expressed and simply state

$$(1) \quad F(a_1).$$

This proposition is true by hypothesis. Consider now a second collection a_2 made up of two grains of wheat and a series of these collections such that each of them differs from the immediately preceding one only by having one grain more than the other. So we get a succession of individuals

$$(\dagger) \quad \langle a_1, a_2, a_3, \dots, a_n \rangle.$$

This succession does not need to be infinite. It is enough that it is sufficiently large to allow us to state that its last element is not F (or that it is F). Therefore, we assert

$(\dagger\dagger) \quad \neg F(a_n).$

This statement is true for the same reason that (1) is true, namely by hypothesis. For a sufficiently large n it is difficult to deny that a_n is a heap. Thirty million grains of wheat reasonably form a heap of wheat, and if you are uncertain let us increase n as much as you like. At the end we should get a heap, if there are heaps in the world.

To generate the paradox we need a second assumption – namely, we must suppose that, in general, if a_j (with $1 \leq j \leq n$) is F , then a_{j+1} is also F . For instance, if the collection of j grains of wheat is not a heap, neither is the collection of $j + 1$ grains. In other words, we assume that each pair of contiguous elements of succession (\dagger) is constituted by elements that cannot be distinguished as far as F is concerned. Call this assumption the *indiscriminability thesis*. We can express it by means of the schema:

$(IT) \quad \forall j(F(a_j) \rightarrow F(a_{j+1})).$

It is easy to see that, by using (1) and an appropriate number of instances of (IT), we reach the negation of $(\dagger\dagger)$. We can construct the argument shown in Figure 12.1.

I	(1)	$F(a_1)$	by hypothesis
	(2)	$F(a_1) \rightarrow F(a_2)$	by (IT)
	<hr/>		
	(3)	$F(a_2)$	by <i>Modus Ponens</i>
	(4)	$F(a_2) \rightarrow F(a_3)$	by (IT)
	<hr/>		
	(5)	$F(a_3)$	by <i>Modus Ponens</i>
	.		
	.		
	.		
	(p)	$F(a_{n-1})$	by <i>Modus Ponens</i>
	($p + 1$)	$F(a_{n-1}) \rightarrow F(a_n)$	by (IT)
	<hr/>		
	($p + 2$)	$F(a_n)$	by <i>Modus Ponens</i>

Figure 12.1

Thus, we have a contradiction because $(p + 2)$ is the negation of $(\dagger\dagger)$, which was accepted by hypothesis. The same pattern applies to the descending Sorites.

A couple of comments are in order. Let us call premise (1) the “*categorical premise*” or “*assumption*” of the argument, and premises such as (2), (4), ($p + 1$) the “*conditional premises*.” First comment: Premises such as (3), (5), and (p) are not proper assumptions of the argument because they are obtained by *Modus Ponens* and may be eliminated. Let us call them the “*intermediate premises*.” A more compact way of presenting the argument would be as shown in Figure 12.2.

I*

(1)	$F(a_1)$	by hypothesis
(2)	$F(a_1) \rightarrow F(a_2)$	by (IT)
(4)	$F(a_2) \rightarrow F(a_3)$	by (IT)
.		
.		
.		
(n)	$F(a_{n-1}) \rightarrow F(a_n)$	by (IT)
<hr/>		
(n + 1)	$F(a_n)$	

Figure 12.2

We consider the expanded version of the paradox only because it corresponds more closely to the iterative questions put forward in Galen’s presentation of it.

Second comment: (IT) does not work as an explicit premise of the argument. It has the function of ensuring that we can rely on the conditional premises, and in fact we need it to justify them. We cannot suppose that we are able to verify them by mere inspection if their number is quite large. In this respect, our reconstruction of the argument is very similar to the Galen text, where the idea that one is allowed to assert F of each element of (\dagger) is justified by the remark, applied to the special case of the heap, that one single grain of wheat added to a nonheap of wheat cannot constitute a heap. This is the same as our (IT) thesis.

I do not think that it is necessary to stress that neither the categorical nor the conditional premises of our argument are logical laws. Their truth, or at least plausibility, depends on the choice of the predicate F and of the individuals a_1, \dots, a_n . As we have seen, the individuals must be ordered and form a series. In the case of the heap, the series is immediately formed by the way in which the collections of grains of wheat are constituted, namely by addition (or subtraction) of one grain. I think that it is in this sense that the following statement of Galen must be interpreted:

(B) According to what is demanded by the *logos*, there must not be such a thing in the world as a heap of grain, a mass or satiety, neither a mountain, nor strong love, nor a row, nor strong wind, nor city, nor anything else which is known from its name and idea to have a measure of extent or multitude, such as the wave, the open sea, a flock of sheep and herd of cattle, the nation and the crowd.

(Galen, *On Medical Experience* 16, 1.114; R. Walzer translation)

What “is known from its name and idea to have a measure of extent or multitude” is the soritical predicate as applied to the ordered series of individuals a_1, \dots, a_n . These must be such that a measure can be applied to them, and this requires that they can be counted, that is, put in a one-to-one relation with an initial segment of the natural numbers. There is no reason to think that the soritical predicates themselves must represent quantitative notions,¹⁹ and our sources make it clear that purely qualitative concepts were also submitted to soritical treatment. Sextus, for instance, reports that Chrysippus had to defend the notion of apprehensive presentation from soritical attacks.²⁰ What is peculiar to soritical predicates is rather that they satisfy (IT). This means that a predicate F is soritical if any pair of contiguous individuals to which F applies cannot be distinguished with respect to F .²¹

One might object to our reconstruction of the structure of the Sorites by pointing out that we have introduced conditional premises, whereas in the Galen passage no such premises are used. In Galen we have a dialectical situation in which somebody is asked to assent to a series of questions such as: “Is one grain a heap?”, “Are two grains a heap?”, and so on, and no mention of the conditional premises is made. However, in the second part of the passage, by explaining why one must answer “No” to any question of the form “Are n grains a heap?” he says that if we have answered “No” to the question “Are $n-1$ grains a heap?” we must also answer “No” to the question “Are n grains a heap?” because no addition of one grain to a collection of n grains that is not a heap makes that collection a heap. This explanation makes it plausible to introduce the conditional premises, and by the same token it shows that (I) is nearer than (I*) to the ancient formulation of the Sorites. One is not asked to assent to the categorical and the conditional assumptions, which are not in question, because they are either true by hypothesis or based on (IT). One is asked to assent to what is derived step by step by *Modus Ponens* from the categorical and conditional assumptions.

It should be clear that (I) is not a single argument, but rather a succession of arguments. However, a passage of Diogenes Laertius suggests that the Stoics sometimes presented the Sorites in the form of a single argument. The version reported by Diogenes is as follows:

(C) It is not the case that two are few and three are not also; it is not the case that these are few and four are not also (and so on up to ten thousand); but two are few; therefore ten thousand are also.

(Diogenes Laertius 7.82)²²

We know that Chrysippus in some cases reformulated conditionals as negated conjunctions,²³ and this may mean that the Stoics took the conditionals of the Sorites as weak implications – more or less what we call “material implications.” If we put the negated conjunctions in the form of implications, we obtain (I*). It should be noted in passing that the type of conditional involved in the Sorites does not affect the argument. All we need to generate the paradox is a conditional that allows for the application of *Modus Ponens*. It is by applying this rule several times that we are able to arrive at the conclusion of (I).

V

Once we cast some light on the structure of the Sorites, we can easily see why it appears to be so compelling. When premise (1) and thesis (IT) are stated, the argument becomes straightforward, because it is simply based on *Modus Ponens*, which is one of the most elementary rules of deduction. The moral of this analysis is clear. From the point of view of logic in general, and Stoic logic in particular, there is nothing that suggests that argument (I) is formally invalid. But a contradiction does arise, so we must suppose that there is something wrong with (I). Therefore, because the inconsistency does not seem to depend on the logic of the argument, it must depend on the premises that are assumed. Premise (1) seems to be based on immediate observation and therefore can hardly be rejected. The intermediate premises depend on *Modus Ponens*, and they are as safe as their premises. Thus, we must look at one of the conditional premises. They are justified by (IT). Therefore, it seems that we must reject the validity of (IT). If (IT) is not valid, then we can state:

$$(ITN) \quad \exists j(F(a_j) \wedge \neg F(a_{j+1})).$$

This move allows us to reject at least one of the conditional premises of (I). So there is a grain of wheat the addition of which makes a collection that is not a heap into a heap, and there is a hair the subtraction of which makes a nonbald man bald. That would be plausible if we take “heap” to mean “a collection of n elements,” where n is a fixed number. Therefore, whereas a collection of $n-1$ grains is not a heap, a collection of n grains is a heap. But this seems to be the meaning we attribute neither to “heap” nor, in general, to soritical predicates. These predicates are intrinsically imprecise and vague, and so we may think that for this reason they cannot admit the sharp treatment imposed by logic. It is only when we make soritical predicates precise that we can avoid the paradox.

Does the Stoic analysis of the paradox develop along these lines? To answer this question we must consider the evidence at our disposal, which is, as we will see, rather disappointing. Apart from a very obscure passage in the *Logical Investigations* (*Logika Zetemata*) of Chrysippus, where the text is so corrupted that no good sense can be extracted from it,²⁴ we have three passages to consider, one from Cicero and two from Sextus. Let us start with Sextus. By comparing the Stoic attitude to paradoxes with the skeptical one he says:

(D) And if Chrysippus and his fellow dogmatists say that when the Sorites is being propounded one should, while the argument is proceeding, stop and suspend judgement to avoid falling into absurdity, how much appropriate it is for us Sceptics, when we suspect absurdity, to give no hasty approval of the premises asked but rather to suspend judgement about each until the completion of the whole series of requests concerning the argument.

(Sextus Empiricus, *Outlines of Pyrrhonism* 2.253)

It seems clear here that in the case of a Sorites, Chrysippus recommended refraining from assenting to at least some premises put forward by the adversary. The point is repeated by Sextus elsewhere when he says:

(E) For in the case of the Sorites, when the last apprehensive presentation lies next to the first non-apprehensive and it is hard to distinguish from it, Chrysippus and his school say that in the case of presentations where the difference is small in this way, the wise man will stop and fall quiet, but in cases where it strikes him as greater, he will assent to the one as being true.

(Sextus Empiricus, *Against the Professors* 8.416)

Once more the idea is advanced that the wise man, when faced with a Sorites, must keep silent without assenting to any part of the argument. However, here things are slightly more complicated than in the preceding passage, because the situation seems to be as follows. Suppose that, for the individuals a_1 - a_{30} , it is very clear that $F(a_1)$, $F(a_2)$, $F(a_3)$, and similarly that $\neg F(a_{30})$, $\neg F(a_{29})$, $\neg F(a_{28})$, where F is a soritical predicate. As the distance between the positive and negative cases decreases, the wise man, according to Chrysippus, must be careful and at the very end stop answering.

Let us conclude our review of the evidence at our disposal by quoting a passage from Cicero:

(F) “But Sorites are vicious.” Then destroy them, if you can, in order that they be not harmful for they will be unless you are careful. “We have already been careful,” he said, “for Chrysippus holds that when you are being questioned step by step, e.g., as to whether three are few or many, you should fall quiet a little while before you come to many (that is what they call ἡσυχάζειν).”

(Cicero, *Prior Academics* 93)

Again, the remedy suggested by Chrysippus seems to be that we must stop answering before we enter the dangerous area of the argument.

There is a standard interpretation of these texts that says that, after all, the Sorites is a *reductio ad absurdum*. If we consider deduction (I), for instance, we must admit that it ends in a conclusion that contradicts ($\dagger\dagger$). So we must say that at least one of its premises is false, and someone may add that at least one of the conditional premises is false. Because the wise man knows by logic that one of the premises is false, he must refuse to assent to that premise. But the predicates involved in the Sorites are to some extent vague, and he is not able to locate where the false premise lies. This is the background against which one is asked to understand Chrysippus's suggestion: The wise man should begin his analysis of the Sorites by answering in some of the clear cases and then refrain from engaging himself in the nonclear cases. If he does not stop early enough, he will be led to admit something false. To use the words that Cicero puts in the mouth of the Stoic discussing with Carneades, the wise man “like a clever charioteer, shall pull up <his> horses, before <he> gets to the end, and all the more so if the place where the horses are coming to is steep.”²⁵

However attractive this interpretation may be, it does not seem completely convincing. At the very least, it consists in attributing a denial of (IT) to Chrysippus. This means that there are no predicates that make adjacent pairs of individuals indistinguishable, and there is a magic grain of wheat that turns a collection of grains into a heap. To make this plausible, we must reform our notion of heap and make it precise by putting a limit on it, stating that we get a heap when we reach a collection of, say, 30,545 grains of wheat. But if this was the Chrysippean answer to the Sorites, why did he order his sage to keep silent after a few questions, rather than simply suggesting that he should use an exact definition for the notion of heap?

One might try to answer this objection by pointing out that the notion of heap is imprecise, not because it does not imply a limit, but because nobody knows where the limit is. By adding grains to a collection step by step, we reach a point where this collection turns from a nonheap into a heap, but nobody knows where this point lies. But can we really believe that a heap of wheat is an exact quantity of grains, the number of which nobody knows exactly? Moreover, why should we not perceive the threshold when we come across it? The notion of heap either has or does not have a boundary. If there is a boundary, as the rejection of (IT) seems to imply, it should be, at least in principle, graspable. To claim that there is a definite point at which a collection becomes a heap means that there is a precise line that separates the true conditional premises of (I) from the false ones. Why should the first false conditional premise be known as false by the Stoic wise man?

Let us try an approach that is more respectful of our intuitions about heaps. What we know of the Stoic answer to the Sorites is that the wise man at a certain point must refrain from answering soritical questions. But there is a further piece of information that emerges by considering the moment at which, according to Chrysippus, the wise man must keep silent. Cicero, in text (F), says that the wise man should stop answering after “a little while before you come to many.” But a few lines after commenting on the metaphor of the charioteer, he says:

(G) Thus, he <sc. Chrysippus> said, “I pull myself up in time, and I don’t go on answering your captious questions.” If you’ve got hold of something clear but won’t answer, you are acting arrogantly; if you haven’t, then not even you see through the matter. If that is because it is obscure, I agree; but you say that you don’t go as far as what is obscure – so you stop at cases that are clear. If you do that simply in order to be silent, you gain nothing; for what does it matter to the man who is after you whether he catches you silent or talking? But if up to 9, say, you answer without hesitation that they are few, and then stop at the tenth you are withholding assent from what is certain and perfectly plain and you don’t let me do that in cases that are obscure.

(Cicero, *Prior Academics* 94)

If we maintain that the Chrysippean position is faithfully reported here, we must say that Chrysippus believed that the wise man had to withhold his assent before things become obscure. With respect to (I) this means that the wise man must

refrain from assenting to some of the premises of (I) before they become obscure. Which are the premises to which he should stop assenting? Cicero's text says clearly that he must stop answering questions such as "Is three a few?" Therefore, he must refrain from assenting to one of the intermediate premises of (I).

Now the question is: On what does the obscurity of the intermediate premises depend? Well, my point is that it cannot depend on the preceding conditional premises. The reason why we accept that if two grains do not form a heap then three grains do not form one either is exactly the same as the reason we accept that if 2 million grains do not form a heap, then 2 million and one grains do not form one either. This is because, to make the point with Galen, "I know of nothing worse and more absurd than that the being and not-being of a heap is determined by a grain of corn." In this way, the intuitive notion of "heap" is preserved.

What explains the obscurity of the intermediate premises of (I) is the following simple fact. Although it is pretty clear that one grain is not a heap of wheat, and that, say, 10,000 grains are a heap, one might be doubtful about considering 7,000 grains a heap. In other words, what we may consider to be typical of soritical predicates is that they have a degree of certainty that varies in a continuous way. With respect to deduction (I), the situation I imagine is this. Suppose that one is asked to admit $F(a_1)$ and say that one grain of wheat is not a heap. The answer is, of course, that it is not a heap, and so $F(a_1)$ really is true – let us say 100 percent true. Then take premise (2), the first conditional premise, which is supposed to be true by the indiscriminability thesis. By *Modus Ponens* we immediately get (3), $F(a_2)$. Now $F(a_2)$ is also surely true, but perhaps a little less so than at $F(a_1)$ – let us say 99 percent true. Two grains of wheat are not a heap, but this is not as clearly so as before. By repeating the procedure a sufficient number of times we get propositions that are less and less true. Then Chrysippus's suggestion becomes clear: Stop the process before the dark precipice of less true propositions appears.

The idea is quite simple. Soritical predicates admit of degrees of truth in the sense that they are more or less true of the objects to which they apply. These degrees vary in a continuous way. What happens when we submit propositions formed by these predicates to the laws of logic is that the laws of logic hold, but locally, namely in the short distance. If we apply *Modus Ponens* in a chain like (I), we can safely do so only if we do not repeat the process too many times. That is all. What is sufficient is to stop in time. In time for what? In time for not deriving a false or a fairly obscure proposition from clearly true premises.

It is not difficult to give a precise semantical basis to the idea we are proposing by using the machinery of fuzzy sets. It is not necessary to go into the details here. For our purposes, it is sufficient to say that we should think that a conditional proposition must be rejected not only when its antecedent is true and its consequent is false but also when its antecedent is much more clearly true than the consequent. If the difference in the degree of truth is just one step, we can accept the conditional, but when the difference increases, we must abandon it. This helps us understand our intuition that the conditional premises of (I) are safe and cannot be rejected. When the degree of truth of the antecedent of a conditional is, say, just one step truer than the degree of the consequent, the conditional will be acceptable and its

degree of truth will be equal to the degree of the consequent. What we have said with respect to conditionals can be applied to inferences. If the conclusion of the inference is much less true than the truest of the premises, then the inference must be abandoned. This allows us to reject (I).

This way of solving the Sorites paradox has a price. We must give up the idea that *Modus Ponens* preserves truth in the long run. Its repeated application in a deductive chain may create problems when soritical predicates are involved. So the Sorites limits the omnipotence dream of logicians, and they are compelled to acknowledge areas where logical laws cannot be applied without restrictions. Although we do not know how Chrysippus faced this problem, his general attitude toward paradoxes clearly shows that he had abandoned the simplistic view that solving a paradox just means finding the mistake concealed by linguistic ambiguities or other tricks of this sort. More realistically, to some extent he took the view that an answer to the main paradoxes implies that we must give up some of our commonsense beliefs, such as our faith in *Modus Ponens*. From this point of view, his perspective is modern. It is not without sacrifices that we free ourselves from paradoxes.

Notes

- 1 Diogenes Laertius 2.108.
- 2 Simplicius, *On Aristotle's Physics* 1108.18–28.
- 3 For the former view see Döring 1972: 111, and for the latter Sedley 1977: 89.
- 4 See Moline 1969: 398.
- 5 Aspasius, *On Aristotle's Nicomachean Ethics* 56.25–57.7; Anonymous, *On Aristotle's Nicomachean Ethics* 140.6–12.
- 6 The only passages where an implicit reference to the Sorites may be made are *Sophistical Refutations* 24, 179b34–37 and *Physics* 8.3, 253a14–26. But *Sophistical Refutations* 24, 179b34–37 refers to an argument presented at 179a35 that has nothing to do with the Sorites, and the argument in the passage from the *Physics* is based on a principle of proportion, which is not soritical either.
- 7 Aristotle, *Topics* 1.1, 100b26–101a1.
- 8 Diogenes Laertius 7.196–197.
- 9 Diogenes Laertius 7.192.
- 10 See Barnes 1982: 41n48. Sextus Empiricus, *Against the Professors* 1.168–169 uses a soritical argument against Dionysius Trax's definition of grammar.
- 11 Diogenes Laertius 7.197.
- 12 Galen, *On Medical Experience* 16, 2.115; Simplicius, *On Aristotle's Physics* 1177.2–4.
- 13 Sextus Empiricus, *Against the Professors* 8.416–417.
- 14 Galen, *On Medical Experience* 16, 2.115.
- 15 Galen, *On Medical Experience* 17, 1.114.
- 16 Galen, *On Medical Experience* 17, 1.114–115.
- 17 Horace, *Epistles* 2.1, 34–49.
- 18 Galen, *On Medical Experience* 20, 3.124–125.
- 19 Pace Burnyeat 1982: 318–320, who considers Galen's way of characterizing soritical predicates not sufficiently general, because it would be limited only to quantitative terms. Modern analysis would show that nonquantitative terms, for instance, “red,” can also generate soritical arguments. But to form a soritical argument with respect to the predicate “red,” we must distinguish a multiplicity of red patches. It is true that these patches are different from one another not in virtue of a quantitative difference.

However, this multiplicity of red patches must be in some way connected with a quantitative element, at least in the sense that the multiplicity must be ordered, because it has to be put in a one-to-one relation with a segment of the natural numbers. Therefore, the ancient notion of soritical predicate does not differ greatly from ours.

20 Sextus Empiricus, *Against the Professors* 7.416–417.

21 Barnes 1982: 31–32.

22 I follow Egli 1967: 8 and 55 in reading μυρίων and μύρια instead of δέκα and δέκα.

23 Cicero, *On Fate* 8.15.

24 As preserved in *PHerc.* 307, 9.17–22. There is a discussion of this passage in Barnes 1982: 9n68. (For more on *PHerc.* 307, see Chapter 13, endnote 6.)

25 Cicero, *Prior Academics* 94.

13 The Liar paradox and the Stoics

I

The history of the ancient versions of the Liar paradox is easily traced. We know that it was probably invented by Eubulides,¹ and a version of it may have been known by Aristotle.² A development of Aristotle's view can be found in the anonymous commentary on the *Sophistical Refutations* traditionally ascribed to Alexander of Aphrodisias.³ But it is surely Chrysippus who dedicated the most effort in ancient times to solve this paradox. He devoted an astonishing number of works to the Liar paradox, as the catalog of his books preserved by Diogenes Laertius shows. At least seven titles mention the Liar paradox,⁴ and we may think that some of the others were dealing with the same problem.⁵ Additionally, Chrysippus's *Logical Investigations* contain a few lines dedicated to the Liar paradox.⁶ Unfortunately, the text is so corrupted and obscure that it is difficult to extract a sense from it.

Despite the attention that ancient authors paid to the Liar paradox, we do not know in detail, at least as far as the Stoics are concerned, the solution they offered to the paradox, nor, moreover, the exact formulation on which they debated. With this in mind, it is not surprising that some commonplaces are widespread among scholars and often repeated in the literature, which to some extent depend on Rüstow's book, *Der Lügner*, dating back to the beginning of the twentieth century.⁷

The first commonplace concerns the formulation of the Liar paradox. It is quite usual to read in textbooks of logic that the ancients were unable to express the paradox in a form as strict and inescapable as the modern (and medieval) one. Besides, the solution that is normally ascribed to Chrysippus is the one that medieval logicians ascribed to the *cassantes* – that is, to people who thought that the main proposition of the Liar paradox is nonsensical because it yields a contradiction. I will try to challenge both views by claiming that at least one ancient version of the Liar paradox is truly paradoxical and that Chrysippus can hardly be counted among the *cassantes*.

II

Let us start with the modern formulation of the Liar paradox. Suppose that I say:

- (1) what I said yesterday is false.

To assign a truth value to this proposition, the obvious thing to do is to consider the propositions I uttered yesterday and see whether they are true or false. If they are all false, then (1) is true; otherwise, it is false. Suppose now that I say:

(2) what I am saying is false.

Again, to assign a truth value to (2) one has to consider what I am saying. In this case, however, the expression “what I am saying” can be taken in different ways. It may be that it simply means “what I have just said.” In this case, we have the same situation as before. If what I have just said is false, then (2) is true; otherwise, it is false. But suppose that “what I am saying” is taken to include not only what I have just said but also (2) itself. Here we have two possibilities. If at least one of the propositions other than (2) is true, then we can easily conclude that (2) is false. But what happens if all propositions different from (2) are false? If all propositions different from (2) are false, one might be tempted to say that (2) is true. If (2) is true, then it is false, because not all propositions to which (2) refers are false. But if (2) and all other propositions to which (2) refers are false, then (2) must be true. In conclusion, if (2) is true, it is false, and if it is false, it is true – a contradiction. The same contradiction can be more easily produced if we imagine that (2) is the only proposition I utter. In this case the proposition is immediately self-referential and a contradiction is generated without needing to consider the other possible propositions uttered by me. This simplified version of the paradox appears in the earliest medieval formulation of it: by Adam of Balsham in his *Ars disserendi* dated to 1132:

(A) whether he says the truth who says nothing but that he is speaking falsely.⁸

This simplified version, however, is not found in the extant ancient texts.

III

Let us now consider the ancient formulations of the Liar paradox. If we leave aside a rather obscure and complicated passage of Aristotle’s *Sophistical Refutations* in which it is not clear to what extent Aristotle refers to the Liar paradox,⁹ then the oldest formulation of the paradox is probably the one found in Cicero’s *Academica*. The passage is as follows:

(B) Clearly it is a fundamental principle of logic that what is pronounced (this is what they call “proposition” (ἀξιῶμα), that is “*effatum*”) is either true or false. Then, are the following propositions true or false: “If you say you are speaking falsely and you tell the truth about it, you are speaking falsely [. . .] you tell the truth?” You claim that these propositions cannot be explained.¹⁰

The personal pronoun “you” at the end of the translation addresses the Stoics, and we may think that the formulation of the Liar paradox refers directly to the Chrysippean one. Before considering the evident lacuna of the transmitted text and the possible ways to fill the gap, let us pause to examine our translation of the verb

mentiri. It is normally rendered as “to lie,” which is, of course, a possible translation. But one might also render the Latin as “speaking falsely” – as the Greek ψεύδεσθαι, which is behind the Latin, allows. The advantage of this translation is that we avoid all the problems connected with the psychological act of lying. We may lie when we say something that is the opposite of what we believe, and what we say by lying may be true if our beliefs are false. By taking *mentiri* as “speaking falsely,” we are faced with the simpler situation of someone who utters false propositions, and this is a necessary condition for building up the paradox.

Let us return to the lacuna between “you are speaking falsely” (*mentiris*) and “you tell the truth” (*verum dicis*). The gap has been filled up in various ways by the editors. On the basis of an old correction of the Codex Vossianus according to which “*et*” is put between “*mentiris*” and “*verum dicis*,” the proposal has been made to delete “*idque verum dicis*” so that the text would be:

(I) *si te mentiri dicis, mentiris <et> verum dicis.*¹¹

But it is strange to fill a gap by deleting an expression found in all the manuscripts. There is no paleographical reason to erase “*idque verum dicis*,” and we must, moreover, be suspicious about the “*et*.”

A much more common way of solving the problem is by inserting “*an*” in the lacuna between “*mentiris*” and “*verum dicis*.”¹² So the text would be:

(II) *si te mentiri dicis idque verum dicis, mentiris <an> verum dicis?*

The scholars who recommend (II) do not refrain themselves from quoting Gellius’s presentation of the Liar paradox.¹³ But, once again, it is hard to believe that one might ask with respect to (II) whether *haec* (in my view, the propositions expressed in the text) are true or false. A question is neither true nor false. It is probably with this objection in mind that Plasberg, followed by Hülser,¹⁴ has proposed the following integration of the passage:

(III) *si te mentiri dicis idque verum dicis, mentiris <et, si te mentiri dicis, idque mentiris> verum dicis.*

Here the intriguing *haec* finds an obvious explanation because it refers to two different propositions that can be represented by means of

(a) if you say that you are speaking falsely and you tell the truth about it, then you are speaking falsely

and

(b) if you say that you are speaking falsely and you speak falsely about it, then you are telling the truth.

Of these propositions, it is asked whether they are true or false.

What is perplexing in this reconstruction is that, if we stick to (a) and (b), no real logical paradox arises. To see the point, let us give a name to the “you” of which Cicero is speaking and call her “Calpurnia.” Suppose that Calpurnia says in her perfect Latin:

(c) *mentior*.

If this proposition refers to other propositions not including (c) itself, no difficulty arises, as we have seen. Therefore, we must imagine that Calpurnia, by saying “*mentior*,” means that (c) is also included among the propositions about which she speaks falsely. On the other hand, we have no hint that (c) is the only proposition uttered by Calpurnia. Therefore, the most natural way of interpreting (c) is by thinking that Calpurnia is referring to a set of propositions among which there is also (c) itself. As we have seen, to generate a real paradox in this situation one must state that all the propositions different from (c) that fall under the scope of (c) are false. Now this condition is not stated if the Plasberg–Hülser text is adopted. Therefore, although conditional (a) must be accepted, there is no reason to accept conditional (b), which is simply false if the hypothesis is made that one of the propositions other than (c) is true. In other words, suppose that Calpurnia utters (c) and

(d) $2 + 2 = 4$.

Then the antecedent of (b) is true but its consequent is false because it is not true that all statements uttered by Calpurnia are false. Therefore, no paradox arises from (c), and (c) itself is simply false. It is difficult to believe that this is the formulation of the Liar paradox that Chrysippus had in mind when he wrote his numerous treatises on this subject. The paradox would be solved too easily.

But if we fill the gap in Cicero’s text differently, a strong paradox arises. A new possibility is offered by the following reading:

(IV) *si te mentiri dicis idque verum dicis, mentiris <et, si mentiris,> verum dicis*.

From a paleographical point of view, the corruption can be explained very easily. So (IV) is the conjunction of two conditionals, the former of which is (a), and the latter is

(e) if you are speaking falsely, you tell the truth.

Suppose as before that in uttering (c) Calpurnia refers to whatever she is saying – namely to a set of propositions that include but is not limited to (c). It is easy to become convinced that (IV) represents an adequately paradoxical formulation of the Liar paradox. As the antecedent of (a) states, Calpurnia says that whatever

she is saying is false and that by saying it she tells the truth. Therefore, (c), being something she has said, is false. But if all her utterances are false propositions and she speaks falsely by stating (c), she tells the truth about it, and consequently (c) is true. A contradiction is generated.

The difference between (III) and (IV) is that in (III), (a) and (b) are taken as independent propositions, whereas in (IV) the antecedent of (e) is the same as the consequent of (a). This allows us to say that the claim that (c) is false, as is implied in the antecedent of (e), is made under the condition that Calpurnia is saying that she is lying and that she is telling the truth about it. This implies that every statement other than (c) that Calpurnia may have made and (c) itself are false. Therefore, (c) is true. The reasoning becomes really paradoxical.

An important part of our interpretation of Cicero's formulation of the Liar paradox is played by the clause

(f) *idque verum dicis*

which appears both in (III) and (IV) and which we have translated as

(f₁) and you tell the truth about it.

A more direct rendering of (f) might be:

(f₂) and you say that this is true,

which may be taken to mean:

(f₃) and you say that (c) is true.

At this point it should be clear that (f₃) could not guarantee that a contradiction arises. If (f) is taken to mean (f₃), it cannot play the role that, as we have seen, (f) must play in order to get a real paradox, because (f₃) does not guarantee that all propositions to which (c) refers are false. This means that the contradiction is generated if we suppose that Calpurnia's statement (c) refers only to (c) and to nothing else. But without any explicit statement, we cannot make such an assumption. Therefore, it is safer to interpret (f) as (f₁).

This way of interpreting (f) is confirmed by other formulations of the Liar paradox found in other ancient authors. This is Gellius's way of putting it:

(C) Then it was inquired what was the solution of this sophistry: When I am speaking falsely and I say that I am speaking falsely, am I speaking falsely or am I telling the truth?¹⁵

What is interesting here is that the paradox is formulated by stating, first, the hypothesis that I am really speaking falsely (*mentior*) and, second, by then

immediately giving the paradoxical proposition (*dico me mentiri*). In other words, Gellius states two clauses:

(g) I am really speaking falsely (*mentior*)

and

(h) I say that I am speaking falsely (*dico me mentiri*).

These two clauses are also present in other versions of the Liar paradox.¹⁶ The obvious way of interpreting the relation between (g) and (h) is to suppose that (g) puts a condition on (c). Then (g) and (h) can be combined into

(j) I say (c) under the assumption that I am really speaking falsely.

Why is the assumption that I am really speaking falsely made explicit? The reason, I believe, depends on the implicit interpretation of (c). If (c) had to be interpreted as an immediately self-referring proposition, there would be no need to add the condition (g) to get the paradox. On the other hand, by taking (c) as including other propositions other than (c), condition (g) is required for the construction of the paradox.¹⁷ Therefore, Gellius's formulation of the Liar paradox confirms our interpretation of the Cicero passage and the filling of the lacuna as proposed in (IV).

After this discussion, one might be tempted to propose an even more conservative reading of the *Academica* text by simply stating

(V) *si te mentiri dicis idque verum dicis, mentiris <et> verum dicis*.

Once more, if we give to (f) the meaning of (f₁), we obtain a real paradox. If Calpurnia utters (c) and what she is saying is the case, in the sense that it refers to a set of false propositions among which there is (c) itself, then she is really speaking falsely because all propositions to which (c) refers are false and (c) is one of them. But, by the same token, she tells the truth because she is really speaking falsely, it being the case that all propositions to which (c) refers and (c) itself are false. Because I am interested in the Liar paradox and not in reconstructing the text of the *Academica*, I can take (IV) and (V) as equivalent for my point, leaving to philologists the choice between the two possibilities.

IV

Once we have explained the way in which the Stoics, and probably Chrysippus, formulated the Liar paradox as a real paradox, we must examine what kind of solution was proposed for it. But did Chrysippus have a solution for the Liar paradox? To judge from the number of works he devoted to the subject, one has the impression that he at least attempted to give an answer to this puzzle. This

impression is confirmed by a passage in Plutarch in which the latter claims that Chrysippus's solution to the Liar paradox overtly contradicted some usual views about logic. Plutarch says:

(D) But, as Cato said that save for the famous Caesar no one while sober and of sound mind had entered upon public affairs for the purpose of ruining the commonwealth, so it seems to me that this man <sc. Chrysippus> exerts the utmost diligence and cleverness in subverting and overthrowing common experience. So, on occasion anyway, even the man's devotees themselves testify when they quarrel with him about the Liar. For, my dear, what kind of motion of demonstration or what preconception of belief is not subverted by denying that a conjunction formed of contradictories is patently false and again by asserting on the contrary that some arguments the premises of which are true and the inferences of which are correct still have the contradictories of their conclusions true as well? The octopus is said to gnaw off its own tentacles in wintertime; but the dialectic of Chrysippus docks and destroys its own most important parts, its very principles, and what conception among the rest has it then left free of suspicion.

(Plutarch, *On Common Notions*
1059 D–E = *FDS* 1213 = *SVF* II 250)¹⁸

In this passage two points are apparently made against Chrysippus:

- (i) He would have refused to qualify as false the conjunction of a contradictory pair – namely admitted that there is at least a contradiction (the conjunction of a proposition with its denial) that is not false.
- (ii) He would have admitted arguments with true premises, which are sound from a logical point of view and nevertheless are such that the negation of their conclusion is true.

I guess that the first Plutarchean criticism may be expanded as follows. The Liar paradox shows that its main proposition, namely

(c*) I am speaking falsely

is or, better, involves a contradiction, because it turns out that it is at the same time true and false. However, Chrysippus denied that this proposition is false, and in this way he would have accepted a contradiction that is not false.

The second criticism can be reconstructed along the same pattern. There is a sound argument by which one can show that (c*) is false. This means that we have a set of premises A_1, \dots, A_n , from which the negation of (c*) is correctly derived. Because these premises are supposed to be true, we can deduce that the negation of (c*) is true, and therefore that (c*) is false. But Chrysippus would have maintained that (c*) is not false. Thus, Plutarch seems to imply that Chrysippus held that (c*) is true. In this way, he would have subverted the notion of sound

inference by admitting correct inferences in which the premises are true together with the negation of their conclusion.¹⁹

I am not sure that this is the point to which Plutarch alludes in his compressed criticism, because his objection can easily be met and his point appears on the whole quite naïve. It is difficult to believe that Chrysippus would have admitted that the premises from which the truth and falsity of (c*) is derived are true. His point might easily be taken as a kind of *reductio ad impossibile* of the hypothesis that a truth value can be assigned to (c*). In other words, from the fact that (c*) is not false, we are not allowed to derive that (c*) is true. But without this implication the whole argument presented by Plutarch collapses.

However, Plutarch's testimony is important because it shows that Chrysippus, probably not without contrasts with other members of his school, maintained that the Liar paradox's main proposition, our (c*), cannot be qualified as false. By considering this claim in light of the subsequent discussion of the Liar paradox in the Middle Ages, we may appreciate its philosophical relevance. Chrysippus cannot be counted among the forerunners of the so-called *restringentes* – namely the people who believed (c*) to be a false proposition on the basis that it yields a contradiction. If it is taken as a point in favor of the *restringentes*, Plutarch's criticism, although clumsily construed, has a philosophical respectability.

Shall we count Chrysippus among the *cassantes* – namely the people who believed that the Liar paradox's critical statement is not a proposition because it is meaningless? In favor of this view, one might argue as follows. If Chrysippus denied that (c*) is false, one can hardly believe that he thought that (c*) is true. Therefore, he should have admitted that (c*) is neither true nor false so that it is not a proposition.

This interpretation finds an indirect confirmation in Alexander of Aphrodisias. Commenting on an Aristotelian locus whereby attributing to a subject a predicate the conclusion can be drawn that the subject has contrary predicates,²⁰ he explicitly reports that it may be used to prove that (c*) is not a proposition. His point is as follows:

(E) By using this *locus* it is possible to reject the view that “I am speaking falsely” is a proposition. For, if it is taken as a proposition, it will be at the same time true and false, as is proven. But this is impossible, since these predicates are contrary. Therefore, “I am speaking falsely” is not a proposition. For if one had admitted that it is a proposition, the principle would be rejected that every proposition is either true or false. For if this principle holds, also that proposition would be true or false. But whatever truth value may be attributed to it, it will have the opposite value. For if “I am speaking falsely” is supposed to be true, it appears to follow that the same person is speaking falsely. For he is telling the truth by saying that he is speaking falsely. But again telling the truth is false. For the same reason he is speaking falsely again by saying that he is speaking falsely, because he is telling the truth.

(Alexander of Aphrodisias, *On Aristotle's Topics* 188.19–28 = *FDS* 1183)

It is the first part of this passage that interests us. Here a view is stated according to which the statement of the Liar paradox – namely (c*) – is said not to be a proposition. This claim is supported by a kind of *reductio ad impossibile*. If (c*) is a proposition, then we can prove that it is both true and false. This is against the Principle of Bivalence.²¹ Therefore, we must give up the hypothesis that (c*) has a truth value and conclude that it is not a proposition. In fact, if one assumes that (c*) is a proposition, then a contradiction follows – namely, that (c*) is both true and false. Therefore, (c*) is not a proposition. Unfortunately, Alexander does not tell us who actually used this sort of *reductio*. It is clear that he is not referring to the Peripatetic standard view about the Liar. The Peripatetics tried to solve the paradox by claiming that (c*) is true and false in different senses. This was the way in which Aristotle's possible allusion to the Liar paradox in the *Sophistical Refutations* 25 was interpreted.²² May we say that he is hinting at the Stoic view?

To answer this question we must return to Cicero's *Academica*. After having formulated the Liar paradox, Cicero continues by stating the view of his adversaries, the Stoics. According to them, a proposition such as (c*) must be counted among the *inexplicabilia* – namely the ἄπορα, the usual Greek qualification of paradoxes. What is interesting is what Cicero believes to be the aporetic aspect of the Liar paradox according to the Stoics. He says:

(F) If propositions of this sort cannot be explained and for them no criterion can be found according to which you can answer the question of whether they are true or false, what happens with the definition of a proposition as that which is true or false.

(Aristotle, *Sophistical Refutations* 25, 180b2–7)²³

Three preliminary remarks are in order here. First, I take “*ista* [propositions of this sort]” to refer to sentences like (c*), the critical statement of the Liar paradox. Second, “*iudicium* [criterion]” is the standard translation for κριτήριον in the Latin philosophical literature.²⁴ Third, the expression “no criterion can be found according to which you can answer the question of whether they are true or false [*nec eorum ullum iudicium invenitur, ut respondere possitis verane an falsa sint*],” taken by itself, looks ambiguous, and it may be interpreted in two ways. The first interpretation, which can be called *epistemic*, is as follows: (c*) has a truth value, but we are unable to determine it. In other words, (c*) is in itself true or false, but we do not know which one of the two truth values must be attributed to it. The second interpretation is *ontological* and it consists in denying that (c*) has a truth value, because there is no criterion for assigning such a value to it in the sense that its truth conditions do not allow such an assignment. A little reflection shows that we must adopt the ontological interpretation; otherwise, we may not be able to understand why it is said to be inconsistent with the definition of proposition. Actually, an undecidable proposition does not necessarily fail to satisfy the definition of proposition, because it may be taken as a proposition that is either true or false, although it is not known which value it has.²⁵

What follows passage (F) confirms our interpretation. Cicero compares the Stoic view about paradoxes to the Epicurean claim that propositions about future contingent events are neither true nor false. He says:

(G) But the farthest length they go is to demand that these unexplainables (*haec inexplicabilia*) should be taken as exceptions. My advice to them is to apply to some tribune. They will never get that saving clause from me. For they will not get Epicurus, who despises and laughs at the whole of dialectic, to admit what we could express as follows: “Hermarcus tomorrow either will be alive or will not be alive,” since dialecticians state that every disjunction of the form “either so or not so” is not only true, but also necessary [. . .]. Therefore let the dialecticians, that is, Antiochus and the Stoics, do battle with him, since he overthrows the whole of dialectic. For if a disjunction formed by contrary members (I say that two propositions are contrary if one is the negation of the other) can be false, none is true.

(Cicero, *Prior Academics* 97 = *FDS* 1212 = *SVF* II 219)

The comparison with Epicurus leads us to think that aporetic propositions are said by Cicero to be different from other propositions because they do not fall under the Principle of Bivalence, and this means that they lack a truth value. The case of Epicurus is put forward in order to stress that the Stoics have an incoherent behavior. On the one hand, they attack Epicurus for having allowed exceptions to the Principle of Bivalence, and, on the other hand, they are themselves adopting the view that there are propositions such as the Liar paradox that are not governed by this law of logic. The point is clear. Epicurus claimed that the Principle of Bivalence admits exceptions, because it does not hold for future contingent propositions. The Stoics, while criticizing Epicurus, allowed the same for another kind of propositions. Therefore, a proposition such as (c*) has no truth value, and it is the ontological interpretation of text (F) that must be endorsed.

According to this interpretation, Cicero’s text contains an important piece of information, because it implies that (c*) can be said to be neither false, as Plutarch reports, nor true. At the same time, the comparison with Epicurus shows that the view attributed by Cicero to the Stoics is different from the position described by Alexander in text (E). In the latter passage, the claim was made that (c*) is not a proposition, because no proposition can be both true and false. The argument presupposes the general validity of the Principle of Bivalence, and (c*) does not constitute an exception to it because it is not a proposition. In the view outlined by Cicero, (c*) is a proposition although it lacks a truth value, and therefore the general definition of proposition and the Principle of Bivalence are jeopardized.

V

Let us return to our main question: Is Chrysippus a forerunner of the medieval *cassantes*? To solve this problem it is essential to decide whether the critical statement of the Liar paradox, our (c*), must be taken as a proposition or not, because

it is typical of the solution proposed by the *cassantes* to hold that (c*) is not a proposition because it is meaningless.²⁶

So our question becomes: According to Chrysippus, was (c*) a proposition or not? The point depends on whom we believe. If we take Alexander as reporting Chrysippus's view, then Chrysippus may be thought to be on the side of the *cassantes*, at least in the sense that the Liar paradox was solved by him by claiming that (c*) is not a proposition. But if we adopt this interpretation, we must distrust Cicero and think that his criticism of the Stoics is completely out of place because it is based on a total misunderstanding. If Chrysippus had really held that (c*) was not a proposition, how could Cicero have charged him with having proposed a solution of the Liar paradox that is inconsistent with his definition of proposition? Cicero's main point is that (c*), according to Chrysippus, is neither true nor false and, nevertheless, it is a proposition, although in the Stoic view every proposition is either true or false by definition.

If we are compelled to choose between the two testimonies, I think that we must side with Cicero for one simple reason. Cicero clearly claims that what he is saying is directed toward Chrysippus, whereas we have no hint that the view referred to by Alexander is the Stoic view. It is difficult to reply to this observation by saying that Alexander's evidence is consistent with the general Stoic position, whereas the view attributed to Chrysippus by Cicero makes his solution of the Liar paradox inconsistent with other tenets of his logic because *contra factum non valet argumentum*. But in the Chrysippean approach to the Liar paradox there is an important philosophical insight that is overlooked in the *cassantes*'s solution so that their position becomes very weak.²⁷ What makes a proposition such as (c*) paradoxical is not its own form or intrinsic structure, but the truth conditions that allow it to be evaluated. Because truth conditions may change according to the history of the external world, it may happen that one and the same proposition is evaluated as false or paradoxical according to different external situations. Therefore, there is no point in considering (c*) meaningless, because in many circumstances it has not only a clear-cut meaning but also a definite truth value. But sometimes the facts are such that it is impossible to assign a truth value to (c*), and this depends on the empirical situation to which the proposition refers, not on its meaning, which is not affected by the history of the world.

Of course, this approach to the Liar paradox would imply a revision of the notion of proposition, and Cicero is right in pointing to the inconsistency between the standard Stoic definition of proposition and the Chrysippean view about the Liar paradox. It may be that Chrysippus was aware of the problem and that he had an answer to it, even if we do not know it. At any rate, what his perspective shows is that he took the Liar paradox, and paradoxes in general, very seriously. Ancient philosophers can be split into two categories with respect to their attitude to paradoxes. The majority of them thought that they were worth considering in order to provide cultivated men with a way of replying to the attacks of sophists and dialecticians. A skilled logician can easily find a solution to these puzzling arguments, which are *ἄπορα* or *inexplicabilia* only for people who are not trained in logic. But there was also a minority of philosophers who thought that the paradoxes

could not easily be dismissed by detecting the obvious ambiguity that lies at their bottom. They took them seriously, being aware that they show that some of our basic ideas about truth are obscure and that they cannot be solved without being prepared to give up some of our received views and tenets. I am inclined to think that Chrysippus belonged to this minority.

Notes

- 1 Diogenes Laertius 2.108 (= *DM* 50; 51A; 64).
- 2 Aristotle, *Sophistical Refutations* 25, 180b2–7.
- 3 Ps-Alexander of Aphrodisias, *On Aristotle's Sophistical Refutations* 171.16 ff (= *FDS* 1210).
- 4 Diogenes Laertius 7.196 (= *FDS* 194; = *SVF* II 15): (i) Περὶ τῆς εἰς τὸν ψευδόμενον εἰσαγωγῆς πρὸς Ἀριστοκρέοντα α'; (ii) Λόγοι ψευδόμενοι πρὸς εἰσαγωγὴν α'; (iii) Περὶ τοῦ ψευδομένου πρὸς Ἀριστοκρέοντα ζ'; (iv) Πρὸς τοὺς νομίζοντας καὶ ψευδὴ καὶ ἀληθῆ εἶναι α'; (v) Πρὸς τοὺς διὰ τῆς τομῆς διαλύοντας τὸν ψευδόμενον λόγον πρὸς Ἀριστοκρέοντα β'; (vi) Περὶ τῆς τοῦ ψευδομένου λύσεως πρὸς Ἀριστοκρέοντα γ'; (vii) Πρὸς τοὺς φασκόντας τὰ λήμματα ἔχειν ψευδῇ τὸν ψευδόμενον λόγον α'.
- 5 Rüstow 1910: 63–65 has interestingly related other titles of the Chrysippean works to the same question of the Liar paradox.
- 6 Chrysippus's *Logical Investigations* were first edited in Crönert 1901: 548–579. A new edition has been proposed by von Arnim in his collection of the Stoic fragments (*SVF* II 298a), and K. Hülser (*FDS* 698) has made some new suggestions based on a new reading of the *PHerc.* 307 carried out by Livia Marrone and partly anticipated in Marrone 1982: 13–18. What is relevant for the Liar paradox is contained in cols. 9–11.
- 7 Rüstow 1910. See also Bochenski 1968⁴: 101–102; Spade 1973.
- 8 Adam Balsamiensis Parvipontani, *Ars disserendi (Dialectica Alexandri)* in Minio-Paluello 1956: 107: “*An verum dicat qui nihil nisi se mentiri dicit.*” See also Spade 1973.
- 9 See Aristotle, *Sophistical Refutations* 25, 180b2–7.
- 10 Cicero, *Prior Academics* 95 (= *FDS* 1212 = *SVF* II 196): *nempe fundamentum dialecticae est, quidquid enuntietur (id autem appellant ἄξιώμα, quod est quasi effatum) aut verum esse aut falsum. Quid igitur haec vera an falsa sunt: “si te mentiri dicis idque verum dicis, mentiris [. . .] verum dicis”? haec scilicet inexplicabilia esse dicitis.*
- 11 See Plasberg 1980²: *apparatus criticus*.
- 12 See Reid 1984²: 290–291; Rüstow 1910: 89.
- 13 Gellius, *Attic Nights* 18.2.9–10 (= *FDS* 1209). See text (C).
- 14 See Plasberg: *apparatus criticus*; K. Hülser, *FDS* 1708.
- 15 Aulus Gellius, *Attic Nights* 18.2.10 (= *FDS* 1269): *Quaesitum ibidem, quae esset huius quoque sophismatis resolutio: cum mentior et mentiri me dico, mentior an verum dico?*
- 16 See Ps-Acro, *Scholia vetustiora in Horati: Epistula* 2.1.45, ii. 281 ff. Keller (= *FDS* 1215): *Dico me mentiri et mentior, verum igitur dico*; Anonymus, *Paraphrasis of Aristotle's Sophistical Refutations* 25, 58.29–33 (= *FDS* 1218): Ἐγὼ ψευδόμενος λέγω ἀληθῆ λόγον, ὅτι ἐγὼ ψευδόμεμαι; and possibly Placidus, *Liber glossarum*, 38.18; 95.14 (= *FDS* 1217): *Diximus de philosophis qui dicunt si dico mentiri et mentitur verum dico.* The passage is amended into *si dico <me> mentiri et mentior, verum dico.*
- 17 Cavini 1993: 98–99 thinks that Cicero's passage in the *Academica* offers grounds to think that (c) is the only proposition uttered by the Liar. In this perspective, however, it is difficult to assign a meaning to clause (f) in Cicero's formulation of the paradox and especially to clause (g) in Gellius's version.
- 18 The text and the translation are those of Cherniss (Cherniss 1976), apart from the fact that I maintain ὦ ἀριστε at 1959 D, where Cherniss, followed by Hülser, prefers the

Wytttenbach correction ἀορίστως, whereas Tony Long and David Sedley in *LS* change ἀορίστως into ἀορίστων. But it is not clear to me why a reference to indefinite propositions should be introduced here.

- 19 {The argument that Mignucci ascribes to Chrysippus as understood by Plutarch seems to presuppose that True(not-A) implies False(A) and that not-False(A) implies True(not-A).}
- 20 See Aristotle, *Topics* 2.7, 113a24–32.
- 21 {Bivalence is tacitly assumed in the sense that a proposition is either only true or only false.}
- 22 The interpretation of this passage is in Ps-Alexander, *On Aristotle's Sophistical Refutations* 171.16 ff. (= *FDS* 1210).
- 23 Cicero, *Prior Academics* 95 (= *FDS* 1212): *si ista explicari non possunt nec eorum ullum iudicium invenitur, ut respondere possitis veram an falsa sint, ubi est illa definitio, effatum esse id quod aut verum aut falsum sit?*
- 24 Elsewhere κριτήριον is rendered by *terminatio* (Cicero, *On Goals* 5.27) or *norma* (Cicero, *Posterior Academics* 1.42).
- 25 Cavini 1993: 107–109 thinks that text (F) allows us to claim that according to Chrysippus, the Liar proposition is ungrounded, in the sense that it has an arbitrary truth value. But if (c*) has after all a truth value, there are reasons to believe that it is at the end both true and false. To equate the Liar paradox to the so-called Truth Teller, where we cannot really decide what truth value the critical proposition of the Story has without deriving a contradiction, is such a mistake that one can hardly attribute it to Chrysippus. The same holds for the view proposed by Cavini 1993: 106–107 according to which Plutarchus's testimony would show that “the Chrysippus or the Speaking Falsely Argument proves to be a piece of fallacy παρὰ τὴν λέξιν.” This interpretation is based on no textual evidence and it is a purely speculative construction, which is in many ways weak. The weakest point is constituted by Cavini's reconstruction of the Liar paradox. He systematically confuses metalogical with logical assertions. He thinks that the Liar paradox has proposition

(C) Δίων λέγει ὅτι < Δίων > ψεύδεται

as its starting point and

(C₁) Δίων ἀλεθεύει

and

(C₂) Δίων ψεύδεται

as its consequences. According to him (C₁) is equivalent to

(C_{1.1}) What Dion says in saying that he is speaking falsely is true

whereas (C₂) might be taken in two different ways, i.e. , as equivalent either to

(C_{2.1}) Not: Dion says what he is really doing

or to

(C_{2.2}) Dion is really doing what he says.

(C_{2.2}) would not be inconsistent with (C_{1.1}), and so the Liar paradox would be solved by distinguishing the meaning one must give to (C₂). I do not think that (C_{1.1}) is an adequate interpretation for (C₁). When in the Liar paradox argument we infer (C₁), what we mean is that (C₁) is true in its obvious sense, which is given by stating that it is true if all propositions to which (C₁) refers are true. Because the proposition to which (C₁) refers according to Cavini is (C₂) itself, then (C₁) means that (C₂) is true. On the other

hand, the conclusion (C₂) implies that we claim that (C₂) is true, and (C₂) is true if and only if at least one proposition to which (C₂) refers is false. Because (C₂) is the only proposition in question, then to state (C₂) entails that (C₂) is false. Therefore, we have a contradiction, and this contradiction does not depend on the meaning of (C₂) but is a consequence of the way in which its truth conditions are stated. There is no hint in the texts that Chrysippus distinguished different meanings of propositions such as (C₂), nor that his presentation of the Liar paradox has anything to do with the complicated metalanguage used by Cavini in his reconstruction of the paradox. I am still convinced that Chrysippus had in mind the real Liar paradox – a paradox that cannot easily be solved without reconsidering some basic views about truth.

26 See, for instance, De Rijk 1966: 104–105, n. 1.02: “When you say (c*), ‘*nihil dicis*’”

27 Thomas Bradwardine considers the *cassantes* “*nimis asinarii*” to deserve a detailed discussion (see Roure 1970: 295).

Part V

Relatives



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14 Relatives in Plato

I

Talking about relations with reference to the ancient world is vaguely provocative. It is enough to peruse any modern manual of the history of logic to find the affirmation that classical antiquity did not work out a theory of relationships. This claim is certainly true if it means that there is no calculus of relations analogous or comparable to what was elaborated by Gottlob Frege in antiquity. But it is false if it means that it is not possible to find a body of reflections, sometimes stimulating and acute, that gives rise to an embryonic theory of relations. Although these reflections are certainly not an alternative to the logic of relations elaborated by Frege, they are of great interest because they allow us to clarify certain philosophical assumptions that are at the root of our modern understanding of the nature of relations. Reflecting on them can help, therefore, to highlight the prerequisites of the Fregean position – prerequisites that are not universally perceived, let alone clearly articulated, by the logicians who have concerned themselves with the theory of relations.

In Plato we find, for the first time in the history of Western thought, many of the ideas that will not only constitute the starting point of the subsequent debates on the nature of relations in Greek philosophy but also the main ideas that will be further elaborated in the medieval tradition and in at least a part of the modern one. In this chapter, I will focus on a few texts from the *Phaedo*, the *Symposium*, and the *Parmenides* that can help outline certain aspects of Plato's conception of relations. Without any pretension of being exhaustive, I set for myself the task of casting some light upon the main lines of Plato's conception of relatives. Needless to say, to fully grasp the complexity of Plato's position a more extensive, and indeed more in-depth, study of the Platonic corpus would be needed.

II

I begin this chapter by examining a text that has given rise to a great deal of discussion among scholars. This text is found in the *Phaedo*, where Plato is concerned with the causality of ideas. To understand the meaning of this passage, we must first clarify the ontology that it presupposes.

In expounding his theory of ideas, Plato argues that ideas exist and that the things down here, that is, the things that fall within the scope of sense-experience, are what they are because they participate in ideas. It is on the poetic metaphor of participation (to put it with Aristotle) that we must dwell.¹ Let us assume that we have, on the one hand, a set of forms to which we refer by means of “ $|F|$,” “ $|G|$,” “ $|H|$,” . . . and, on the other hand, a set of individuals to which we refer by means of “ a ,” “ b ,” “ c ,” The relation that holds between the elements of these two sets is a relation of participation.² To indicate that the individual a partakes of the form $|F|$ we can write:

$$(1) \quad \text{Par}(a, |F|).$$

It remains implicit in the treatment of *Phaedo* 99 D–102 E (but it is made explicit in *Parmenides* 133 C 3–5) that the forms are not *in* the individuals that participate in them. In other words, individuals do not *have* or *possess* the forms. If we use “ $\text{Imm}(x, y)$ ”³ to illustrate this point, we can write

$$(2) \quad \text{Par}(a, |F|) \rightarrow \neg \text{Imm}(|F|, a).^4$$

However, Plato states that Simmias *has* largeness (*Phaedo* 102 C 2) or that Socrates *has* smallness (*Phaedo* 102 C 4). This manner of speaking suggests that we must distinguish largeness itself, namely the form of largeness (or smallness), from the largeness (or smallness) that is present in us.⁵ The latter expression is not an alternative way to refer to the form of largeness: My largeness is present in me, whereas the form of largeness is not present in me.⁶ There is, however, a correspondence relation between forms and “things in us.” We can refer to this relation by saying that the things in us are instances or imitations of the forms. For example, the largeness that is in us is an instance of largeness itself. Without entering into further specifications that would force us to examine a problem that is extraneous to the intent of this chapter, from now on, in order to indicate the relation between the largeness that is in me and the form of largeness, I will say that the largeness in me is an instance of largeness itself, whatever the meaning of “being an instance of” may be.

It is easy to see that the largeness in Simmias, which is an instance of largeness itself, is an individual entity. The largeness that is present in Simmias cannot be the same as the largeness that is present in Phaedo. The reason is that they are features that are immanent in the different things, so they have to be different in the different things.⁷ It is therefore natural to think that the expression τὸ ἐν ἡμῖν μέγεθος, “largeness in us,” refers to the largeness instantiated in the various individuals. Each instance is in relation with largeness itself but is different from all the other instances.⁸ We can use “ f ,” “ g ,” and “ h ” to indicate the instances in us that correspond to the forms, and we can give them the name of “immanent properties.”⁹ We can then say that if an individual a has an immanent property f , there exists a form $|F|$ of which f is a copy. If “ $\text{Ist}(x, y)$ ” stands for “ x is an instance of y ,” we can write:

$$(3) \quad \text{Imm}(f, a) \rightarrow \exists |F| \text{Ist}(f, |F|).^{10}$$

For an individual a to participate in the form $|F|$ entails having an immanent property f that corresponds to $|F|$. In other words,

$$(4) \quad Par(a, |F|) \rightarrow \exists f(Imm(f, a) \wedge Ist(f, |F|)).^{11}$$

We can even reinforce (4) as follows:

$$(4A) \quad Par(a, |F|) \leftrightarrow \exists f(Imm(f, a) \wedge Ist(f, |F|)).$$

In this way, (4A) is a sort of definition of the notion of participation.¹²

We are offered another characterization of the relation of participation. Summing up the results of the discussion offered up to this point, Plato says:

(A) Each of the forms is something and the other things, partaking of these, get their name from them.

(Plato, *Phaedo* 102 A 10–B 3)

The same point is made in *Parmenides* 130 E 5–131 A 2:

(B) Well, tell me, do you think that, as you say, there are ideas, and these other things that partake of them are named from them – as, for instance, those that partake of likeness become like, those that partake of largeness large, those that partake of beauty and justice just and beautiful? Certainly, said Socrates.

(H. N. Fowler translation, with modifications)

In Plato, a predicate *denominates* an object if, and only if, that predicate is true of that object.¹³ Thus, for example, “man” constitutes a denomination of Peter in the sense that it is a predicate that is true of Peter. The idea seems to be that if an individual a partakes of a form $|F|$, then there exists a predicate F that is semantically related to $|F|$ and F is true of a . For example, if a partakes of largeness itself, then the predicate “large” semantically related to the form of largeness is true of a . Plato does not elaborate on the relation between form and predicate. More directly, he does not say whether a predicate corresponds to each form or whether the predicate to be associated with the form is unique.¹⁴ He is content to state that there exists a semantic relation between forms and predicates. We can state this relation as follows:

$$(5) \quad Par(a, |F|) \rightarrow \exists F(Rel(F, |F|) \wedge F(a)).$$

“ $Rel(x, y)$ ” expresses the relation in virtue of which we say that x is semantically related to y .¹⁵ It seems natural to strengthen (5) as follows:

$$(5A) \quad Par(a, |F|) \leftrightarrow \exists F(Rel(F, |F|) \wedge F(a)).$$

In this way, we obtain a characterization of the notion of participation alternative to the one expressed in (4A). Needless to say, the meaning of (5) and (5A)

depends on how the relation *Rel* is understood. A possible way to understand this relation is to suppose that a predicate *F* is semantically related to a form $|F|$ if, and only if, the relation between the name of *F* and that of $|F|$ is the same as the relation between the adjective “large” and the noun “largeness.” Alternatively, we can think of something more sophisticated and imagine that *F* is semantically related to $|F|$ if, and only if, there is an immanent property in the reference of *F* that is in a correspondence relation with $|F|$.¹⁶ In this way, we would be able to give a characterization of the relation *Rel* that does not refer to language. If we use “*Rif*(*x*, *y*)” to indicate the relation in virtue of which we say that *x* refers to *y*, we can write:

$$(6) \quad \text{Rel}(F, |F|) \leftrightarrow \exists f(\text{Rif}(F, f) \wedge \text{Ist}(f, |F|)).^{17}$$

If we accept (6), we can legitimately assume that if a predicate has within its reference an immanent property, then this predicate is true of the thing in which the referred property is present. In a parallel way, if *F* has *f* within its reference, and *F* is true of *a*, then *f* is immanent in *a*. As a result, we obtain:

$$(7) \quad \text{Rif}(F, f) \rightarrow (F(a) \leftrightarrow \text{Imm}(f, a)).^{18}$$

Formulas (6) and (7) allow us to establish a link between (4A) and (5A). Given *Rel*(*F*, $|F|$), they enable us to derive the second member of (4A) from the second member of (5A). Moreover, we can add:

$$(8) \quad \text{Imm}(f, a) \leftrightarrow \exists F(\text{Rif}(F, f) \wedge F(a)).$$

In other words, we can state that *f* is a property immanent in *a* if, and only if, there exists at least one predicate *F* that has *f* within its reference and *F* is true of *a*. Given *Rel*(*F*, $|F|$), (8) combined with (6) and (7) allows us to derive the second member of (5A). In other words, (6), (7), and (8) give us the conditions under which the two characterizations of the notion of participation expressed in (4A) and (5A) are equivalent. Plato never says that they are equivalent. Nor is he committed to (6), (7), or (8). And yet, because Plato provides us with two approaches to the notion of participation, it is helpful to see under which conditions these two approaches are equivalent.

III

Armed with these general considerations, we can now return to the *Phaedo*, where Plato is concerned with the formulation of a consequence of his doctrines of ideas:

(C) Socrates asked: “Now if you assent to this, do you not, when you say that Simmias is larger than Socrates and smaller than Phaedo, say that there is largeness and smallness in Simmias?” “Yes.”

(Plato, *Phaedo* 102 B 3–6;
H. N. Fowler translation, slightly modified)

From Plato's point of view, to say that

(1*) Simmias is larger than Socrates

entails that Simmias partakes of the form of largeness (*Phaedo* 100 E 5–6). We can use “ a^* ” to refer to Simmias and “ $|G|^*$ ” to refer to the form of largeness. From (1*) we can derive:

(9) $Par(a^*, |G|^*)$.

Moreover, from (9), by means of (4), we can derive:

(10) $\exists g(Imm(g, a^*) \wedge Ist(g, |G|^*)$.¹⁹

In other words, if Simmias partakes of largeness itself, Simmias possesses an immanent property that is an instance of largeness itself. This means that Simmias possesses a definite largeness. We can refer to this property that Simmias possesses by using “ g^* .” As a result, we can write:

(11) $Imm(g^*, a^*)$.

Now let us turn to the proposition

(2*) Simmias is smaller than Phaedo.

In this case too we can conclude that Simmias partakes of smallness. If we posit that “ $|P|^*$ ” refers to the form of smallness, we obtain:

(12) $Par(a^*, |P|^*)$.

On the basis of the considerations already advanced for the form of largeness, we obtain:

(13) $Imm(p^*, a^*)$.

Of course, (11) and (13) cannot be accepted without qualification. There is an obvious sense in which I cannot say that the same thing is both large and small and that it has both largeness and smallness. If we take “largeness” and “smallness” as referring to immanent properties that are also contrary properties, we cannot say that they are immanent in the same object. Nor can we say that this object is both large and small. In fact, Plato goes on as follows:

(D) “But,” said Socrates, “you agree that (a) the statement that Simmias is larger than Socrates is not true as stated in those words. (b) For Simmias is not by nature larger than Socrates by reason of being Simmias, but by reason

of the largeness that he happens to have (οὐ γάρ που πεφυκέναι Σιμμίαν ὑπερέχειν τούτῳ, τῷ Σιμμίαν εἶναι, ἀλλὰ τῷ μεγέθει ὃ τυγχάνει ἔχων); (c) nor is he larger than Socrates because Socrates is Socrates, but because Socrates has smallness relatively to his largeness.” “True.” (d) “And again, Simmias is not smaller than Phaedo because Phaedo is Phaedo, (e) but because Phaedo has largeness relatively to Simmias’ smallness.” “That is true.”

(Plato, *Phaedo* 102 B 8–C 8; H. N. Fowler translation, slightly modified)

The text is difficult, and all its parts have been objects of a great deal of scholarly discussion. Let us begin with (a). Almost all interpreters agree that this clause does not mean that the proposition

(3*) Simmias is larger than Socrates

is false.²⁰ However, this agreement disappears as soon as scholars try to establish what this clause means in a positive way. Héctor-Neri Castañeda, who has dedicated several essays to the Platonic theory of relations and has extensively discussed our passage, believes that (3*) is said to be not true in the sense that it does not express everything that a fully adequate ontological analysis of it would require. By his lights, there would be an inadequacy in (3*) that would not compromise its truth but would make difficult to identify the ontological components that give it meaning.²¹ Another possible reading of clause (a) consists in supposing that Plato simply wants to say that (1*) and (3*) are nonperspicuous propositions because if they are interpreted in a certain way, they give rise to a contradiction, since it should be concluded that one and the same thing is both large and small or it has both largeness and smallness at the same time.

There is no consensus on how to read clause (b) either. John Burnet and David Gallop base their interpretation of the passage on the opposition between πεφυκέναι and τυγχάνειν.²² They read this opposition as entailing a distinction between essential and accidental attributes. This reading is criticized by Castañeda.²³ According to Castañeda, the formal structure of the clause is

(*) Not p because A but rather because B .

In other words,

(**) Not p because A but p because B .

As a result, (*) is equivalent to

(***) p not because A but rather because B .

On this reading, regardless of how πεφυκέναι is to be understood, the meaning of clause (b) is πεφυκέναι Σιμμίαν ὑπερέχειν, not by reason of being Simmias, but by reason of the largeness that τυγχάνει ἔχων. If this reading is correct, *Phaedo*

102 C 1–2 entails that Simmias πεφυκέναι ὑπερέχειν in virtue of having largeness. This would rule out the reading that invokes the distinction between essential and accidental properties.

Although the interpretation proposed by Castañeda is compatible with the Greek, it is not the only possible reading of the passage. A great deal depends on the scope of the operator “not” that opens the Greek clause. If the scope is πεφυκέναι Σιμμίαν ὑπερέχειν τούτῳ, then the interpretation proposed by Castañeda cannot be avoided. But if the scope is limited to πεφυκέναι, the sentence οὐ γάρ που πεφυκέναι Σιμμίαν ὑπερέχειν τούτῳ could mean that it is not by nature that Simmias is larger than Socrates. In this case, τῷ Σιμμίαν εἶναι would explain under which conditions Simmias is by nature larger than Socrates. In other words, Simmias would be by nature larger than Socrates by reason of his being Simmias. However, Simmias is larger than Socrates not because of his being Simmias but because of the largeness he happens to have. This second reading entails a distinction between essential and accidental properties.

To my ear, the reading proposed by Castañeda is more natural. But even if we accept that the structure of clause (b) is captured by (**), nothing prevents us from continuing to read in it a contrast between being Simmias (τῷ Σιμμίαν εἶναι), that is, what Simmias is by nature, and the largeness that Simmias happens to have (τῷ μεγέθει ὃ τυγχάνει ἔχων). In other words, the clause can be taken to mean that the fact that Simmias is larger than Socrates does not depend on what Simmias is by nature, but on the accidental presence of largeness in Simmias. Still, we can wonder whether considering the largeness that is in Simmias an accidental property helps us solve the following puzzle: How can largeness and smallness, if they are contraries, be both true of Simmias? Invoking the fact that the relation between Simmias and largeness is accidental does not give us an answer to this question. Largeness and smallness, if they are contraries, cannot belong to the same object, regardless of how the relation between the object and its contraries is to be understood. Of course, if largeness and smallness are accidental properties, they can belong to the same object at different times. But what is difficult to explain is how they can be immanent properties in the same object at the same time. The hypothesis from which Plato moves is that (1*) – Simmias is larger than Socrates – and (2*) – Simmias is smaller than Phaedo – are true of Simmias at the same time. Hence, to stress that largeness and smallness are accidental properties of Simmias does not solve the puzzle.

Having established that the standard reading that invokes the distinction between accidental and essential properties is not enough to solve the puzzle does not give us a plausible reading of clause (b), what we need is a fresh start. Let us begin by observing that the expression “being Simmias” (however it is to be understood) cannot explain what is stated in (3*). In other words,

- (i) It is not by reason of being Simmias that Simmias is larger than Socrates.

It is possible to understand this statement by supposing that Plato, when he says that Simmias is larger than Socrates not by reason of being Simmias, means to say that the reason for this fact is not to be sought in the particular way in which

Simmias is constituted. In other words, Simmias is not larger than Socrates because he has a particular bodily constitution (for instance, he has longer legs). Rather, he is larger than Socrates by means of the largeness that he has. To (i), we have therefore to add:

- (ii) Simmias is larger than Socrates by means of the largeness that Simmias has.

We have already been told that if two individuals (let us refer to them with “*a*” and “*b*”) are such that *a* is larger than *b* by a head, it is not legitimate to say that the reason why *a* is larger than *b* is a head:

(E) “And large things are large and larger things larger by largeness, and smaller things smaller by smallness?” “Yes.” “And you would not accept the statement, if you were told that one man was larger or smaller than another by a head, but you would insist that you say only that every larger thing is larger than another by nothing else than largeness, and that it is larger by reason of largeness, and what is smaller is smaller by nothing else than smallness and is smaller by reason of smallness.”

(Plato, *Phaedo* 100 E 5–101 A 5;
H. N. Fowler translation, slightly modified)

This passage allows us to clarify the meaning of (ii), while at the same time avoid a false reading that derives from the ambiguity of the word μέγεθος. One might think that when Plato says that the largeness of Simmias is the reason of his being larger than Socrates, by “largeness” he means the size of Simmias. If so, Simmias would be larger than Socrates by virtue of his size. But it is immediately evident that the size of Simmias alone is not enough to conclude that Simmias is larger than Socrates. We also need to know the size of Socrates. Following Plato, we can say that the fact that Simmias is larger than Socrates depends not only on the largeness in Simmias but also on the smallness in Socrates. It is the size of Simmias compared to the size of Socrates that allows us to say that Simmias is larger than Socrates. When he states (ii), Plato refers to the largeness that Simmias has in relation to Socrates. Therefore, the immanent property that Simmias possesses by participation of the form of largeness is not an absolute property, but rather is relative to the smallness of Socrates.

This reading finds confirmation in clause (c). There we are told that Simmias is larger than Socrates not because Socrates is Socrates, but rather because Socrates has smallness in relation to the largeness of Simmias. This means that the smallness of Socrates is not an immanent property that Socrates has, regardless of any reference to other things; rather, it is a relative property, that is, a property that Socrates has in relation to the largeness of Simmias.

But in what sense is the smallness of Socrates relative to the largeness of Simmias? To answer this question, Castañeda has suggested that between the largeness of Simmias and the smallness of Socrates there exists a relation such that the two immanent properties are connected instances, respectively, of the form of

largeness and the form of smallness. Consequently, from his point of view, there are immanent properties that obtain only if they are paired with other immanent properties. The largeness of Simmias belongs to Simmias only in relation to the smallness that is proper to Socrates.²⁴

This idea captures an important aspect of the Platonic theory of relations. However, it also presents a serious inconvenience. The strategy that Plato adopts in solving the puzzles consists in arguing that g^* and p^* , respectively, the largeness of Simmias and his smallness, are different but not contrary properties. Castañeda postulates that the immanent properties that constitute the connected pairs of instances corresponding to the forms are different from one another. In other words, an immanent property can be a member of one and only one pair. This means that Simmias's being large in relation to Socrates is the particular property of Simmias that expresses his largeness relative to the smallness of Socrates. This is different not only from the property that Phaedo has in relation to Simmias but also from the property that Simmias has in relation to someone else (for instance, Plato). Hence, Simmias would have an immanent largeness k connected to the smallness k of Socrates and a smallness j connected to the largeness j of Phaedo. Therefore, a smallness k and a largeness j would belong to Simmias at the same time, and they would not be contrary properties.²⁵

The general interpretation of the Platonic notion of relationship proposed by Castañeda does not directly explain, nor does it allow us to explain, why Simmias ought to have as many immanent properties as the relations that are true of him. Nothing prevents us from thinking that the same immanent property is connected to several immanent properties. Consider the case of Socrates, who is smaller than both Simmias and Phaedo. Why should the smallness of Socrates connected to the largeness of Simmias be different from the smallness of Socrates connected to the largeness of Phaedo? Plato's argument requires that these two immanent properties be different. And yet the explanation that Castañeda offers of the connection between the smallness of Socrates and the largeness of Simmias, and the connection between the smallness of Socrates and the largeness of Phaedo, does not explain why.²⁶ If the smallness of Socrates is a property that is present in Socrates in concurrence with the presence of largeness in Simmias, I do not see why the presence of smallness in Socrates cannot be concurrent not only with the presence of largeness in Simmias but also with the presence of largeness in Phaedo. But, then, the same smallness of Socrates would be concurrent with both the largeness that is present in Simmias and the largeness that is present in Phaedo.

In an analogous way, there is no reason to think that the smallness of Simmias in relation to the largeness of Phaedo, and the largeness of Simmias in relation to the smallness of Socrates, are not two immanent properties that are contrary to one another, even if they are present in Simmias concurrently with the presence of two distinct immanent properties in different individuals. If we can suppose that it is the same smallness of Socrates that is relative to the largeness of Phaedo and to the largeness of Simmias, then we are authorized to conclude that the reference to Phaedo or to Simmias does not make the smallness of Socrates different. The smallness of Socrates is relative to some largeness, and this is enough to identify

the smallness of Socrates. But then the smallness of Simmias, whether or not it is compared to the largeness of Phaedo, is always the smallness of Simmias. The same can be said of the largeness of Simmias, whether or not it is compared to the smallness of Socrates. So nothing prevents us from thinking that the smallness of Simmias is contrary to his largeness. For the largeness of Simmias not to be contrary to his smallness, it is necessary to posit a condition stronger than the simple relation of immanent properties. If an immanent property f is related to an immanent property g , and the relation means that f cannot belong to an individual without g belonging to another individual, we cannot conclude that f related to g is different than when it is related to h (where, of course, $\neg(g = h)$ and $\neg(f = h)$).

My hypothesis is that Plato, when he says in clause (c) that Simmias is larger than Socrates because Socrates has the property of smallness in relation to the largeness of Simmias, alludes to the fact that Simmias and Socrates possess two distinct (and related) immanent properties. Simmias possesses the property of largeness in relation to the smallness of Socrates, and Socrates the property of smallness in relation to the largeness of Simmias. This reading suggests that when we say that Simmias is larger than Socrates, we do not refer to the immanent property of largeness taken as an absolute property, namely a property that Simmias possesses by reason of being Simmias, but rather to a relative property, namely the largeness that Simmias has in relation to the smallness of Socrates. What we actually single out is not the property of being large, but the property corresponding to the expression “largeness of Simmias in relation to the smallness of Socrates.” This immanent property is different from the immanent property corresponding to the expression “smallness of Socrates in relation to the largeness of Simmias.” Whereas the first belongs to Simmias but is not present in Socrates, the second belongs to Socrates but is not present in Simmias. Moreover, the smallness of Simmias in relation to the largeness of Phaedo is a property that is different from, but is not contrary to, the largeness of Simmias in relation to the smallness of Socrates.

The central idea in my interpretation is that immanent properties like the one to which the expression “the largeness of Simmias” refers must be treated as relational properties. This means that they imply a reference to some relation. Thus, for example, the largeness of Simmias, which is an immanent property possessed by Simmias, is relative to the immanent property possessed by someone else. If we use “ $Lr(x, y)$ ” to indicate the relation that allows us to say that x is larger than y , we can characterize the property of being larger than the smallness of Socrates that is possessed by Simmias. This is the property that, if it is possessed by Simmias, makes him larger than Socrates. If we refer to the largeness of Simmias in relation to Socrates with “ g^* ,” we can write:

$$(14) \quad g^* = \text{uxEnt}(\text{Imm}(x, a^*), Lr(a^*, b^*)).^{27}$$

In an analogous way, with reference to j^* , the smallness of Socrates in relation to Simmias, we can write:

$$(15) \quad j^* = \text{uxEnt}(\text{Imm}(x, b^*), Lr(a^*, b^*)).^{28}$$

The representation of p^* – namely the smallness of Simmias in relation to the largeness of Phaedo – and k^* – namely the largeness of Phaedo in relation to the smallness of Simmias – is no longer a problem. In the first case we can write:

$$(16) \quad p^* = \text{uxEnt}(\text{Imm}(x, a^*), \text{Lr}(c^*, a^*)).^{29}$$

In the second case, we can write

$$(17) \quad k^* = \text{uxEnt}(\text{Imm}(x, c^*), \text{Lr}(c^*, a^*)).^{30}$$

It is easy to see that g^* and p^* , if they are understood as suggested in (14) and (16), are not contrary properties. So it is legitimate to assume both (11) and (13), because no contradiction follows from both assumptions.

It is time to return to clause (a) in order to answer the following question: In what sense are propositions (1*), (2*), and (3*) not true? I suggest understanding the statement as a declaration of the nonperspicuity of these propositions due to the fact that they lend themselves to a falsifying analysis. The falsifying analysis consists in assuming, in analogy with what happens to the other predicates, that the predicates “large” and “small” implied in (1*), (2*), and (3*) refer to absolute immanent proprieties in that they are contrary to each other. But if we take “large” and “small” as referring to relative immanent properties, we can avoid the conclusion that contrary properties belong to Simmias. If the properties possessed by Simmias are not an absolute largeness and an absolute smallness, but rather a largeness relative to the smallness of Socrates and a smallness relative to the largeness of Phaedo, then nothing prevents us from saying that Simmias is both large and small. When we say this, we only say that Simmias is large in relation to Socrates and is small in relation to Phaedo.

Plato does not deny that from the truth of (1*) and (2*), it follows that we can say that Simmias is both large and small. On the contrary, he is quite explicit on this point:

(F) “Then Simmias is called small and large, when he is between the two <sc. Socrates and Phaedo>, surpassing the smallness of the one by exceeding him in height, and granting to the other the largeness that exceeds his own smallness.”

(Plato, *Phaedo* 102 C 10–D 2; H. N. Fowler translation, slightly modified)

To explain this statement, it is sufficient to assume that “large” and “small” are relative terms so that “large” means “larger than someone” and “small” means “smaller than someone.” In fact, Plato seems to derive

(4*) Simmias is large

from

(1*) Simmias is larger than Socrates

and

(5*) Simmias is small

from

(2*) Simmias is smaller than Phaedo

These derivations are possible only if “large” and “small” have the proposed meaning.

Therefore, if (4*) means:

(4'*) Simmias is larger than someone

and (5*) means:

(5'*) Simmias is smaller than someone

we can conclude that there is no reason to think that (4*) and (5*) are incompatible. More to the point: If this is the correct interpretation of what Plato says, we can also conclude that he is operating not only with clear and firm distinction between relative and absolute properties but also with a clear and firm distinction between absolute and relative predicates.

IV

I hope to have established that Plato admits the existence of relational properties that are immanent in individuals such as “the largeness of *a* in relation to the smallness of *b*.” I argued that a property of this sort should be interpreted as a property possessed by *a* that expresses a relation to a property possessed by *b*. Plato admits the existence of relational terms that correspond to relational properties. In this section, I would like to look more closely at the kinds of relations that are involved in relational properties and relational terms.

Let us turn to a passage from the *Symposium*, where Socrates, in praising the exordium of Agathon’s speech, says:

(G) “Are we so to view his character as to take Love to be love of some object, or of none? My question is not whether he is love of a mother or a father – how absurd it would be to ask whether Love is love of mother or father – but as though I were asking about our notion of ‘father,’ whether one’s father is a father of somebody or not. Surely you would say, if you cared to give the proper answer, that the father is father of a son or of a daughter, would you not?” “Yes, of course,” said Agathon. “And you would say the same of the mother?” He agreed to this too. “Then will you give me just a few more answers,” said Socrates, “so that you may the better grasp my meaning?”

Suppose I were to ask you, ‘Well now, a brother, viewed in the abstract, is he brother of somebody or not?’” “He is,” said Agathon. “That is, of a brother or of a sister?” He agreed. “Now try and tell me about Love: is he a love of nothing or of something?” “Of something, to be sure.”

(Plato, *Symp.* 199 D 1–E 8; H. N. Fowler translation)

The impression is that the focus in this passage is on terms such as “father,” “mother,” and “brother.” Socrates argues that “love” belongs to the same class of terms. What distinguishes the term “father” is that ὁ πατήρ ἐστὶ πατήρ τινος (199 D 5), the τινός of which ὁ πατήρ ἐστὶ πατήρ is immediately specified in our text as υἱός γε ἢ θυγατρὸς (D 6–7). In other words, a father is always a father of an individual. We can read the Greek in two ways. We can take ὁ πατήρ as referring to the general term “father.” In this case, ἐστὶ denotes identity. Alternatively, we can give ἐστὶ the usual copulative function and assume that ὁ πατήρ refers to all human beings who are fathers. Either way, what matters is that Plato argues that human beings who are fathers or the term “father” are characterized by the fact that they exhibit a certain relation with something else. It is evident that a term is relative not because it involves any relation with something else, but because it is in a determinate relation with it. We can represent this relation formally. If we take ὁ πατήρ as the predicate “being a father,” we can represent this predicate by means of “ $\lambda xF^*(x)$.” Moreover, we can use “ $RF^*(x, y)$ ” to represent the relation that is characteristic of $\lambda xF^*(x)$. So we can write:

$$(18) \quad \lambda xF^*(x) = \lambda x\exists yRF^*(x, y).^{31}$$

If we adopt the second interpretation of ὁ πατήρ, we obtain:

$$(19) \quad F^*(x) \leftrightarrow \exists yRF^*(x, y).^{32}$$

We do not have to decide between (18) and (19) because they are logically equivalent. Instead, we should try to clarify the nature of the link between the predicate “being a father” (or the human beings who are fathers) and the relation of paternity. My hypothesis is that the link in question is semantic. We cannot understand “father” unless we refer to the relation of paternity. If “father of” is part of the meaning of “father,” we have to say that if someone is a father, then he is the father of someone. The link that exists between “father” and “father of” is such that if “father” is true of a , then “father of” is also true of a . And this is equivalent to saying that there exists someone of whom a is the father.

At this point, we are in the position to generalize on the basis of the examples Plato gives us in the *Symposium*. Let us concentrate on the more familiar of the two characterizations offered earlier, namely (19). We can say that a general term is relative – “ $Rel(F)$ ” – if, and only if, it is characterized by a relation that is part of its meaning. In other words,

$$(20) \quad Rel(F) \leftrightarrow \forall x(F(x) \leftrightarrow \exists yR(x, y) \wedge Mn(R, F))^{33}$$

where “ $Mn(x, y)$ ” is the relation in virtue of which x determines (contributes to determining) the meaning of y . I will use “ RF ” to indicate the relation that determines the meaning of “ F .”³⁴

However, Plato also refers to relative terms in another way. In the *Statesman*, for instance, he says that the larger is larger than the less (283 D 11–12). A similar expression is found in the *Republic*, where we are told that the double is double of the half (IV, 438 C 1–2). This way of presenting things can be seen as an alternative way to qualify the relation that characterizes the meaning of relatives. To say that the double is double of the half is equivalent to saying that double things make up the domain of the relation “being double of” that has the things that are half as its codomain. If we posit “ $Db(x)$ ” for “ x is double” and “ $Hf(x)$ ” for “ x is half,” we can write

$$(21) \quad Db(x) \leftrightarrow \exists y(RDb(x, y) \wedge Hf(y)).^{35}$$

It is clear, however, that in order to qualify a relative term, it is enough to have the relation that characterizes its meaning. We can use this relation to determine both the relative term and its correlative, regardless of whether we have a name to indicate the latter. Consider again the predicates “being double” – $\lambda x Db(x)$ – and “being half” – $\lambda x Hf(x)$. If we can say that

$$(22) \quad \lambda x Db(x) = \lambda x \exists y RDb(x, y)^{36}$$

is a definition of “being double,” we can also say that

$$(23) \quad \lambda x Hf(x) = \lambda y \exists x RDb(x, y)$$

is an adequate characterization of “being half.”³⁷ In an analogous way, we can use the relation “being half of” to define not only “being half” but also “being double.” In this sense, “ a is double of half” does not add any new information to “ a is double of something” because both expressions are different ways to express the logical fact that “being double” is equivalent to “being double of something.”

If this way of characterizing relative terms is the one used by Plato, or at least it remains faithful to it, it is not immediately obvious that we can attribute to him a confusion between what today are called “incomplete attributive predicates” and “incomplete relative predicates.”³⁸ “Beautiful” and “heavy” are examples of incomplete attributive predicates. When we say, “John is heavy,” we mean “John is a heavy man.” In other words, we understand the first assertion only because the context allows us to complete it by supplying the appropriate substantive. The case of incomplete relative predicates cannot be reduced to this first case. More directly, they cannot be completed by adding a substantive. In this second case, we have to make the relation explicit. Now when Plato is concerned with relative predicates, he does not focus on any incomplete predicate, but only on those incomplete predicates that imply a relation to something. Hence, he does not confuse attributive and relative terms. It is nevertheless true that he sometimes

classifies as relative terms that are today considered attributive terms. For example, “large” and “small” are regarded as relatives. The same happens in the *Republic* (VII, 523 E) where “soft” and “hard” are classified among the relative terms. But this does not mean that Plato is confused about the general distinction between attributive and relative terms.

V

So far I have concentrated on how the notion of relatives can be applied to immanent properties and predicates. A difficult passage from the *Parmenides* allows us to extend this notion to forms. In the first part of this dialog, Plato offers a close analysis of his theory of ideas as expounded, *inter alia*, in the *Phaedo*. By reflecting on concepts such as participation, separation, and resemblance, Plato deduces a series of contradictions within the theory of ideas with a particular reference to the relation between ideas and sensible things. The passage that interests us is an argument for the unknowability of ideas. Very roughly, the argument goes something like this: If ideas were knowable, there would be a relation between our knowledge and ideas. But two things, if they are related, have to be homogeneous in the sense that ideas have to be related to ideas, properties to properties, and individuals to individuals. But individuals and properties are not homogeneous with ideas, so there cannot exist a relation of knowledge between ideas and individuals. As a result, ideas cannot be known by us (*Parmenides* 133 B 4–134 C 3). We do not need to engage in a detailed discussion of this argument. Rather, I would like to focus on the following passage:

(H) Then those ideas that are what they are in relation to one another have their own nature in relation to themselves, and not in relation to the likenesses, or whatever we choose to call them, which are among us, and from which we receive certain names as we participate in them. And these things, which have the same names with the ideas, are likewise in relation only to themselves, not to the ideas, and, belong to themselves, not to the like-named ideas.

(Plato, *Parmenides* 133 C 8–D 5; H. N. Fowler translation)

Here we find the three (or four) kinds of entities that we have already found in the *Phaedo*. Next to ideas or forms, we find things among us (τὰ παρ’ ἡμῖν) and individuals that are bearers of predicates. It is easy to see that the things among us are those things that we have called immanent properties. We have to understand what Plato means by “ideas that are what they are in relation to one another.” This expression is far from being crystal clear. It is telling that Socrates too does not understand it, so Parmenides clarifies his point with the following example:

(I) “For instance,” said Parmenides, “if one of us is master or slave of anyone, he is not the slave of the master itself, nor is the master the master of the slave itself; each is a human being and is master or slave of a human being,

but mastership itself is mastership of slavery itself, and likewise slavery itself is slavery of mastership itself. But the things that we find among us do not have power ($\delta\acute{\upsilon}\nu\alpha\mu\iota\nu$) in relation to them, nor they <have power> in relation to us; they, as I say, are what they are in relation to one another, and the things among us are in relation likewise to themselves. You understand what I mean, do you not?"

(Plato, *Parmenides* 133 D 6–134 A 1; H. N. Fowler translation, modified)

In this passage, Plato concentrates on the relation between master and slave. Following what we have seen in dealing with relative predicates, we can say that if it is true that a is a master ($M^*(a)$), then a is master of someone. In other words,

$$(24) \quad M^*(a) \rightarrow \exists y RM^*(a, y)$$

where " $RM^*(x, y)$ " stands for " x is master of y ." Plato adds that if a master must be master of someone and the master is an individual, the master is the master of an individual. Therefore, we can complete (24) with

$$(25) \quad M^*(a) \rightarrow \exists y (RM^*(a, y) \wedge Ind(y))$$

where " $Ind(y)$ " stands for " y is an individual." A parallel claim is made with respect to the form of mastery. The form of mastery, $|M|^*$, is in a certain relation (let us call it " $R|M|^*$ ") to something different from itself, and this must be another form, that is, the form of slavery. In other words,

$$(26) \quad x = |M|^* \rightarrow \exists y (R|M|^*(x, y) \wedge For(y))$$

where " $For(y)$ " stands for " y is a form." The final part of our passage allows us to extend the point to properties. If " m^* " refers to the property of being a master, and this property is immanent in an individual, then we can say that there exists another property, namely the property of being a slave for a determinate individual, and this property is related to m^* through Rm^* :

$$(27) \quad x = m^* \rightarrow \exists y (Rm^*(x, y) \wedge Prop(y)),$$

where $Prop(y)$ means " y is a property."

Two general remarks can be made on the basis of (25)–(27). The first concerns the very notion of relatives. The example of master and slave shows that we can speak of relatives not only with respect to predicates but also with respect to immanent properties and forms. Just as there are relative predicates, there are also relative properties and relative forms. In the case of forms, we can say that $|F|$ is a relative form if, and only if, it is connected to something through a relation that contributes to offering an answer to the question "What is $|F|$?" In other words,

$$(28) \quad Rel(|F|) \leftrightarrow \exists x (R(|F|, x) \wedge Mn'(R, |F|)).^{39}$$

In an analogous way, a property f is relative if, and only if, it is related to another property via a relation R that contributes to establishing what f is:

$$(29) \quad Rel(f) \leftrightarrow \exists x(R(f, x) \wedge Mn''(R, f)).^{40}$$

My second remark concerns what I want to call the Principle of Homogeneity of the Relatives, or more briefly (PHR). The most natural, and indeed obvious, way to understand (PHR) is this: If an entity x entertains a relation R with an entity y , the nature of y depends on the nature of x , in the sense that if x is an individual, then y is an individual, and if x is an immanent property or a form, then y is an immanent property or a form. In other words,

$$(30) \quad R(x, y) \rightarrow ((Ind(x) \rightarrow Ind(y)) \wedge (Prop(x) \rightarrow Prop(y)) \wedge (For(x) \rightarrow For(y))).$$

The unknowability of ideas can be derived from (30). The reason is that (human) knowledge is an immanent property and as such cannot be anything but knowledge of an immanent property. But ideas are not immanent properties, so they cannot be known by us.

One might object that (30) is manifestly implausible because it implies the impossibility of establishing any relation not only between individuals and ideas but also between individuals and immanent properties and between immanent properties and ideas. As a consequence, we would have no reason to posit the relation of participation between individuals and ideas, and we would not even say that immanent properties belong to individuals. Moreover, talking about immanent properties would no longer make sense, because a property f can only be said to be immanent in relation to another immanent property g but certainly not in relation to an individual. Last but not least, the argument that Plato offers in the *Parmenides* is meant to lead to the conclusion that the knowability of ideas is not compatible with (PHR) and that if we have to choose between (PHR) and the principle that ideas can be known by us, we should abandon (PHR) because Plato cannot give up the latter principle. All this should alert us that when Plato offers (PHR), he supports something different from (30).⁴¹

In light of this objection, one might think that the homogeneity Plato requires does not concern the elements of the domain and the codomain of a given relation, but rather the domain and the codomain themselves. In other words, if the domain of a relation is a form, the codomain is also a form, and if the former is a predicate, the latter is also a predicate. The advantage of this position is that it would not prevent the possibility of a relation between entities that are nonhomogeneous. For instance, we could have relations between properties of individuals and properties of forms (if forms can have properties) or individuals and forms if there are predicates that are true of the former and predicates that are true of the latter. Although a weaker version of (PHR) can only be welcome, at least three considerations count against it. First, there is no textual evidence that supports a weaker version of (PHR). Second, a weaker version of (PHR) does not allow us to reach the conclusion that Plato would like to derive from it, namely the unknowability

of forms. Third, the example of master and slave shows that Plato is committed to the original formulation of (PHR).

VI

I would like to conclude this chapter with some general considerations. First of all, the analysis conducted so far shows that Plato had some awareness of the distinction between, on the one hand, predicates, properties, and forms that we call absolute and, on the other hand, predicates, properties, and forms that are relative. Relative predicates are those that correspond *grosso modo* to incomplete terms, that is, terms that, in order to be understood, require a semantic integration by reference to the corresponding relation. For example, “father” cannot be understood except by reference to “son” or “daughter” or, more exactly, to the relation “being father of.” In other words, the *meaning* of “father” is complete only by reference to the relation of paternity. Something analogous is true for relative properties and relative forms. Plato does not really elaborate on the nature of these relations. However, the little he tells us seems to go in the direction of what Aristotle says (for instance, in *Categories* 7) and of what much of the ancient tradition will say after Aristotle.⁴² What is important to emphasize is that these results are generally disregarded by the interpreters of Plato, who are ready to accuse him of not having clear ideas on the topic of relations and having often confused absolute and relative predicates. That this happens cannot be denied. However Plato, though painstakingly, introduces the distinction between absolute and relative predicates, and he deserves credit for this achievement. But he does so in a way that is not the one that logicians from Frege onward have adopted. Frege analyzes a statement such as “Simmias is larger than Socrates” in a way that does not consider the grammatical structure of the statement. In Frege’s analysis, both Simmias and Socrates function as logical subjects of the relational predicate “being larger than.” There are very good reasons to accept this way of putting things. For one, it becomes easier to develop a logic of relations in this way. By contrast, Plato takes Simmias to be the logical subject of the statement “Simmias is larger than Socrates.” In his view, Socrates is part or specification of the predicate “being larger than.” The insight that “being larger than” applies to a pair of individuals is not lost in his approach. In this sense, “father” is different from “man” because the former, but not the latter, entails a reference to a relation and therefore to something else in which the relation is grounded. But this other thing is not regarded as a logical subject of the predicate “being father of.” This approach has the merit of accounting for two other facts that we have to consider when we analyze the statement “Simmias is larger than Socrates.” First, Simmias alone is the focus of the linguistic act that consists of the assertion that Simmias is larger than Socrates. Second, what is predicated of Simmias is “being larger than *Socrates*.”⁴³

Notes

- 1 Aristotle, *Metaphysics* 1.9, 991a20–23. For the various expressions that Plato uses in *Phaedo* 99 D–102 E to refer to the relation of participation, see Ross 1961⁶: 227–229. Cf. also Nehamas 1972–1973: 464–465.

- 2 Unless otherwise indicated, I will add an asterisk after the letter to refer to specific entities. So “ $|F|$ ” and “ a ” can be taken as constants.
- 3 “ $Imm(x, y)$ ” stands for “ x is immanent in y .”
- 4 In other words: If an individual a partakes of a form $|F|$, then it is not the case that $|F|$ is immanent in a .
- 5 See *Phaedo* 102 D 6–7, where αὐτὸ τὸ μέγεθος is distinguished from τὸ ἐν ἡμῖν μέγεθος.
- 6 On the distinction between ἔχειν and μετέχειν in the *Phaedo*, see Fujisawa 1974: 34 ff.
- 7 Fine 1983: 234.
- 8 Ross 1961⁶: 30; Vlastos 1969: 291–325.
- 9 I will add an asterisk – “ f^* ”, “ g^* ”, and “ h^* ” – to refer to individual immanent features.
- 10 In other words: If an immanent property f is present in a , then there exists at least one form $|F|$ of which f is an instance.
- 11 In other words: If an individual a partakes of a form $|F|$, there exists at least one individual property f that is present in a and is an instance of $|F|$.
- 12 This characterization of participation rests on the conviction that immanent properties are ontologically distinct from their respective forms. This reading has an illustrious tradition (a full list of adherents to this position can be found in O’Brien 1967: 201n1). However, it is challenged in O’Brien 1967: 201–202; Guthrie 1975: 353–358; De Rijk 1986: 52–59. These authors maintain that largeness itself and largeness in us are to be distinguished as two different states of the same largeness. I confess that I find it difficult to get their point. I have no difficulty accepting that the tiger that performed yesterday at the Circus Togni is the same as the tiger that a few years ago wandered in the forests of the Himalayas. In this case, we are dealing with two states (or situations) of the same individual. But no one would easily accept that this tiger is to be identified with the two-footed animal that shouted “Forza Juve” under my window last night. Some of the properties of the latter are not compatible with some of the properties that identify a tiger as a tiger. The point is whether transcendence is an identifying characteristic of Platonic forms, that is, a characteristic without which a Platonic form is no longer a Platonic form. It seems to me rather adventurous to claim that, for example, largeness itself, which is immutable (*Phaedo* 78 D 3–5), divine, eternal, and intelligible (*Phaedo* 80 A 10–B 1), is the same as the largeness in us, which is immanent and perishable. The difference between immanent properties present in perishable things and transcendent forms does not seem to be merely a difference in status or situation. For a discussion of the problem of separation of forms from things, I refer the reader to Fine 1984: 31–87. At any rate, the interpretation of the relation of *Par* that I have offered can be adapted to the hypothesis that Platonic forms are immanent.
- 13 See Vlastos 1969: 85n29. That this implies that predicates are treated by Plato as proper names is controversial. See Fine 1977: 277–279; Nehamas 1979: 100–101.
- 14 One might ask whether our passage implies that the same predicate applies to a form and to the things that possess the properties that are instances of that form. In this case, the predicate “large” would be true not only of large things but also of largeness itself. *Phaedo* 102 B 6–7 can be read as committing Plato to this view.
- 15 In other words: If an individual a partakes of a form $|F|$, then there exists at least one predicate F that is semantically related to $|F|$ and that predicate is true of a .
- 16 See Allen 1965: 45–46.
- 17 In other words: If a predicate F is semantically related to a form $|F|$, then there exists an immanent property f that falls within the reference of F and is an instance of $|F|$. Of course, f cannot constitute the totality of the reference of the predicate F because f is an individual entity and F is a general term. We can think that the reference of F is the set constituted by all the immanent properties f of the same kind. So we can say that an immanent property f falls within the reference of F if f belongs to the reference of F .
- 18 In other words: A property f falls within the reference of a predicate F only if F is true of an individual a if, and only if, f is immanent in a . Formulas (5A), (6), and (7) do

not imply that there is a form that corresponds to each predicate. It is not clear to what extent Plato is committed to this claim (he seems to accept it at *Republic* X 596 A–B). On this point, see Owen 1986: 172; Nehamas 1972–1973: 477–478.

- 19 In other words: There exists at least a property g that is immanent in Simmias and is an instance of the form of largeness.
- 20 Prauss 1966: 88–91 is the exception to the rule, but his overall reading of the passage is not plausible.
- 21 See Castañeda 1978: 43. Castañeda finds confirmation of his reading in the meaning that ἀληθές would have in our passage. In his view, Plato would not be claiming that the ordinary relational sentences of Greek (or English) are not true. They are indeed true but are not ontologically perspicuous. See, in particular, Castañeda 1982: 158n9.
- 22 Burnet 1980¹³: *ad locum*. Gallop 1975: *ad locum*. David Gallop has restated his position in Gallop 1976: 155. Among the scholars who have sided with Burnet, I recall Bluck 1955: 118–119; O’Brien 1967: 199–200; Dorter 1982: 141, White 1981: 6–7; Fine 1983: 237, and Loriaux 1969–1975: 111–112 (with hesitations). *Contra* Hackforth 1972²: 155.
- 23 Castañeda 1978: 48–49.
- 24 Castañeda 1972: 467–480. *Contra* Gallop 1976: 149–163; White 1976: 79n16; Matthen 1982: 90–100. Against Matthen, and in defense of Castañeda, McPherran 1983a: 298–306 (for Matthen’s reply, see Matthen 1984: 304.312). McPherran has developed the view defended by Castañeda in McPherran 1983b: 149–164; McPherran 1984: 292–302; McPherran 1986: 233–252.
- 25 Castañeda 1982: 131–134.
- 26 Here is how Castañeda (in Castañeda 1972: 471) describes the relation of the immanent properties: “some facts consist of a particular instantiating, or participating in, a Form: they are *single-pronged*. Other facts are *multiple-pronged*: they consist of an array of Forms each instantiated by one particular, where these instantiations do not by themselves constitute facts.” (Italics in the original.)
- 27 The expression “ $ux(. . .)$ ” stands for “that unique x such that,” and the expression “ $Ent(x, y)$ ” means “ x entails y .” Therefore, the largeness of Simmias in relation to Socrates is that unique property immanent in Simmias entailing that Simmias is larger than Socrates. {Formula (14) – like formulas (15), (16), and (17) – implies that a property that satisfies the condition $Ent(Imm(x, a^*), Lr(a^*, b^*))$ exists and is unique. This would require the specification of a theoretical and logical context in which identity and relations among properties are accounted for in such a way that justifications can be provided for statements of existence and uniqueness concerning properties.}
- 28 In other words, the smallness of Socrates in relation to Simmias is that unique property immanent in Socrates entailing that Simmias is larger than Socrates.
- 29 In other words, the smallness of Simmias in relation to Phaedo is that unique property immanent in Simmias entailing that Simmias is smaller than Phaedo.
- 30 In other words, the largeness of Phaedo in relation to Simmias is that unique property immanent in Phaedo entailing that Phaedo is larger than Simmias.
- 31 In other words, being a father is the same as being someone who entertains the relation of paternity with someone.
- 32 In other words, someone is a father if, and only if, there exists someone of whom he is the father.
- 33 In other words, a predicate F is relative if, and only if, for any x , F is true of x , if and only if, there exists a y to which x entertains a relation that expresses the meaning of F .
- 34 If we adopt the second characterization offered earlier, namely (18), we will write:

$$(20A) \quad Rel(F) \leftrightarrow \lambda x F(x) = \lambda x \exists y (R(x, y) \wedge Mn(R, F)).$$

Formula (20A) is equivalent to formula (20).

- 35 In other words, an x is double if, and only if, there exists at least one y such that x is double of y and y is half of x .
- 36 In other words, being double is the same as being one of the things that are double of something.
- 37 In other words, being half is the same as being one of the things of which something is double.
- 38 See Brentlinger 1972: 69–73.
- 39 In other words, a form $|F|$ is relative if, and only if, $|F|$ entertains a relation R with something and R contributes to establishing the meaning of $|F|$. “ $Mn'(x, y)$ ” is a relation and refers to the relation between a relation and a form such that the relation expresses (or contributes to expressing) what the form is. The relation “ $Mn'(x, y)$ ” should not be confused with the relation “ $Mn(x, y)$.” I introduced the latter to refer to the relation between a relation and a predicate. I will abbreviate “ $Mn'(R, |F|)$ ” with “ $R|F|$ ”.
- 40 In other words, an immanent property f is relative if, and only if, it entertains a relation R with something and R contributes to establishing what f is. In the case of immanent properties, “ $Mn''(x, y)$ ” plays the role that “ $Mn(x, y)$ ” plays for predicates and “ $Mn'(x, y)$ ” for forms. I will abbreviate “ $Mn''(R, f)$ ” with “ Rf ”.
- 41 Mueller (1983): 3–7.
- 42 {See Chapters 15 and 16.}
- 43 I owe both points to Nino B. Cocchiarella.

15 Aristotle's definitions of relatives in *Categories* 7

I

Chapter 7 of Aristotle's *Categories* is dedicated to a study of relatives that are called “πρός τι.” At the very beginning a characterization, or definition,¹ of them is given:

(A) We call *relatives* all such things as are said to be just what they are, *of* or *than* other things, or in some other way *in relation to* something else.

(Aristotle, *Categories* 7, 6a36–37; J. Ackrill translation)

This definition is followed by some examples. Aristotle says:

(B) For example, larger (τὸ μείζον) is said to be what it is (τοῦθ' ὅπερ ἐστίν) *than* something else (it is said to be larger *than* something); and double (τὸ διπλάσιον) is said to be what it is *of* something else (it is said to be double *of* something); similarly with all other such cases.

(Aristotle, *Categories* 7, 6a37–b2;
J. Ackrill translation, slightly modified)

It is important for the interpretation of these passages to ask what τὸ μείζον and τὸ διπλάσιον mean, because what we have to count as relatives depends on this question. An obvious answer is to take τὸ μείζον and τὸ διπλάσιον as meaning “what is larger” and “what is double,” as Ackrill suggests in his translation. Then, the first example given in (B) would mean that an object *a* that is larger than an object *b* is called what it is, namely larger, with reference to *b*. Consequently, *a* has to be called a relative according to the definition proposed by (A) and, in general, a relative is whatever stands in a relation to something. However reasonable this view may be, it entails a consequence that is paradoxical if it is considered in light of Aristotle's ontology and the meaning that he assigns to the categories. Everything stands in an identity relation to itself and, according to the interpretation of (A) we are discussing, it has to be called a relative. The category of relatives would then have the same extension as the class of existing things. In order to avoid this difficulty one would have to deny that identity is a relation, saying, for instance, that a relation holds only if two different terms are

related and not if two occurrences of the same term are in question. This is not a very happy move, but there is an even worse consequence. Consider the proposition:

(*) a is taller than b .

Take a to stand for an individual substance, Aldous. According to the hypothesis, one should say that Aldous is a relative, being taller than someone. But Aristotle explicitly denies that an individual substance can be put among the relatives (*Categories* 7, 8a16–18), because Aldous is not what he is, namely Aldous, *of* or *than* something else. One might try to escape this objection by saying that Aldous is not a relative if he is considered *qua* Aldous, but he is a relative if he is considered *qua* taller. In general, if F stands for a predicate such as “taller,” “larger,” “double,” “son,” “slave,” and so on, and a is F , one could say that a is a relative *qua* F . According to this view, things that are called relatives are individual things considered under a given predicate.

I do not think that this interpretation captures the real sense of Aristotle's words, if we give the *qua*-expression its natural meaning. In my view, “ a *qua* F is G ” means simply that a is G because it is F , namely that “ a is G ” can be inferred from “ a is F .”² Then, if it is true to say that Aldous *qua* taller is a relative, it follows that Aldous is a relative – against Aristotle's explicit denial. Of course, one might try to give a different meaning to the *qua*-expression. Whatever its sense may be, it should be such that “ a *qua* F is G ” does not imply “ a is G ,” and I find it difficult to believe that any such sense can be given.

Perhaps another road should be explored. It is useful to distinguish two senses of the word “relative.” On the one hand, we can say that a is a relative because a is related to something else. In this sense, Aldous, being taller than someone else, can be called a relative. On the other hand, “relative” could be taken to mean relational, and from this point of view, Aldous is not a relative, because “Aldous” by itself does not imply a reference to something else. That is what I think Aristotle means when he says that an individual substance is not what it is “*of* or *than* something else.” “Aldous” does not express by itself any relation to something, and consequently Aldous is not a relative in the sense of relational. It is a property such as “larger,” “double,” “similar,” “son,” or “slave” that is relational, because being larger or double or a son implies a reference to something else. Of course, the two meanings of “relative” are strictly connected: a is related to something if, and only if, it possesses a relational property. Aldous has a childhood relation to someone if, and only if, he is a son. In my view, Aristotle is interested in defining and discussing relatives in the sense of relational properties in *Categories* 7. If he had in mind the other sense of “relative,” he would not exclude individual substances from the relatives. He does not explicitly distinguish the two senses of the term, even if he sometimes uses this distinction. More precisely, he sometimes plays with the connection between the relational character that some properties have and the corresponding situation of being related to something in which individuals that have these properties are. An example of that is offered in the following passage:

(C) All things then are relative which are said to be just what they are, of or than something else – or in some other way in relation to something else. Thus a mountain is said to be large in relation to something else (the mountain is said to be large in relation to something; and similar (τὸ ὁμοῖον) is said to be similar to something; and the others of this kind are in the same way spoken of in relation to something.

(Aristotle, *Categories* 7, 6b6–11; J. Ackrill translation, slightly modified)

The first part of the passage repeats the definition of relatives given in (A). What interests us is the example of the mountain. One might think that what has to be counted as a relative is the mountain: It is the mountain that is what it is, namely large, with respect to something else. But I do not believe that Aristotle would have considered a mountain to be in the category of the relatives, nor does it make any sense to say that a mountain is a relative *qua* large. What it means to be a relative is to have the property of being large or, more simply, to be large. Being large means large with respect to something, just as similar is always similar to something. Because that is the case with the predicate “large,” whatever is said to be large is said to be so with reference to something else. Thus a mountain is said to be large with respect to something else, and it is related to something else without being a relative in the Aristotelian sense.

If this interpretation is adopted, terms such as “τὸ μείζον” and “τὸ διπλάσιον” in (B) or “τὸ ὁμοῖον” in (C) do not refer to the things that are larger or double or similar, but rather stand for the predicates or the properties being larger, being double, and being similar. When Aristotle says that larger is said to be larger with respect to something else, he means that “larger” in its use implies a reference to something else. It is also entailed by this interpretation that the expression “τοῦθ’ ὅπερ ἐστίν,” which occurs in the definition of relatives and in texts (B) and (C), does not hint at metaphysical entities such as essences, but has a plain meaning. Larger is said to be τοῦθ’ ὅπερ ἐστίν, that is, is said to be larger with respect to something else. The example of τὸ ὁμοῖον in (C) shows clearly that this is the meaning we have to give here to this expression Aristotle uses elsewhere to refer to essences.³

The view that “larger” means larger than something else might sound strange to us. One of the reasons for our feeling uneasy with it is that we are trained by modern logic to distinguish sharply between properties and relations, and we are inclined to take an expression such as “*a* is larger” as an incomplete proposition that cannot be said to be either true or false. Aristotle’s view is different. According to him, “*a* is larger” is a proposition in which a relative property is involved. Because “being larger” means “being larger than something,” the proposition “*a* is larger” means “*a* is larger than something.” Therefore, “*a* is larger” may be said to be true or false in the same way as “*a* is larger than something.” Perhaps Aristotle’s position was influenced by examples of relatives such as “slave” in which there is no incompleteness of meaning when they are predicated of something.

To sum up, I take Aristotle’s definition of relatives to mean exactly that a property *F* is said to be a relative property if, and only if, it can be expanded into a relation that determines *F* univocally. Let us try to express this point more formally.

Call “P1” the class of relatives identified by Aristotle’s definition and consider a property F . One possible way to express a property formally is by means of the so-called “*lambda*-operator,” namely λ . A term of the form $\lambda xF(x)$ denotes the class of all x that satisfy $F(x)$. Let $\lambda x\lambda yR_F(x, y)$ be a relation that determines a relative property F , for instance, “to be a slave of” with respect to the relative property “being a slave.” Let us call it the “constitutive relation” of the property. Then Aristotle’s definition can be formalized as follows:

$$(1) \quad \lambda xF(x) \in \text{P1} \text{ iff } \lambda xF(x) = \lambda x\exists yR_F(x, y).$$

Of course, “ $\lambda x\exists yR_F(x, y)$ ” can be a very complex term – for instance, “ $\lambda x(\exists yS(x, y) \wedge G(x) \wedge \dots \wedge T(x))$.” What is essential is that there must be a reference to a specified relation that alone or together with other properties characterizes the relative at issue.

It is quite obvious that the identity that appears in (1) cannot be interpreted as extensional equivalence.⁴ If it were so, any property with the same extension as $\lambda xF(x)$ would be a relative property, and that is false. Suppose that “ $\lambda xF(x)$ ” stands for “being a lover.” This property is surely a P1-property: being a lover is being a lover of someone or something. Suppose now that all men with dark eyes, and no others, are lovers and that “ $\lambda xG(x)$ ” stands for being a man with dark eyes. By hypothesis $\lambda xF(x) = \lambda xG(x)$. If the identity in (1) was interpreted as extensional equivalence, one could infer $\lambda xG(x) = \lambda x\exists yR_F(x, y)$ and therefore conclude that $\lambda xG(x)$ is a P1-property. But of course “being a man with dark eyes” is not a relative property. Thus, the identity in (1) has to be taken not extensionally but intensionally. We can stress this important fact by pointing out that the relation $\lambda x\lambda yR_F(x, y)$, which is used for defining the relational property $\lambda xF(x)$, is a well-determined relation, the choice of which depends intrinsically on F . Consequently, for different F s different relations have to be taken. Even if it holds extensionally that $\lambda xG(x) = \lambda x\exists yR_F(x, y)$, it does not follow that $\lambda xG(x)$ is a P1-property, because identity interpreted as extensional equivalence is not sufficient to guarantee that the property is intrinsically linked to its constitutive relation.⁵

II

Aristotle does not clarify the nature of the link between a relative property and its constitutive relation. As we have seen, it is surely an intensional connection that involves the senses both of the property and of the relation. But how senses are implied is not explicitly stated by him. Shall we leave the problem here? Perhaps progress can be made if the definition of P1-relatives at the beginning of *Categories* 7 is compared with another definition of relatives. The latter is discussed at the end of the same chapter. Let us call the relatives characterized by this second definition “P2-relatives.”

The demand for a new definition of relatives comes from the problem raised by parts of secondary substances. Both primary substances – individuals like Coriscus and Callias – and secondary substances – species and genera such as dog and

animal – have parts. In the first case, the parts are individual parts of substance (for instance, this hand or this head); in the second, we are dealing with nonindividual parts such as being a hand or being a head. The following problem comes to light. Primary and secondary substances are not P1-relatives, and the same is true of parts of primary substances (*Categories* 7, 8a13–24). But parts of secondary substances seem to be relative, at least in the sense of P1-relatives. A head is the head of someone and a hand is a hand of someone. If “ $\lambda xT(x)$ ” stands for being a head, we are allowed to assert

$$(2) \quad \lambda xT(x) = \lambda x\exists yR_T(x, y)$$

and therefore to conclude that $\lambda xT(x)$ is a P1-property (*Categories* 7, 8a24–28). But Aristotle does not want to draw this conclusion. If parts of secondary substances were relative, it would be difficult to maintain that secondary substances are not relative; and if secondary substances are relative, primary substances have to be admitted to be so. A major commitment of Aristotle’s ontology would be compromised. Aristotle points out that it will not do to deny that parts of secondary substances are P1-relative. According to him, another strategy must be followed, consisting of restricting the initial definition of relatives. If some other condition is added to it, one would perhaps be able to exclude the parts of secondary substances from the class of relatives (*Categories* 7, 8a28–33). The problem is clearly stated by him. A condition must be given that allows us to isolate a subclass of the class of P1-relatives in which the parts of secondary substances are not contained (*Categories* 7, 8a33–35). It follows immediately from this way of putting things that P2-relatives are conceived of as a proper subclass of P1-relatives: Every P2-relative is also a P1-relative, but there are P1-relatives that are not P2-relatives.

After this long introduction, we should be in a position to assess Aristotle’s new definition of relatives. But his words are rather disappointing and obscure. He says:

(D) those things are relative for which being is the same as being somehow related to something (τὸ εἶναι ταὐτόν ἐστι τῷ πρὸς τι πῶς ἔχειν).

(Aristotle, *Categories* 7, 8a31–33; J. Ackrill translation)

If we had only this statement at our disposal for building an interpretation of the meaning Aristotle assigns to P2-relatives, it would be very difficult to understand what he wants to say. Some authors think that internal relations are hinted at by the new definition, but I do not believe that this view is tenable.⁶ “Head” is not a P2-relative term; on the other hand, it can hardly be denied that a head has an internal relation to what it is the head of if “internal” means “essential” or “necessary.”⁷ According to Aristotle, a dead man is not a man (*On Interpretation* 11, 21a22–23) and a part of a man, such as an eye or a finger or a head, cannot be severed from him without ceasing to be an eye or a finger or a head (*Metaphysics* 7.10, 1035b23–25; *On the Soul* 2.1, 412b19–22). On the other hand, can we accept

the traditional interpretation that distinguishes the first from the second definition on the basis that the first refers to what is said and the second to what is?⁸ But if so, to what extent is this view consistent with Aristotle's claim that P2-relatives constitute a proper subclass of the class of P1-relatives? What does it mean to say that there are relatives *secundum dici* that are not relatives *secundum esse*, and why is "head" a relative *secundum dici* but not *secundum esse*, while "double" is both *secundum esse* and *secundum dici*? It is difficult to give satisfactory answers to these questions. The point of asking them is to show that the meaning of "being" in the second definition is far from clear, and without further information from Aristotle, it is useless to try to solve the problem.⁹

However, Aristotle considers what he says to be a consequence of the second definition, which perhaps helps clarify its distinction from the first definition. This consequence is strongly tied to the definition of P2-relatives, as is shown by the fact that it is mentioned in one of the two other places in which P2-relatives are considered, namely in *Topics* 6.4, 142a26–31.¹⁰

III

After giving the second definition of relatives that we have already discussed, Aristotle says:

(E) It is clear from this that if someone knows any relative definitely (εἰδῆ τι ὁρισμένως τῶν πρὸς τι) he will also know definitely that in relation to which it is spoken of (κακεῖνο πρὸς ὃ λέγεται ὁρισμένως εἴσεται).

(Aristotle, *Categories* 7, 8a35–37; J. Ackrill translation)

A preliminary remark must be made before we attempt an interpretation of the whole passage. One might ask what the expression "εἰδέναι ὁρισμένως," which occurs twice here, and frequently in the context, means. I suspect that the adverb "ὁρισμένως" does not play a very important role, a suspicion that is confirmed by the fact that it does not occur in the parallel passage of *Topics* 6.4, 142a26–31. I am inclined to believe that it should be explained by reference to the point Aristotle makes at 8b9–15. But I will return to this question later. For the time being, it is enough to observe that it is not necessary to take the presence of ὁρισμένως as support for the traditional interpretation of the text. Many scholars think that what is meant here, or at least implied, is that knowing that *a* is *F*, where *F* stands for a P2-relative, entails that *one knows the precise b* to which *a* is related.¹¹ For instance, if I know that *a* is double, I must also know the precise number *b* of which *a* is the double.¹²

I do not believe that this view can be accepted. It is so obviously false that it is difficult to believe that Aristotle could have been committed to it. Take a number, say 1,515,798. I can immediately state that it is the double of some number, even if I do not know exactly what this number is. In order to say that it is the double of 757,899 I need to calculate it. Nevertheless, before carrying out the calculation, I can assert that the number at issue is the double of some number. Analogous

counterexamples to this interpretation can be found by considering other instances of P2-relatives, such as “son” or “slave.” There are surely cases in which one can know that *a* is a slave without knowing exactly of whom *a* is a slave. Of course, the fact that a view is false is not a sufficient reason for denying that it is held by Aristotle. Even a great philosopher can be guilty of absurdities. But charity requires us to attribute to him an absurd view only if there is no alternative. My claim is that there is an alternative in this case. The adverb “ὀρισμένως” supports the traditional interpretation only if it is taken to specify what is known. But it could also be taken to specify the epistemic state, for instance, “definite” and “indefinite understanding.” When I will comment on *Categories* 7, 8b7–15, I will try to substantiate this interpretation.

The traditional interpretation could be defended by pointing out that in (E) Aristotle says (or at least implies) that if one knows that *a* is *F* (*F* being a P2-property), then one must know that to which *a* is related. But “to know that to which *a* is related” can only mean “to know the entity (entities) to which *a* is related.” Therefore, even without relying on the meaning of “ὀρισμένως,” one could hold that Aristotle’s words provide plain evidence for the traditional view. Is that so? Let us consider the point carefully. Suppose that *a* is a slave of *b*. If we are acquainted with this fact, we surely know of whom *a* is a slave and therefore that to which *a* is related by means of the relation “being a slave of.” The problem is whether we know that to which *a* is related by means of the relation “being a slave of” only if we know of whom *a* is a slave. If this question is answered positively, the traditional interpretation becomes inescapable and Aristotle is guilty of holding an absurd view. But I am not sure that we are compelled to give a positive answer to the question. We are compelled if we take the expression “know *x*” in the sense of “be acquainted with *x*.” But I do not see any compelling reason for interpreting the verb “to know” in (E) in this way. It seems to me quite reasonable to take “to know that to which *a* is related” to mean “to have a piece of information about that to which *a* is related” or, if you prefer, to know something about that to which *a* is related. Then we cannot know that *a* is a slave without having some piece of information about the person whose slave *a* is. The minimal piece of information required to know that *a* is a slave is that we know that there is someone to which *a* is related by means of the relation “being a slave of.” Then to know that *a* is a slave implies at least to know that the person of whom *a* is a slave exists. Of course, we often know more about the master of *a*. Perhaps we are able to describe, and to identify, this person on different occasions. But sometimes all we know about him is that he exists, and that is enough to allow us to say that *a* is a slave. It is with reference to this minimal knowledge of that to which *a* is related that I will try to formalize Aristotle’s definition of P2-relatives.

Before doing that, another preliminary question, which is only apparently naïve, must be addressed. Aristotle speaks of knowledge of a relative. As we have seen, a relative is a property. Now it is more or less clear what is meant by saying that one knows a proposition or even an individual thing. But what does knowledge of a property amount to? As far as I can see, there are three main possible interpretations. The first can be easily dismissed. It consists in assuming that “knowing a

relative property" means knowing its definition. If this view is adopted, the thesis expressed in (E) might be formalized in the following way. Suppose that $\lambda xF(x)$ is a P2-property, that " $C(x, y)$ " stands for " y is the definition of x ," that " t " is a term, and, finally, that " n knows that A " (" n " standing for an individual knower and " A " for a proposition) is expressed by " $K_n(A)$." Then Aristotle's claim could be represented by the following formula:

$$(3) \quad K_n(C(\lambda xF(x), t)) \rightarrow K_n(t = \lambda x\exists yR_F(x, y)).$$

But I do not believe that this position is tenable. According to Aristotle, what is said by (3) should be such that it could be applied to every P2-relative but not to all-relatives. In particular, (3) should not hold for parts of secondary substances, such as being a head. One should therefore be able to state a definition of "being a head" without mentioning, or even knowing, that it is intrinsically linked to something by a constitutive relation. But a definition of "being a head" in which no reference is made to its being a part of a living body can hardly be thought to be adequate by Aristotle's standards. Remember that, for him, a head severed from its body is no longer a head and therefore no longer satisfies the definition of being a head, because two homonymous properties do not have the same $\lambda\acute{o}\gamma\omicron\varsigma\ \tau\eta\varsigma\ \omicron\upsilon\sigma\acute{\iota}\alpha\varsigma$ (*Categories* 1, 1a1–6). It is true, however, that in *Topics* 6.4, 142a26–28 Aristotle says that being a double cannot be defined without being a half. From this one can infer that, according to him, if $\lambda xF(x)$ is a P2-relative, then $\lambda x\exists yR_F(x, y)$ must be included in its definition. But that does not prove that the same is true in the case of P1-relatives. Charity requires us to attribute to Aristotle a view that is obviously false only if it is explicitly stated by him or at least is an immediate consequence of what he says.¹³

There are two other possible interpretations of the expression "knowing a relative property" that must be considered. According to one of them "to know a property F " means "to know of F that" In the other view, our expression amounts to "to know that F is true of" Although many scholars prefer the latter interpretation without even considering the former, I think that the problem is worth examining. Let us first try to clarify the different interpretations of (E) that the two options make possible and compare the strength of the resulting theses attributed to Aristotle. According to the first hypothesis ("to know F " = "to know of F that . . ."), Aristotle's statement could be translated as follows:

$$(4) \quad K_n(\phi(\lambda xF(x))) \rightarrow K_n(\phi(\lambda x\exists yR_F(x, y)))$$

where " $\lambda xF(x)$ " is a P2-property and " $\phi(x)$ " is a property of properties. If " $\lambda xF(x)$ " is taken to stand for being double – a P2-property in Aristotle's view – then (4) means that if n knows that being double has a property ϕ , it follows that n knows also that being double of something is ϕ . A different result is obtained if one assumes that "to know F " means "to know that F is true of" Aristotle's thesis then becomes:

$$(5) \quad \forall x(K_n(F(x)) \rightarrow K_n(\exists yR_F(x, y))).$$

What (5) says is that if any a is known to have a relative property, for instance, a is known to be a slave, then a is known to be related to something by the constitutive relation of the property itself. In our example, it is known that a is a slave of someone.

We are tempted to prefer (5) to (4). But one must be suspicious of temptations of this kind, because they depend on our acquaintance with first-order logic, and we are not allowed to transfer our own logical habits to Aristotle. In order to choose between (4) and (5) we must look at the texts and see which of them fits Aristotle's words better. This will be a long and, I am afraid, boring job. In order to prepare for (but also to delay) it, let us make some remarks about the different strengths of the two formulas and their philosophical meaning. It is easy to see that (5) is a particular case of (4). To show this, it is sufficient to take as ϕ the property of being true of a . Then if $\lambda x F(x)$ is known to have the property of being true of a , $\lambda x \exists y R_F(x, y)$ will also be known to have it. But this is precisely what (5) states. What then does (4) fully, and (5) partially, assert? We can try to explain this by reference to the identity that is involved in the definitions of relatives. Take a P2-relative, say $\lambda x F(x)$. Because it is a P2-relative by hypothesis, it is also a P1-relative. Consequently it satisfies (1) and therefore

$$(6) \quad \lambda x F(x) = \lambda x \exists y R_F(x, y)$$

holds. It is well known that the simple fact that $t = s$ does not allow us to substitute " t " for " s " in every context. Even if it is true that

Tully = Cicero

from

n knows that Cicero denounced Catiline

it does not follow that

n knows that Tully denounced Catiline.¹⁴

Now (4) expresses a particular case of the thesis that $\lambda x \exists y R_F(x, y)$ can be substituted for $\lambda x F(x)$ within the scope of the verb "to know." The same is said by (5), but with the restriction that substitution inside the scope of to know is admitted only with reference to the property of being true of something. From this point of view what is stated in text (E) contributes to an understanding of the definition of P2-relatives that, taken by itself, sounds quite mysterious.

IV

The last remarks of the preceding section have a twofold result. First, they show that the philosophical questions involved in Aristotle's view are far from being

trivial or unimportant. The question of substitution in cognitive contexts has been a subject of lively philosophical debate for many years, and no general agreement on the basic issues has been reached yet.¹⁵ It might therefore be interesting to consider Aristotle's position in order to get new material for the discussion. Second, the contrast between (4) and (5), which at first sight seemed striking, is softened by the observation that (5) can be taken as a particular case of (4). Even if one chooses the wrong one, it does not follow that the resulting interpretation will be completely useless.

However, the question of the aptness of (4) or (5) as interpretations of Aristotle's words still has to be examined. Let us first quote the passage that immediately follows text (E). Aristotle says:

(F) This is obvious on the face of it. For if someone knows of a certain "this" (τόδε τι) that it is one of the relatives, and being for relatives is the same as being somehow related to something, he knows that also to which this is somehow related. For if he does not in the least know that to which this is somehow related, neither will he know whether it is somehow related to something.

(Aristotle, *Categories* 7, 8a37–b3; J. Ackrill translation, slightly modified)

The reference of the expression "a certain this" is the key problem for the interpretation of the whole passage. It is very natural to suppose that it refers to an individual thing, say a . Because relatives are properties, to say that "a certain this is one of the relatives" (τόδε τι ὅτι τῶν πρὸς τί ἐστιν) can only mean that a has a relative property. That may sound difficult, but it is confirmed by 8b4–5, where a particular case is given by means of the expression "τόδε τι . . . ὅτι ἔστι διπλάσιον." Here "τόδε τι" surely refers to an individual and "διπλάσιον" exemplifies one τῶν πρὸς τι. The last sentence of the passage must be given a corresponding interpretation: "neither will he know whether it is somehow related to something" (οὐδ' εἰ πρὸς τί πως ἔχει εἴσεται: 8b2–3) expresses in general what "neither does he know whether it is double at all" (οὐδ' εἰ ἔστι διπλάσιον ὅλως οἶδεν) says with reference to the case of "double," and it has to be taken to mean "he will not know whether a has a relative property." Aristotle's claim in the first part of the passage is, then, that if someone (our Mr. n) knows that an individual thing, a , has a relative property, $\lambda xF(x)$, and this property is a P2-property, then n also knows that to which a is somehow related. As we have seen, the minimal interpretation of " n knows that to which a is related" is that n knows that there is something to which a stands in the relation that constitutes $\lambda xF(x)$. Therefore, a sensible formalization of the first part of our passage is:

$$(7) \quad K_n(F(a)) \rightarrow K_n(\exists y R_F(a, y)).$$

The second part of the passage (8b2–3) does not add any relevant information, because it simply states the contrapositive of (7):

$$(7') \quad \neg K_n(\exists y R_F(a, y)) \rightarrow \neg K_n(F(a)).$$

If n does not know that there is something to which a is related, n does not know that a has the corresponding relative property.

As we have already indicated, Aristotle continues by giving some examples of his main thesis. He says:

(G) The same point is clear also in particular cases. For example, if someone knows definitely of a certain “this” that it is double he also, by the same token, knows definitely what it is double of; for if he does not know it to be double of some individual neither does he know whether it is double at all.

(Aristotle, *Categories* 7, 8b3–7; J. Ackrill translation, slightly modified)

By itself the passage is clear. What Aristotle has stated in text (F) is here restated with reference to the P2-relative property double. He says that if n knows that a is a double, then n must also know that of which a is the double. According to our interpretation that means that n must know at least that there is something of which a is the double. Therefore, if “ $D(x)$ ” stands for “ x is a double,” the first part of the passage can be formally interpreted as follows:

$$(8) \quad K_n(D(a)) \rightarrow K_n(\exists y R_D(a, y)).$$

It is easy to see that the structure of text (G) strictly parallels that of (F). As in the case of (F), two sections can be identified in our passage. The first (8b4–6) asserts (8), which corresponds to (7) in (F). Lines 8b6–7, the second section of (G), have the same role as lines 8b2–3 in (F). Because they express the contrapositive of (7), it must be concluded that 8b6–7 states the contrapositive of (8):

$$(8') \quad \neg K_n(\exists y R_D(a, y)) \rightarrow \neg K_n(D(a)).$$

With this in mind, let us turn to the Greek text in order to solve a question that, although philological in essence, has a philosophical interest.

The two oldest manuscripts, *Ambrosianus* L93 (IX cent.) and *Marcianus* 201 (X cent.), offer different readings of 8b6. The *Marcianus* has

$$(m) \quad \epsilon\iota \gamma\alpha\rho \mu\eta\delta\epsilon\nu\omicron\varsigma \tau\omega\nu \acute{\alpha}\phi\omega\rho\iota\sigma\mu\acute{\epsilon}\nu\omega\nu \omicron\iota\delta\epsilon\nu \alpha\upsilon\tau\omicron \delta\iota\pi\lambda\acute{\alpha}\sigma\iota\omicron\nu$$

whereas the *Ambrosianus* gives:

$$(a) \quad \epsilon\iota \gamma\alpha\rho \mu\acute{\eta} \tau\iota\nu\omicron\varsigma \tau\omega\nu \acute{\alpha}\phi\omega\rho\iota\sigma\mu\acute{\epsilon}\nu\omega\nu \omicron\iota\delta\epsilon\nu \alpha\upsilon\tau\omicron \delta\iota\pi\lambda\acute{\alpha}\sigma\iota\omicron\nu.$$

Text (m) is preferred by all modern editors, and almost all translators presuppose it evidently for reasons depending on sense.¹⁶ In themselves, both (m) and (a) are ambiguous, because the scope of the negation operator is not clearly fixed. Text (m) can mean:

$$(m1) \quad \text{one knows that none of the individuals . . .}$$

or

(m2) of none of the individuals one knows that

Similarly, there are two possible translations of (a):

(a1) one knows that some of the individuals are not . . .

or

(a2) it is not the case that one knows that some of the individuals

Consequently, there are two formalizations for each reading:

(M1)	$K_n(\neg \exists y R_D(a, y))$	corresponds to	(m1)
(M2)	$\neg \exists y K_n(R_D(a, y))$	corresponds to	(m2)
(A1)	$K_n(\exists y \neg R_D(a, y))$	corresponds to	(a1)
(A2)	$\neg K_n(\exists y R_D(a, y))$	corresponds to	(a2)

(M1) and (A1) are ruled out by the context in an obvious way. In fact, almost no translator adopts (M1), whereas (A1) is ignored because none of them uses text (a).¹⁷ For the most part they say something that more or less corresponds to (M2).¹⁸ On this view, then, Aristotle says something that can be formalized as follows:

(8'') $\neg \exists y K_n(R_D(a, y)) \rightarrow \neg K_n(D(a))$.

(8'') is surely meant to be an explanation of (8). Therefore, (8) must be derivable from (8''). And it is derivable on condition that something like

(9) $\exists y K_n(R_D(a, y)) \rightarrow K_n(\exists y R_D(a, y))$

is implicitly stated. In fact, (9) is quite a reasonable assumption, whereas its converse is very doubtful.¹⁹ But Aristotle does not make any assertion that corresponds to (9), and one may well wonder why such a complicated detour needs to be made. It would be much easier and more straightforward to read (a) with the *Ambrosianus*, to give it the meaning expressed by (A2) and then to construe consequently (8') as an explanation of (8) in the way in which (7') is meant to confirm (7).

If the interpretation of texts (F) and (G) that we have proposed is accepted, the choice between (4) and (5) can easily be made. It is clear that (F) and (G) are intended to confirm the main thesis proposed in (E). Suppose that what is stated in (E) is formalized by (4). Then (F) and (G) must be taken to be, at most, a partial confirmation of (E), because we are supposing that they give an argument for a special case of (4), namely the case represented by (5). On the other hand, if (5) is considered the better formalization of (E), then (F) and (G) can be taken to offer an argument (whatever its force may be) for Aristotle's thesis as a whole. It is

therefore preferable to suppose that what is meant in (E) is something like what is expressed by (5). Even if Aristotle had in mind in (E) what we have represented by means of (4), the part of (4) that interests him and on which he concentrates his efforts is (5). Therefore, we must take (5) as the best candidate for expressing his thesis.

V

Aristotle next gives an example that strictly parallels the example stated in text (G). He says:

(H) Similarly, if he knows of a certain “this” that it is more beautiful, he must also, because of this, know definitely (ὠρισμένως) what it is more beautiful than. (He is not to know indefinitely (ἀορίστως) that this is more beautiful than an inferior thing. This is a case of supposition (ὑπόληψις), not knowledge. For he will no longer strictly (ἀκριβῶς) know that it is more beautiful than an inferior thing, since it may so happen that there is nothing inferior to it.) It is plain, therefore, that anyone who knows any relative definitely (ὠρισμένως) must know definitely (ὠρισμένως) that also in relation to which it is spoken of.
(Aristotle, *Categories* 7, 8b7–15; J. Ackrill translation)

The general meaning of this passage does not add anything to what we already know. The example shows once more that Aristotle does not make any distinction between sentences such as “ a is a slave” and “ a is more beautiful,” where we would be prepared to give to the first, but not to the second, the status of a proposition. What is interesting, however, is the remark in the parenthetic sentence (8b9–13), for on it depends our understanding of the adverb “ὠρισμένως” (or “ἀφορισμένως”) with which Aristotle specifies the kind of knowledge with respect to which (5) is asserted to hold. We must first consider the traditional interpretation, which has already been shown to be unnecessary. As we have said, the received view assumes (i) that knowing a relative definitely means knowing that a specified individual, say a , has a relative property F and (ii) that knowing definitely that to which a is related means knowing a specified b to which a stands in the constitutive relation characteristic of F . Therefore, Aristotle would state:

$$(10) \quad K_n(F(a)) \rightarrow K_n(R_F(a, b)).$$

Now if (10) holds, one could assume that n 's universe of knowledge is closed under elementary logical consequences and therefore say that if n knows that $F(a)$, he also knows that $\exists y R_F(a, y)$ because this is a logical consequence of $R_F(a, b)$. This view is not inconsistent with the interpretation we are discussing. It can be still maintained that $K_n(\exists y R_F(a, y))$ is not in itself definite knowledge, because the precise individual to which a stands in the relation R is not mentioned. Aristotle would simply rule out that n can know $\exists y R_F(a, y)$ starting from knowledge of $F(a)$ and without first knowing $R_F(a, b)$. In other words, a situation such as

$$(11) \quad K_n(F(a) \wedge \exists y R_F(a, y)) \wedge \neg K_n(R_F(a, b))$$

would be rejected by Aristotle. But let us look at the reasons he gives and try to explain by means of them why (11) does not hold. If $\neg K_n(R_F(a, b))$ obtains, $K_n(F(a) \wedge \exists y R_F(a, y))$ cannot be qualified as real knowledge in the case in which there is nothing to which a is related by $\lambda x \lambda y R_F(x, y)$. But that is nonsense. There is no reason for saying that $\neg \exists y R_F(a, y)$ can be true together with $K_n(F(a) \wedge \exists y R_F(a, y))$, unless one already presupposes that “ K_n ” in this formula refers not to knowledge, but to belief. In this way, Aristotle's argument disintegrates into a *petitio principii*.

We must look for a different interpretation. Aristotle does not explain what definite knowledge is. However, he contrasts knowing $\acute{\omega}\rho\iota\sigma\mu\acute{\epsilon}\nu\omega\varsigma$ with what he qualifies as $\acute{\alpha}\omicron\rho\acute{\iota}\sigma\tau\omega\varsigma$ εἰδέναι (8b9–10), which we might call “indefinite understanding.” Because he gives some characterization of the latter, we can hope by examining it to get some insight into the former. What does Aristotle say, then, about indefinite understanding? First of all (i) that to understand A indefinitely (where “ A ” stands for a proposition) is a supposition (ὑπόληψις) (8b10–11). Furthermore, (ii) to understand A indefinitely is not yet strict knowledge (8b11–12). Finally, (iii) an indefinite understanding of A is consistent with $\neg A$ (8b12–13). It seems to me quite natural to think that indefinite understanding would be less mysterious if we called it “belief.” In fact, a belief can be qualified as a supposition in Aristotle's sense,²⁰ and it becomes knowledge only if certain conditions are fulfilled. Finally, it can surely happen that one believes that A without its being the case that A . Moreover, condition (ii) specifies the kind of belief that is in question here. It might be said that if one knows that A , one also believes that A . Condition (ii) allows us to state that it is not this kind of belief that Aristotle is considering. The belief he is speaking of here is a belief that is not accompanied by knowledge. It is rather the state one is in when one believes that A without knowing that A . Conditions (i) and (iii) are important for a more precise understanding of the meaning of “knowing definitely.” Knowing definitely cannot be classified as a supposition. Because supposition is contrasted with ἐπιστήμη (8b10–11), it can be concluded that knowing definitely is a kind of ἐπιστήμη, whose result is εἰδέναι ἀκριβῶς (8b11–12). I do not think that ἐπιστήμη has the technical meaning with which a reader of the *Analytics* is acquainted. Evidence in support of this is the fact that singular propositions are here said to be known definitely, whereas ἐπιστήμη of the individuals is not admitted in the *Analytics* and elsewhere (*Posterior Analytics* 1.31, 87b33–39. Cf. *Metaphysics* 7.15, 1039b27ff.). I suspect that condition (iii) gives the key to understanding the meaning that must be assigned to ἐπιστήμη and εἰδέναι ὥρισμένως here. This condition says that simply believing that A is consistent with $\neg A$. We might then suppose that there is ἐπιστήμη or definite knowledge when one knows that A and this assumption is not consistent with $\neg A$, that is,

$$(12) \quad K_n(A) \rightarrow A.$$

If this interpretation is accepted, two remarks at once suggest themselves. First of all, the adverb “ὠρισμένως” (or “ἀφορισμένως”) in “ὠρισμένως (ἀφορισμένως) εἰδέναι” does not have a hidden and perverse role, as the traditional interpretation assumes, but simply reinforces “εἰδέναι,” so that the whole expression denotes real knowing. Second, the general task of lines 8b9–13 becomes clear. They are intended to stress that the consequent of (5) cannot be changed from knowledge to belief. If $\lambda xF(x)$ is a P2-property, knowledge of $F(a)$ entails knowledge of $\exists yR_F(a, y)$ because it cannot be the case that one knows that $F(a)$ and merely believes that $\exists yR_F(a, y)$. Aristotle makes his point by considering the following case. Take $P(x)$ to stand for “ x is more beautiful” (where, of course, being more beautiful is a P2-property) and assume that n knows (definitely) that $P(a)$, but only believes and does not know that $\exists yR_P(a, y)$. What Aristotle aims to prove is that this situation cannot occur. In other words, he rejects the formula

$$(13) \quad K_n(P(a)) \wedge B_n(\exists yR_P(a, y))$$

where “ $B_n(A)$ ” stands for “ n believes that A without knowing that A .” His argument reduces to the obvious remark that $B_n(\exists yR_P(a, y))$ is consistent with the assumption that $\neg\exists yR_P(a, y)$ (8b12–13). It is difficult to extract from this a full proof of the falsity of (13). One possible reconstruction of it goes like this. A *reductio ad absurdum* might be imagined to take place. Suppose that (12) holds. Because $B_n(\exists yR_P(a, y))$ is said to be consistent with $\neg\exists yR_P(a, y)$, one might be tempted to derive from this statement and (13) that

$$(14) \quad K_n(P(a)) \wedge \neg\exists yR_P(a, y)$$

holds good. Because real knowledge of $P(a)$ is attributed to n and (12) holds in general, one can infer from (12) and (14) that

$$(15) \quad P(a) \wedge \neg\exists yR_P(a, y)$$

is true. But this is impossible, because $\lambda xP(x)$ is by hypothesis a P2-property. Therefore,

$$(16) \quad P(a) \rightarrow \exists yR_P(a, y)$$

must be admitted to hold always. But (16) is the negation of (15). Thus, a contradiction arises, and (13) has to be rejected. Unfortunately, this argument is wrong because (14) does not follow from the assumption that $B_n(\exists yR_P(a, y))$ is consistent with $\neg\exists yR_P(a, y)$ and from (13). In general, one cannot infer that it is possible that p and r are true from the assumptions that both p and q are true and that q is consistent with r .²¹ Perhaps there are other possible ways of reconstructing Aristotle’s argument that avoid the flaw present in our interpretation, but I confess that I am not able to see any of them. Therefore, a suspicion of inconsistency hovers over the passage.

VI

Aristotle next states an essential point for his main argument: Although P2-relatives satisfy (5), the parts of secondary substances do not. He says:

(J) But as for a head or a hand or any such substance, it is possible to know it – what it itself is – definitely, without necessarily knowing definitely that in relation to which it is spoken of. For whose this head is, or whose the hand, it is not necessary to know definitely. So these would not be relatives. And if they are not relatives it would be true to say that no substance is a relative.

(*Categories* 7, 8b15–21; J. Ackrill translation)

Let me say immediately that the supplement proposed by Ackrill in lines 18–19 seems to me inescapable, and I read with him οὐκ <ἀναγκαῖον> εἰδέναι ἔστι ὁρισμένως.²² The general meaning of the passage is clear. Take $T(x)$ to stand for “ x is a head.” As we have seen, $\lambda xT(x)$ is a P1-relative. Therefore, (2) holds and

$$(17) \quad T(a) \rightarrow \exists y R_T(a, y)$$

is *a fortiori* true. But even if (17) is stated, $K_n(\exists y R_T(a, y))$ cannot be inferred from $K_n(T(a))$. Consequently, substitutivity in the scope of the verb “to know” does not apply to parts of secondary substances. Because (5) is immediately entailed by the definition of “being a P2-relative,” if secondary substances do not satisfy (5), they do not satisfy the definition either. Therefore, if “being a relative” is taken to mean being a P2-relative, it can be concluded that parts of secondary substances are not relative and in general that no substance falls in the category of relatives. The meaning of Aristotle's claim is not difficult to understand. What is difficult is to justify his view or to imagine a set of plausible reasons that might have led him to assert what he asserted. Unfortunately, no hint comes from the text, or at least I have not been able to discover one. We have to be content with speculation.

Let us discuss a difficulty to help us along. One might say that to know that $T(a)$ implies knowing what a head is, namely knowing a definition of being a head. But any reasonable definition of “being a head” must contain a reference to the fact that a head is somehow related to a living body. If we take this relation as the constitutive relation of being a head, it is natural to conclude that knowing that a is a head implies knowing that there is something to which a is related by means of the relation that constitutes the property of being a head. Therefore (5) is satisfied even by parts of secondary substances.

I agree that a reference to something else has to be included in the definition of being a head, but it is far from clear to me that knowing that $T(a)$ implies knowing the definition of being a head. In general, it is plausible to say that in some cases one can know that an individual a has a property F without possessing an exact definition of F . I can truly say that I know that a is a dolphin without being able to give the zoological definition of this mammal. I am inclined to think that

knowledge of many general terms, which must be involved in knowledge of propositions of which they are constituents, does not consist in grasping their definitions. In the case of names for natural kinds, for instance, it is probably sufficient to have that vague understanding of them that allows us to identify them in different situations. I am thinking of something like the stereotypes with which Putnam and Johnson-Laird have acquainted us.²³ Their relevant feature is that they do not determine the extensions of the terms to which they are referred, but only permit us, at least in most cases, to say that a term is used with the same meaning even in different contexts.

An example will make the point a little clearer. Consider, for instance, the term “fish.” It is well known that its extension has changed, and therefore so has its definition. Before Linnaeus, Cetacea were placed among fish. Even Linnaeus, in the first editions of his *Systema Naturae*, held the same view. Only in the tenth edition of his work did he finally include Cetacea among mammals.²⁴ Shall we conclude that the term “fish” has changed its meaning from the first to the tenth edition of Linnaeus’s work? Of course, the answer to this question must be affirmative if we are inclined to think that the meanings of terms determine their extensions. But this assumption is far from obvious and it has been questioned.²⁵ I cannot discuss this complicated issue here. I limit myself to pointing out that it is hard to believe that the scientific discovery that led Linnaeus to classify Cetacea among mammals did modify his linguistic competence. Even if a zoologist adjusts his definition of fish to accord with new observations, it does not follow that the meaning that he assigns to the term “fish” changes correspondingly. The proposition “Cetacea are fish” that we find in the first edition of the *Systema Naturae* can be compared with the statement contained (at least by implication) in the tenth edition: “Cetacea are not fish”; and it is reasonable to say that the last statement is inconsistent with the first. But these propositions can be compared and said to be inconsistent only if their terms are supposed to have the same meaning. If different meanings are attributed to “fish,” no logical relation can be established between the propositions, and one cannot be said to be the negation of the other. If we are inclined to believe that Linnaeus changed his mind about the zoological objects and not about the meaning of words, it becomes quite useful to link meanings to stereotypes. This allows us to identify general terms in contexts where different definitions of them are given. My point is that meanings do not always determine extensions of general terms. Therefore, meanings cannot be identified with definitions, because these do the job of fixing the extensions of the terms to which they apply. What does correspond to meanings, then, from a psychological point of view? Because I do not want to be involved in the questions concerning intensions, I have used Putnam’s expression “stereotype,” but I am not committed to defend it in detail. What is relevant for me is the claim that knowledge of meanings does not always reduce to knowledge of definitions. The objection is that in knowing that $T(a)$, only an understanding of the meaning of “head” is involved, namely an understanding of the stereotype that determines the meaning of this term. It is therefore with reference to such a notion that it must be asked whether knowing that $T(a)$ implies knowing that $\exists y R_T(a, y)$.

Let us now compare the case of a P2-relative, for instance, “ a is big,” $G(a)$, and the case of parts of secondary substances, for instance, “ a is a paw,” $Z(a)$. Aristotle's claim is that $K_n(G(a))$ implies $K_n(\exists y R_G(a, y))$, whereas $K_n(Z(a))$ does not entail $K_n(\exists y R_Z(a, y))$. I am not sure that this point can be defended in every case, but I hope to make it at least plausible by means of the following remarks. Suppose that I do not know whether there is anything to which a is related. Can I reasonably assert that a is big? I am inclined to think that it is only by putting a in relation to something else that I can say that a is big, and the relation that has to be found between a and something else must be the relation of “exceeding a standard size” or something of the like. Similarly, it would be difficult to establish that a is a slave without knowing that a is somehow related to something else. It could be replied that it might happen that n knows that a is a slave without referring to what a is related to if, for instance, he knows that all branded humans in his country are slaves and he sees a brand on a . But having a brand is not something that is always associated with being a slave. There can be slaves who are not branded and branded humans who are not slaves, and that proves that this feature is not essentially linked to the property of being a slave. Therefore, one could answer that being a branded human is not sufficient to constitute the meaning of “slave,” even if in particular cases of social groups it could happen that all and only slaves are branded humans. But the understanding of being a slave that is implied by knowing that a is a slave cannot be simply that slaves are branded humans; otherwise, we should have to deny that one can come to recognize nonbranded men as slaves without changing one's linguistic competence.

We have now to consider the case of “ a is a paw.” We will perform the following thought experiment. Suppose that a human being (let us call him “Aldous”) has lived since his birth in a hut without being allowed to leave it. Aldous masters the English language and is quite learned. But there is a flaw in his education: He does not know anything about animals. He has never seen them, nor does he have at his disposal words for naming them and their parts. Aldous has no direct contact with the outside world until he discovers a small hole in a wall of the hut through which he can take a look outside. What he can see are the paws of some dogs who move around the hut. Unfortunately, he cannot see anything more than the paws of the dogs. He is fascinated by his discovery and decides to dedicate himself to a systematic observation of these new things that by a lucky coincidence he calls “paws.” After a while he becomes able to distinguish one paw from another (the dogs around the hut are always the same) and assigns names to them, say “ s ,” “ f ,” and “ w .” He then starts to write down his observations and makes a list of propositions expressing what he believes he knows about paws. Of course, Aldous wants to know more about the objects he is carefully observing. He succeeds in making the hole through which he is looking at the paws bigger, and he finally sees the whole dogs. He realizes that the objects that he probably considered before independent items are parts of other objects and that his list of propositions about paws must be largely revised. But suppose that his list contained a proposition such as “ s is a paw.” Must this proposition be changed or rejected? I do not think so. This proves that one can know that s is a paw without knowing that a paw is a part of a living body.

It could be objected to this argument that we could adapt our thought experiment to the case of slaves. Suppose that Aldous can see only human beings who are slaves and that he decides to call them “slaves” without knowing the relation they have to other people. If in his list at this stage the proposition “*k* is a slave” appears, it will not be changed when Aldous knows that *k* is a slave of someone. Therefore, if in the case of paws we are able to conclude that one can know that *s* is a paw without knowing its constitutive relation, the same is true also for the case of slaves. Thus, a counterexample to (5) has been found.

Against this difficulty it can be replied that

(i) *s* is a paw

is different from

(ii) *k* is a slave.

Suppose that both (i) and (ii) are in the lists before revision. According to the objection, they must appear in the revised lists too. But I doubt that this can be maintained in the case of (ii) if ambiguity is to be avoided. My point is that, in the case of (i), the frame that determines the meaning of (i) does not change from the unrevised to the revised list. We are still attributing to *s* the property of being a paw, which keeps the same meaning in both occurrences of (i). On the other hand, it is hard to believe that “slave” in the unrevised list has the same meaning as in the revised one, because it is difficult to grasp what its frame is if no mention is made of its relation to something. The point I am trying to make is that our understanding of what being a paw is can be increased and improved by discovering that being a paw is always being a paw of something. The new piece of information joins what is already known, and is linked to it necessarily, but the new information is not essential for fixing the meaning of “paw.” For instance, Aldous could assign a meaning to “paw” with reference to its shape or to some features of its structure. This characterization remains after the discovery that a paw is a part of a living body, even if it, of course, can be complemented by new elements. In other words, even before knowing that being a paw is a relative, we are able to have a stereotype of this entity that identifies it sufficiently. Can we say the same for being a slave? I do not think so, because I cannot see what might constitute the core that identifies the meaning of “slave,” if the information about its being in relation to something is lacking. How can Aldous determine the meaning of “slave” before having enlarged the hole? He might make it with reference to a brand that the humans he believes to be slaves have, or with reference to the color of their skin, or to the presence of chains on their limbs. But it is easy to see that none of these features gives the real meaning of being a slave. We can always think of slaves without a brand or with a differently colored skin or without chains on their limbs. What really secures the meaning of “slave” is being in a certain relation to something. But, by hypothesis, this is not known to Aldous. Therefore, the meanings of “slave” in (ii) are different before and after revision. If Aldous

uses a nonambiguous language in his lists, he cannot simply transfer (ii) from the unrevised to the revised list. The discovery that people that he calls “slaves” have a constitutive relation to other people forces him to make a revision of the meaning of the word “slave” if he wants to use it in the sense in which it is used in English.

Many problems remain. One concerns the nature and meaning of stereotypes. Can they be conceived in the way in which Johnson-Laird has devised them, namely as frame systems in which default values are given?²⁶ And is this view consistent with Aristotle's doctrine about meanings and concepts? I cannot try to answer these questions here. What my attempt to explain Aristotle's view aims at is to show that his position is far from being trivially false, as it is on the traditional interpretation, and that it can be credited with having some philosophical importance. Moreover, his attempt is stimulating because it approaches a modern problem from a different point of view. Nowadays we are accustomed to consider what is entailed by the fact that substitutivity does not hold in cognitive contexts, and we try to explain why it does not obtain. Aristotle is well aware of these restrictions,²⁷ but he is more interested in isolating cases in which substitutivity can be safely applied. Perhaps this change of perspective may help refresh our own patterns of analysis.

Notes

I am indebted to many friends and colleagues for helpful discussion about the problems dealt with in this chapter. Pierdaniele Giaretta, Paolo Leonardi, and Carlo Natali kindly read a first draft of the chapter and contributed substantially to its improvement and refinement. I benefited also from remarks and suggestions that the editor of *Phronesis* made to me when I submitted the original paper. I am grateful to him because his stimulating criticism made me avoid some errors and helped me better explain my views.

- 1 Simplicius, *On Aristotle's Categories* 159.9–11, points out that it is impossible to give a proper definition of relatives, because they are *summa genera* and a *definiens* must contain the genus of its *definiendum*. But Aristotle calls the characterization of relatives that he gives in text (A) a *ὁρισμός* at *Categories* 7, 8a29 and 33.
- 2 This is also the way in which the *qua*-expression is interpreted by Lear 1982: 168–174.
- 3 Sometimes the expression “ὅπερ ἐστὶ” is used by Aristotle to indicate what a thing is in itself, that is, essentially (for example, *Metaphysics* 4.4, 1007a32–33). But it is quite clear that it does not have this technical meaning here (cf. *LSJ*, s.v. ὅπερ, 5a). For an extended analysis of this notion, see Oehler 1985²: 221–234.
- 4 If the identity contained in (1) were interpreted extensionally, it could be defined as follows:

$$\lambda x F(x) = \lambda x \exists y R_F(x, y) \text{ iff } \forall x (F(x) \leftrightarrow \exists y R_F(x, y)).$$

- 5 Perhaps the flavor of Aristotle's position would be better grasped if we compared it with the view of a modern philosopher. Moore 1978⁶ tries to characterize what he calls “relational properties” as follows. According to him, a property such as “to be white” is a relational property “because being of some particular shade of white is so. And my reason for saying this latter is simply that where you have a sense-datum which is the same shade of white all over, this color which it *has* seems to me to be *related* to it in quite a different way from that in which its shape or its size are related to it” (377). The distinction Moore introduces between “white” and “a particular shade of white” does not matter here. Let us speak simply of “white.” It seems to me that in Moore's

argument it is assumed that a property is relational if, and only if, there is a relation of which the property is an argument. “White” is a relational property because it is related to me in some way. Aristotle would admit that “white” can be put in relation to something, but he would not conclude from this that “white” is a relational property because he would say that “white” does not by itself express a relation to anything. This means that a property can be counted as relational only if it is intensionally connected to a well-determined relation that specifies it.

- 6 Oehler 1984: “*Mit dieser Definition erfaßt Aristoteles den Typus von Relationen, der in der modernen Diskussion als intern (essentiell) bezeichnet wird. Eine Relation heißt intern, wenn sie das Wesen der zugrunde liegenden Dinge in begrifflicher Hinsicht beeinflusst*” (248).
- 7 A modern introduction to the doctrine of internal relations can be found in Rescher 1975: 177–191. Interesting remarks are also found in Ryle 1971: II, 85–100; Armstrong 1978: 84 ff.
- 8 The Greek commentators on Aristotle resort to this distinction in order to clarify the difference between P1- and P2-relatives (Ammonius, *On Aristotle’s Categories* 77.28ff; Simplicius, *On Aristotle’s Categories* 198.17 ff.; Olympiodorus, *On Aristotle’s Categories* 100.4–20; Philoponus, *On Aristotle’s Categories* 108.31–109.31), even if they spell it out quite differently. An interesting exception is Syrianus who, according to Elias’s report (*On Aristotle’s Categories* 218.32ff), maintained that text (D) has to be understood as an explanation, and not a revision, of the definition given in text (A). Waitz 1844–1846: I 301 too refers to the pair *secundum dici/secundum esse*, and Ackrill 1963: 101 says that it is “undeniable” that P1-relatives, but not P2-relatives, have to do with what things are called. But they do not attempt to explain the meaning of this distinction.
- 9 The verb “εἶναι,” which appears in the definition of P2-relatives, is linked to οὐσία at *Topics* 6.8, 146b3–4. But that does not help very much.
- 10 The other passage is *Topics* 6.8, 146b3–4, which I have already mentioned in the preceding note.
- 11 Following Porphyry (*Isagoge* 125.34–126.7) many Greek commentators held this view. See Ammonius, *On Aristotle’s Categories* 78.29–79, 2; Olympiodorus, *On Aristotle’s Categories* 112.23–30; Simplicius, *On Aristotle’s Categories* 201.16–20; Philoponus, *On Aristotle’s Categories* 131.6–11. Ackrill 1963: 102 has shown the internal absurdity of this position. A different interpretation is proposed in Oehler 1984: 249–250. Oehler asserts, correctly, that the traditional view does not catch Aristotle’s point, but his alternative explanation seems to me hardly tenable. According to him, knowing definitely that, for instance, *a* is a father means knowing that *a* has a child. An understanding of the latter is always entailed by an understanding of the first. On the other hand, in order to know that *a* is a head it is not necessary to know that *a* is the head of a man or of a dog or of a horse. In this sense knowing that *a* is a head can be said to imply an indefinite knowing of that to which “head” is related, because it is not necessary to state whether it is a dog or a man or a horse. But I must confess that I do not see the point. One could maintain that knowing that *a* is a head does not imply that one knows that *a* is the head of a man or of a dog, but it *does* imply that one knows that *a* is the head of a “headed” (κεφαλοτόν), as Aristotle calls it (*Categories* 7, 7a16–17). In the same way, knowing that *a* is a father does not entail that one knows that *a* is the father of a son or of a daughter, even if it entails (at least according to the hypothesis) that one knows that *a* is the father of a child. Furthermore, why should knowing that *m* is a double entail knowing that there is a number that is the half of *m*? It is hard to believe that, in general, a necessary condition for knowing what the domain of a relation is is that one knows what the domain of the converse relation is. It is, of course, a theorem of logic that every relation has its converse, but that does not imply that the meaning of the one is dependent on the meaning of the other.

- 12 The expression “ a is a double” is odd. We normally use “double” as a name for a function. A function applied to an argument yields a term that is usually preceded by a definitive article. Consequently, a functional term has to be taken as a definite description and the copula by which it is linked to a subject as denoting identity. Aristotle does not distinguish between relations and functions or, therefore, between relational and functional terms. He considers only relational terms.
- 13 Françoise Caujolle-Zaslowsky, in her paper on Aristotle's relatives (Caujolle-Zaslowsky 1980: 190–191), expresses the view that the definition of “head” contains no reference to anything else: “le sens de ‘tête’ ne se définit pas ‘par rapport à (πρός) homme’ (τι): ce n’est pas par rapport à autre chose que la tête est ce quelle est, c’est en tant que telle; tandis que le double n’est pas double en tant que tel, mais par rapport à autre chose – de même que l’esclave n’est pas esclave en tant que tel, mais en tant qu’il a un maître.” But she does not offer any argument in favor of her claim.
- 14 This example is adapted from Quine 1961b: 141–142.
- 15 The bibliography on substitution in cognitive contexts is immense. For an introduction to the problem, see the papers collected in Linsky 1971.
- 16 See Bekker 1831; Waitz 1844–1846; Minio-Paluello 1949. Curiously enough, in the Loeb edition, Bekker's text is adopted, but the translation by Harold Cook seems to presuppose the reading of the *Ambrosianus*: “you cannot know that it is double without knowing that it is double of something specific and definite” (61).
- 17 Kirchmann 1883: I, 21 comes near to (M1) when he says: “Denn wenn er keinen bestimmten Gegenstand kennt, dessen Doppeltes es sein soll, so wird er auch überhaupt nicht wissen, daß es ein Doppeltes ist.” In the same vein, Pesce 1966: 68 proposes: “se la sapesse doppia di niente di definito, non saprebbe affatto che è doppia.” The position of μή in the Greek makes reading (a1), and therefore (A1), hardly possible. I mention it only for the sake of completeness.
- 18 Translations that correspond to (M2) are offered by Edghill 1928: *ad locum*; Ackrill 1963: 23; Apostle 1980: 15; Rolfes 1978: 62; Oehler 1984: 21; Tricot 1959: 40. Other scholars use rather obscure expressions, the meaning of which could more or less be represented by

$$(M3) \quad \neg K_n(\neg \exists y R_D(a, y)).$$

They are Colli 1955: 29; Pelletier 1983: 41; Sanmartín 1982: 54. But it is difficult to grasp what such a formula could mean in this context. I suspect that supporters of (M3) are actually confused advocates of (M2).

- 19 It is reasonable to assume that the intended meaning of $\exists x K_n(A(x))$ is $K_n(A(a))$ for some a . Therefore, if n 's universe of knowledge is thought to be closed deductively, from the fact that n knows $A(a)$ for some a , it follows that n knows $\exists x A(x)$. This proves that (9) holds. But the assumption that n 's universe of knowledge is deductively closed at least under certain conditions is necessary to the argument, and it is far from being a trivial or obvious point. On the other hand, the converse of (9) is even less obvious, because

$$(9') \quad K_n(\exists x A(x)) \rightarrow \exists x K_n(A(x))$$

is doubtful. Take a finite domain of m elements over which x ranges. Then the antecedent of (9') can be translated into:

$$(\#) \quad K_n(A(a_1) \vee A(a_2) \vee \dots \vee A(a_m)).$$

The consequent of (9') is correspondingly represented by:

$$(*) \quad K_n(A(a_1)) \vee K_n(A(a_2)) \vee \dots \vee K_n(A(a_m)).$$

Now if it is reasonable to say that one can know a disjunction without knowing any of its elements, it must be concluded that (#) does not imply (*). This statement is true if the disjunction is taken classically. It can be questioned, on the other hand, if the

disjunction is conceived intuitionistically. But even in this case, we can show that (#) implies (*) only if we make a very strong assumption about the nature of the creative subject, one that makes his behavior far removed from any plausible description of real processes of knowing. See also Dummett 1977: 339–340.

- 20 In a larger sense, “ὀλόληψις” substantivizes the meaning of “ὀπολαμβάνειν” that Bonitz 1955²: 799b26–7 describes as “*sumere ac statuere aliquid pro vero*.” Using the word in this sense Aristotle can say at *On the Soul* 3.3, 427b24–26 that science, opinion, and prudence are varieties of ὀλόληψις. But there is a more restricted sense, in which ὀλόληψις is sharply distinguished from science and denotes a belief that is not the result of knowledge (for example, *Rhetoric* 3.16, 1417b9–10). In this sense ὀλόληψις is taken as synonymous with δόξα (for instance, *Metaphysics* 12.8, 1073a17–18 = 1.3, 984a1–2) and is said to be, like δόξα, liable to error (*Nicomachean Ethics* 6.3, 1139b17–18).
- 21 {In fact, $\exists y R_P(a, y)$ follows from the first conjunct of (13) via (12) and (16). So (14) is not consistent with (13), but because “ $B_n(A)$ ” stands for “ n believes that A without knowing that A ” (13) is not contradictory unless $K_n(\exists y R_P(a, y))$ follows from $K_n(P(a))$. Mignucci discusses this question in a general way.}
- 22 Ackrill 1963: 23n1. This reading is also adopted in Oehler 1984: 21.
- 23 See Putnam 1975a, 1975b; Johnson-Laird 1982: 19 ff.
- 24 Aristotle too, of course, thinks that Cetacea are fish. See, for instance, *History of Animals* 6.12, 566b3 ff.
- 25 This point has been recently defended by Johnson-Laird 1982: 21–26.
- 26 Johnson-Laird 1982: 26–29.
- 27 See, for instance, *Sophistical Refutations* 24, 179a35–b5.

16 The Stoic notion of relatives

I

The fragments of the Stoics explicitly concerned with a theory of relations are few, scattered, and difficult to interpret. The largest of them is preserved in Simplicius's commentary on the *Categories* (*On Aristotle's Categories* 165.32 ff. = *SVF* II 403). It expounds an important distinction that the Stoics made between two kinds of relatives. This doctrine is attributed to the Stoics, but no representative of the school is mentioned. Echoes of it are reflected in some skeptical arguments reported by Sextus Empiricus (*Against the Professors* 8.455–456) and Diogenes Laertius (9. 87–88).¹ There are also some related passages in the *scholia* on Dionysius Thrax's *Ars grammatica* that are supposed to go back to Apollonius Dyscolus (second century AD), where, although the Stoics are not explicitly named, Stoic material is believed to be used and referred to.² There is also a text of Sextus (*M* 8.453–454 = *SVF* II 404) in which a general definition of relatives is attributed by him to the Dogmatists, and reasons can be given for saying that his Dogmatists must be identified with the Stoics. Finally, some passages that tie the name of Chrysippus to questions that we are concerned with here are difficult to interpret, and on closer inspection, they reveal themselves not to pertain to the theory of relatives.³

In the face of this complicated situation in our sources, I will first examine Simplicius's passage. I will try to disentangle it from spurious connections with other parts of the Stoic doctrine that have generated more than one misunderstanding. I will then inquire to what extent the possibly general definition of relatives implied in Simplicius's passage is consistent with the statements reported by the other sources in order to determine whether Simplicius's report can be inserted in a coherent framework.

This sketch of my plan for this chapter shows that I place a great emphasis on Simplicius's passage, and this emphasis might be disputed, because Simplicius is a late authority and no Stoic master of the first generation is mentioned in his report. I will discuss these problems in due course. Whatever their solution might be, it must be pointed out that Simplicius's text is almost the only one in which a relevant aspect of the Stoic doctrine of relatives is expounded and discussed. The other sources are much vaguer and mostly concerned with a general

characterization of the notion of relative. Therefore, it is difficult in this situation not to confer a special position to Simplicius's passage.

II

We could divide Simplicius's passage into three sections. A first section (165.32–166.29) contains an exposition of the Stoic view of the difference between τὰ πρὸς τι and τὰ πρὸς τί πῶς ἔχοντα. A second section (166.30–167.20) offers a criticism of the Stoic position. Finally, in a third section (167.20–36), Simplicius contrasts the Stoic view with the view held by the Peripatetic Boethus of Sidon (first century BC) and criticizes the latter. The first section of the text must be carefully analyzed. I begin my analysis by quoting the first part of it:

(A) The Stoics here enumerate two kinds of relatives rather than one; they put the one kind among τὰ πρὸς τι, and the other among τὰ πρὸς τί πῶς ἔχοντα. They contrast τὰ πρὸς τι with τὰ καθ' αὐτά and τὰ πρὸς τί πῶς ἔχοντα with τὰ κατὰ διαφοράν. They say that the sweet and the bitter and things that dispose in such a way are πρὸς τι, whereas on the right, father and the like are πρὸς τί πῶς ἔχοντα. They say that things that are characterized by some form are κατὰ διαφοράν. Just as the notion of τὰ καθ' αὐτά differs from the notion of τὰ κατὰ διαφοράν, so τὰ πρὸς τι are different from τὰ πρὸς τί πῶς ἔχοντα. The order of the implications is inverted. For τὰ κατὰ διαφοράν belong to τὰ καθ' αὐτά, since the things that are καθ' αὐτά have some differences, as, for example, the white and the black; yet τὰ καθ' αὐτά do not belong to τὰ κατὰ διαφοράν; for the sweet and the bitter have differences by which they are characterized, but they are not καθ' αὐτά, being πρὸς τι. Τὰ πρὸς τί πῶς ἔχοντα, which are opposed to things κατὰ διαφοράν, are at any rate πρὸς τι. For on the right and father are not only τὰ πρὸς τί πῶς ἔχοντα but also πρὸς τι. The sweet and the bitter, being πρὸς τι, are κατὰ διαφοράν, whereas τὰ πρὸς τί πῶς ἔχοντα are contrary to τὰ κατὰ διαφοράν. For it is impossible that τὰ πρὸς τί πῶς ἔχοντα are either καθ' αὐτά or κατὰ διαφοράν, since they depend only on their relation to something else. On the other hand, τὰ πρὸς τι are not καθ' αὐτά, since they are not absolute, but they are at any rate κατὰ διαφοράν, for they are conceived with some characteristic.

(Simplicius, *On Aristotle's Categories* 165.32–166.15 = *SVF* II 403)⁴

Four expressions are introduced in this passage: τὰ καθ' αὐτά, τὰ πρὸς τι, τὰ κατὰ διαφοράν, and τὰ πρὸς τί πῶς ἔχοντα. The relations of their denotata are also stated. Simplicius does not try to define them systematically. He gives a short characterization of τὰ κατὰ διαφοράν, saying that entities characterized by some form are κατὰ διαφοράν (165.37–166.1), but he only gives examples for τὰ πρὸς τι and τὰ πρὸς τί πῶς ἔχοντα. Neither a definition nor an example is offered for τὰ καθ' αὐτά; it is only said that they are ἀπόλυτα, absolute (166.14). Among τὰ πρὸς τι are put the sweet and the bitter (τὸ γλυκὺ καὶ πικρόν: 165.35; 166.10), as well as possession (ἔξις), knowledge (ἐπιστήμη), and perception (αἴσθησις) (166.21).

On the other hand, on the right (δεξιόν: 165.36; 166.9; 166.23, 25), father (165.37; 166.9; 166.25) and son (166.23) are offered as examples of τὰ πρὸς τί πως ἔχοντα. Finally, the white and the black (166.5),⁵ the sweet and the bitter, are said to be among τὰ κατὰ διαφοράν (166.6–7; 166.10–11).

Before trying to give an interpretation of the meaning of our four expressions, let us consider the relations that are said to hold between their denotata and comment briefly on them. The relations can be summarized as follows:

- (a) τὰ πρὸς τι are contrasted (ἀντιδιαίρουσι) with τὰ καθ'αὐτά (165.33–34).
- (b) τὰ πρὸς τί πως ἔχοντα are contrasted with τὰ κατὰ διαφοράν (165.34–35).
- (c) τὰ καθ'αὐτά are κατὰ διαφοράν (166.3–5).
- (d) It is not the case that everything that is κατὰ διαφοράν is also καθ'αὐτό (166.5–8).
- (e) τὰ πρὸς τί πως ἔχοντα are πρὸς τι (166.8–10).
- (f) τὰ πρὸς τί πως ἔχοντα are opposed (ἀντίκειται) to τὰ κατὰ διαφοράν (166.8–9) and are said to be contrary (ἐναντία) to τὰ κατὰ διαφοράν (166.11).
- (g) τὰ πρὸς τί πως ἔχοντα cannot be κατὰ διαφοράν (166.12–13).
- (h) τὰ πρὸς τί πως ἔχοντα cannot be καθ'αὐτά (166.12).
- (i) τὰ πρὸς τι are κατὰ διαφοράν (166.10–11; 166.13–15).
- (k) τὰ πρὸς τι are not καθ'αὐτά (166.13–14).

I take (c) to paraphrase 166.3–4: τοῖς μὲν γὰρ καθ'αὐτά συνυπάρχει τὰ κατὰ διαφοράν. The verb συνυπάρχειν normally means “coexist,” but it is sometimes used by Simplicius as a synonym for ὑπάρχειν in a predicative sense (for instance, 141.8–9: εἶπερ διὰ ταῦτα καὶ τοῖς ἄλλοις συνυπάρχει τὰ παρακολουθοῦντα τῷ ποσῷ). Here the existential meaning cannot be the right one, because the statement is meant to explain the ἀκολουθία, namely the predicative relations, between these notions.⁶ That is confirmed by what is said in (d), where the Greek is:

- (1) οὐ μέντοι τοὺς κατὰ διαφοράν τὰ κατὰ διαφοράν συνυπάρχει (166.5–6).

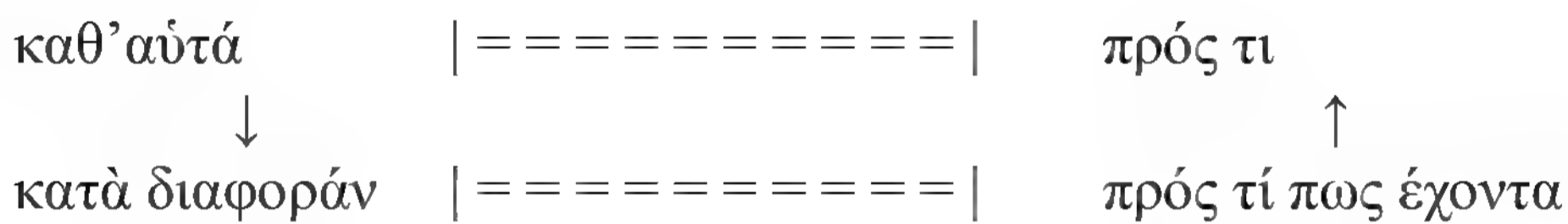
The statement in (1) is justified by means of the pair sweet/bitter, which is said to be κατὰ διαφοράν but not καθ'αὐτά (οὐ μέντοι καθ'αὐτά ἐστι τοιαῦτα: 166.7). Thus,

- (2) there is something which is κατὰ διαφοράν but not καθ'αὐτό

is stated and (1) is said to follow from that. If an existential meaning is attributed to συνυπάρχειν in (1), it must be such that it allows us to infer (1) from (2). Because (d) is logically equivalent to (2), the statement that Simplicius attributes to the Stoics, namely (1), is at least implied by (d). But that can hardly be if συνυπάρχειν is taken to have an existential meaning in (i). Besides, “coexist” expresses a symmetrical relation: If *A* coexists with *B*, *B* coexists with *A*. If συνυπάρχειν here means “coexist,” from the statement that τὰ κατὰ διαφοράν coexist with τὰ καθ'αὐτά, it would follow that τὰ καθ'αὐτά coexist with τὰ κατὰ

διαφοράν. But as we have seen, Simplicius says that in general it is not true that τὰ καθ'αὐτὰ συνυπάρχει τὰ κατὰ διαφοράν. Therefore, the predicative meaning of συνυπάρχειν is in question here, or at least the existential sense of the word.⁷ The context makes it clear that quantification must be added in (d). Quantification could be avoided if συνυπάρχειν and the other terms for the predicative relation were taken to denote the relation of inclusion between classes. On this assumption, (c) would mean that the class of τὰ καθ'αὐτὰ is included in the class of τὰ κατὰ διαφοράν, and (d) that the converse inclusion does not hold. Thus, (c) and (d) together would amount to the statement that the class of τὰ καθ'αὐτὰ is a proper subclass of the class of τὰ κατὰ διαφοράν. A corresponding statement is made in (c) and (d) by using the quantification in (d).

In light of these remarks, we can represent relations (a)–(f) by means of the following schema:



where “| = = |” expresses the relation of contrariety, and “→” inclusion. The schema accounts for Simplicius’s statement that the ἀκολουθία between these notions is inverted (166.3), because the arrows go in the opposite direction. The reason why relations (g)–(k) are not represented in the schema will be apparent later.

It is easy to observe that (b) is equivalent to (f). One might be tempted to say that (b) is also equivalent to (g) (leaving aside the modal qualification of (g)) and that (a) is equivalent to (k). The main reason for proposing these equivalences is 166.11–13, where (g) is given as a justification of (f), at least if we take γάρ (166.12), which introduces (g), in a proper sense. If these equivalences are admitted, the following interpretation of the contrast, opposition, or contrariety that is said to hold between some of the *denotata* of the expressions at issue becomes inescapable. *A* is said to be opposed (contrasted, contrary) to *B* if *A* entails not *B*, and because “*A*” and “*B*” can be taken to stand for classes, as we will see, this means that *A* and *B* have no elements in common. I do not deny that 166.11–13 presupposes an equivalence between (f) and (g). But it is also true that, if this equivalence is systematically adopted throughout the passage, the first part of it states something false. I am inclined to think that the meaning of the relation of contrariety or opposition does not remain the same in the different parts of the text. I will try to substantiate this claim later. It is, however, plain that (a) entails (k) and that (b) (or (f)) implies (g), at least if we ignore the modal qualification of (g).

III

Let us first try to answer a preliminary question: What do τὰ καθ'αὐτὰ and the other related expressions denote? If we consider the case of τὰ καθ'αὐτὰ we might be tempted to say that our problem is easily solved. Things that are καθ'αὐτὰ are characterized as ἀπόλυτα at 166.14. If this last word is taken to refer to the property

of having an independent existence, it is easy to conclude that τὰ καθ'αὐτά are individual things – Aristotle would have called them primary substances. Consequently, the statement in (c) that τὰ καθ'αὐτά are τὰ κατὰ διαφοράν should be taken to mean that individual substances, such as Socrates, are κατὰ διαφοράν, namely somehow qualified, for instance, as white in the case of Socrates. Therefore, according to this interpretation, instances of τὰ κατὰ διαφοράν are white or black things. By analogy, one might be tempted to conclude that when the sweet and the bitter are put among τὰ πρὸς τι or the white and the black among τὰ κατὰ διαφοράν or, again, father and son among τὰ πρὸς τί πως ἔχοντα, what is meant is that sweet or bitter things are examples of πρὸς τι, and white or black things are examples of κατὰ διαφοράν and that a person who is a father or a son is an instance of a πρὸς τί πως ἔχοντα. In this view, instances of τὰ πρὸς τι or τὰ πρὸς τί πως ἔχοντα are taken to be not general terms, but individual things that have certain properties. If we take “ $F(x)$ ” to mean “ x is sweet” and a , an individual thing, to be sweet, namely $F(a)$, what must be put among τὰ πρὸς τι to according to this interpretation is not $\lambda xF(x)$, being sweet, but a , an individual thing that is sweet.

Although it appears at first sight obvious, I do not believe that this interpretation is correct. In order to show this, let us first observe that the relations that are said to hold between the denotata of our four expressions do not hold between their instances. For example, when it is said in (a) and (f) that τὰ πρὸς τί πως ἔχοντα are opposed, or even contrary, to τὰ κατὰ διαφοράν, it is not meant that a relation of opposition or contrariety holds between instances of τὰ πρὸς τί πως ἔχοντα and instances of τὰ κατὰ διαφοράν. Whatever “on the right” and “sweet” might be taken to denote, what is on the right cannot be said to be opposed or contrary to what is sweet. Statements (a) and (f) make sense if we suppose that τὰ πρὸς τί πως ἔχοντα and τὰ πρὸς τι refer here to classes of entities (whatever these entities may be). So it is the class of τὰ πρὸς τί πως ἔχοντα to which the class of τὰ κατὰ διαφοράν is opposed or contrary. In a similar way, when it is said in (c) that τὰ καθ'αὐτά are κατὰ διαφοράν, this statement must be taken to mean that the class of τὰ καθ'αὐτά is included in the class of τὰ κατὰ διαφοράν. The other assertions of the list (a)–(k) have to be interpreted accordingly. Then our problem becomes: What are the members of the classes of τὰ πρὸς τι and of τὰ πρὸς τί πως ἔχοντα?

Consider the relation that is asserted to hold between τὰ πρὸς τί πως ἔχοντα and τὰ κατὰ διαφοράν stated in (b) and (f). Being a relation between classes, it is reasonable to interpret the statement that τὰ πρὸς τί πως ἔχοντα are opposed or contrary to τὰ κατὰ διαφοράν as implying that the intersection of the two classes is empty. Suppose that the elements of these classes are individual things properly qualified. Then, we must say that a thing that is on the right of something is a πρὸς τί πως ἔχον and a thing that is sweet is a κατὰ διαφοράν. But if so, there is no reason to think that the class of τὰ πρὸς τί πως ἔχοντα and the class of τὰ κατὰ διαφοράν are disjointed. One can imagine a lump of sugar that is on the right side of my table. The lump of sugar *qua* on the right side of my table belongs to the class of τὰ κατὰ διαφοράν, and *qua* white belongs to the class of τὰ κατὰ διαφοράν. Therefore, the two classes are not disjointed and cannot truly be said to be opposed. The same can be said with respect to the opposition that is said to

hold between τὰ καθ'αὐτά and τὰ πρὸς τι. If members of the former are individual things and members of the latter things that are in a given relation to something, there is no reason to deny that the two classes can have a nonempty intersection. Moreover, it is relevant to underline that Simplicius's way of speaking does not compel us to conceive τὰ καθ'αὐτά as the class of all individual things or, if you prefer, the class of all Aristotelian primary substances. What might lead one to this interpretation is ἀπόλυτον in 166.14, which may be taken to mean "independently existing." But this is not the only possible sense that this word has. By itself ἀπόλυτον means "absolute," and more than once Simplicius uses it to qualify what is not relative to something else (see, for instance, *On Aristotle's Categories* 108.10–11; 146.18–20). The word ἀπολελυμένον, "unrelated," is similarly used by Alexander of Aphrodisias in a passage included by von Arnim in his collection of the fragments of the Stoics (= *SVF* II 945). But if we give this sense to ἀπόλυτον, a sense that is perfectly consistent with the context, it is not necessary to take τὰ καθ'αὐτά to denote the class of individual things; it may simply mean the class of nonrelative things, "things" here meaning by the universe of entities that Simplicius is in general referring to and that we are trying to identify.

If members of our four classes cannot be individual things, what are they? A possible answer is that they are general terms. Therefore, among the members of the classes of τὰ πρὸς τι and τὰ κατὰ διαφοράν are not sweet things, but being sweet, which we have represented by " $\lambda xF(x)$." We speak here of "general terms," leaving aside the further question of whether we are here dealing with a classification of predicates of properties or classes of entities. This question depends on the way in which we conceive of reference to universals, and it would be outside the scope of this inquiry to consider this problem.⁸ Let us then keep to the rather neutral expression "general term."⁹

On this interpretation, τὰ καθ'αὐτά are general terms that are absolute, that is, not relative. We may suppose that terms such as "man," "book," and "carrot" fall into this class. Of course, the extension of it can be more precisely determined only if we are able to establish what is the extension of the class of τὰ πρὸς τι, and we will try later to accomplish this task. It is more difficult to understand what τὰ κατὰ διαφοράν refer to. A κατὰ διαφοράν term is qualified by being characterized by a form. I take this to mean that such a term has a differentiating power with respect to the things of which it is truly predicated. So being sweet is κατὰ διαφοράν because whatever is sweet, say *a*, is properly qualified by its being sweet in the sense that it marks a possible difference with respect to the states of *a* in which *a* is not sweet. We will later come to this question when we try to explain why τὰ κατὰ διαφοράν and τὰ πρὸς τι, but not τὰ πρὸς τί πῶς ἔχοντα, are said to be κατὰ διαφοράν.

Before going on, let us consider a possible objection to our interpretation of the extension of the four classes. In two places, Simplicius gives ὁ δεξιός as an example of πρὸς τί πῶς ἔχοντα (166.9 and 23). In this case, it seems natural to take ὁ δεξιός to mean a man who is on the right side. Therefore, it should be admitted that elements of the class of τὰ πρὸς τί πῶς ἔχοντα are not general terms, such as "being on the right," but certain individual things, such as those things that are on the right. I agree that, according to my interpretation, τὸ δεξιόν would have been

expected instead of ὁ δεξιός, but I do not think that this difficulty is so impressive that my general interpretation should be abandoned. Simplicius seems to shift from the masculine to the neuter gender without any apparent change of meaning, so that I am not at all sure that ὁ δεξιός must be translated in the two mentioned passages as “the man who is on the right side.” I am inclined to think that ὁ δεξιός means simply “on the right.” The main reason for my claim is found at 167.10–14, where ὁ ἀριστερός καὶ δεξιός appears to be used synonymously with τὸ ἀριστερὸν καὶ δεξιόν. Besides, even if this argument is not accepted, the reasons why I think that members of the four classes are general terms seem to me so strong that I am more prepared to charge Simplicius with infelicitous terminological choices than to admit that elements of our classes are individuals.

IV

We now have to sketch the general lines of the argument proposed by the Stoics and to make some comments on the list of statements (a)–(k) of Section II. The main thesis that Simplicius attributes to the Stoics is that they distinguished τὰ πρὸς τί πῶς ἔχοντα from τὰ πρὸς τι. I take lines 165.33–166.10 to contain the beginning of the argument and lines 166.10–166.29 to be a development of it, which is only partially consistent with its initial move. Two theses are first stated at 165.33–35, namely (a) and (b). If we take “ $\lambda xK(x)$ ” to stand for τὰ καθ’ αὐτά, “ $\lambda xD(x)$ ” for τὰ κατὰ διαφοράν, “ $\lambda xR(x)$ ” for τὰ πρὸς τι, “ $\lambda xP(x)$ ” for τὰ πρὸς τί πῶς ἔχοντα, and “ $O(x, y)$ ” for “ x is opposed to y ,” we can formally translate (a) and (b) as follows:

- (1) $O(\lambda xR(x), \lambda xK(x)).$
- (2) $O(\lambda xP(x), \lambda xD(x)).$

A second step in the argument is made with the remark that τὰ πρὸς τι and τὰ πρὸς τί πῶς ἔχοντα are different in so far as the notion (ἔννοια) of τὰ καθ’ αὐτά is different from the notion of τὰ κατὰ διαφοράν (166.1–3). Perhaps this rather cryptic statement can be articulated as follows. I imagine that the following argumentative pattern is at work. It cannot be true that $a = b$ if the contrary of a is different from the contrary of b . But that is what happens in the case of the classes of R - and P -terms, because their contraries have different notions and therefore are different. Thus, the class of R -terms is not the same as the class of P -terms.

If this analysis of the passage is accepted, we cannot adopt a weak interpretation of the relation of opposition, on which the opposition between two classes is said to be equivalent to their being disjointed, for then the argument does not work. In order to make it sound, we must take the relation of opposition between two classes A and B to indicate that the contrary of A is the same as B , where being the contrary of something must be understood as a function. In other words, a unique term can be truly said to be the contrary of A . That makes the equivalence between (a) and (k) and between (b) (or (f)) and (g) impossible. On the other hand, because the contrary of A is unique, if A and B have different contraries, they are different. Then if “ $C(x)$ ” is taken to express the function of contrariety, a more adequate

representation of the relations that are said to hold between *R*- and *K*-terms and between *P*- and *D*-terms is the following:

$$(1') \quad C(\lambda x R(x)) = \lambda x K(x).$$

$$(2') \quad C(\lambda x P(x)) = \lambda x D(x).$$

The argument can be reconstructed accordingly. Because τὰ καθ' αὐτά and τὰ κατὰ διαφοράν have different notions, they are different. Then

$$(3) \quad \lambda x K(x) \neq \lambda x D(x)$$

can be affirmed. From it and (1') and (2') we obtain

$$(4) \quad C(\lambda x R(x)) \neq C(\lambda x P(x)).$$

Moreover, (4) entails

$$(5) \quad \lambda x P(x) \neq \lambda x R(x)$$

because contrariety is supposed to be a function. Therefore, the class of *P*-terms is not the same as the class of *R*-terms.

The most natural way to interpret the nonidentity of two things that are classes is by saying that they are not coextensive. Then what our argument proves is simply that the class of *P*-terms is not coextensive with the class of *R*-terms. This is confirmed by lines 166.3–10, where it is said that implication (ἀκολουθία) between τὰ πρὸς τι and τὰ πρὸς τί πως ἔχοντα is inverted with respect to implication between τὰ καθ' αὐτά and τὰ κατὰ διαφοράν: τὰ καθ' αὐτά are κατὰ διαφοράν, whereas it is not always the case that τὰ κατὰ διαφοράν are καθ' αὐτά. On the other hand, τὰ πρὸς τί πως are πρὸς τι. In other words, (c), (d), and (e) are stated. We may represent these relations by means of the following formulas:

$$(6) \quad \forall x (K(x) \rightarrow D(x)).$$

$$(7) \quad \exists x (D(x) \ \& \ \neg K(x)).$$

$$(8) \quad \forall x (P(x) \rightarrow R(x)).$$

Formulas (6) and (7) show that the class of *K*-terms is a proper subclass of the class of *D*-terms. By analogy with (7), one might expect it to be further stated that it is not always the case that τὰ πρὸς τι are πρὸς τί πως ἔχοντα,

$$(9) \quad \exists x (R(x) \wedge \neg P(x))$$

which shows, together with (8), that the class of *P*-terms is a proper subclass of the class of *R*-terms. Although this is not explicitly stated in our passage (but is at 167.20–21, where the Stoic position is summarized), I think that the remark about the implications is made in this sense. The class of *K*-terms is to the class of

D-terms as the class of *P*-terms is to the class of *R*-terms. The class of *K*-terms is a proper subclass of the class of *D*-terms, just as the class of *P*-terms is a proper subclass of the class of *R*-terms. Precisely in this sense, the class of *P*-terms is different from the class of *R*-terms.

I have the impression that this conclusion is not clearly drawn, because the progress of the reasoning is disturbed in its last part by another related argument that aims to prove a stronger result and that is tacitly inserted in the body of the first by exploiting a common premise. Let me try to make this point clearer.¹⁰

V

At 166.8–9 Simplicius repeats the observation that τὰ πρὸς τί πως ἔχοντα are opposed to τὰ κατὰ διαφοράν. The same point is made at 166.11 by saying that τὰ πρὸς τί πως ἔχοντα are contrary to τὰ κατὰ διαφοράν. We have already analyzed the meaning of this opposition and seen that it cannot be interpreted in a weak sense. Surprisingly enough, the fact that τὰ πρὸς τί πως ἔχοντα cannot be κατὰ διαφοράν is taken (at 166.11–13) as an explanation of the fact that τὰ πρὸς τί πως ἔχοντα are opposed to τὰ κατὰ διαφοράν, and this implies that the two statements are considered equivalent there. I take this as a clue that Simplicius is here reporting a different argument in which, if a class *A* is said to be opposed (or contrary) to a class *B*, that simply means that *A* and *B* are disjointed.

There is a more important sign that the perspective of the argument has changed. At 166.8–10, we are told that τὰ πρὸς τί πως ἔχοντα are πρὸς τι (namely our (8)). At 166.10–15, something is stated that makes this assertion hardly plausible, namely that τὰ πρὸς τι, but not τὰ πρὸς τί πως ἔχοντα, are κατὰ διαφοράν. We can represent this point by means of

$$(10) \quad \forall x(R(x) \rightarrow D(x))$$

and

$$(11) \quad \forall x(P(x) \rightarrow \neg D(x)).$$

What is expressed in (11) does not constitute anything new. It is a plain consequence of (2). What is really new is (10). Strictly speaking, (8) is not inconsistent with (10) and (11). But if (10) is admitted together with (8) and (11), we obtain

$$(12) \quad \forall x(P(x) \rightarrow D(x))$$

by transitivity from (8) and (10). Consequently, we get from (11) and (12)

$$(13) \quad \neg \exists x P(x)$$

namely that the class of *P*-terms is empty. This consequence would have not been accepted by the Stoics. One is led to think that, at 166.10–15, the meaning of πρὸς

$\tau\iota$ is different from the meaning that this expression has in the preceding lines. The class of $\tau\alpha \pi\rho\acute{o}\varsigma \tau\iota$, which is a subclass of the class of $\tau\alpha \kappa\alpha\tau\grave{\alpha} \delta\iota\alpha\phi\omicron\rho\acute{\alpha}\nu$, is different from the class of $\tau\alpha \pi\rho\acute{o}\varsigma \tau\iota$, of which $\tau\alpha \pi\rho\acute{o}\varsigma \tau\acute{\iota} \pi\omega\varsigma \acute{\epsilon}\chi\omicron\nu\tau\alpha$ are said to be a proper subclass. Let “ R' ” represent this new class of $\tau\alpha \pi\rho\acute{o}\varsigma \tau\iota$. Then, (10) has to be changed into

$$(10') \quad \forall x(R'(x) \rightarrow D(x)).$$

If (10') is substituted for (10), it is clear that the undesired conclusion (13) cannot be derived. It is obvious that the class of R -terms can be taken to be the genus of the class of R - and P -terms. Then, we might expect not only (8), but also

$$(14) \quad \forall x(R'(x) \rightarrow R(x)).$$

Simplicius's statement that the Stoics split the Aristotelian category of relatives into two different genera (165.32–33) does not necessarily imply that R - and P -terms cannot have a common genus.

The price we have to pay in order to avoid the undesired conclusion (13) is that the meaning of one of the basic terms at issue must be changed. That also explains the different result to which the argument that I do not hesitate any longer to call a new argument leads: $\tau\alpha \pi\rho\acute{o}\varsigma \tau\acute{\iota} \pi\omega\varsigma \acute{\epsilon}\chi\omicron\nu\tau\alpha$ no longer form a proper subclass of the class of $\tau\alpha \pi\rho\acute{o}\varsigma \tau\iota$, but rather constitute a class that is disjointed from the class of $\tau\alpha \pi\rho\acute{o}\varsigma \tau\iota$. In this new perspective

$$(15) \quad \forall x(P(x) \rightarrow \neg R'(x))$$

is substituted for (9) as the goal to be reached by the argument. If we suppose that the class of R' -terms is not empty and, at the same time, that R' -terms constitute a subclass of the class of R -terms (namely we admit (14)), (9) follows from (15), and in this sense the result aimed at by the second argument is stronger than the conclusion of the first.¹¹

The argument for (15) is straightforward. Simplicius says that $\tau\alpha \pi\rho\acute{o}\varsigma \tau\iota$ and $\tau\alpha \pi\rho\acute{o}\varsigma \tau\acute{\iota} \pi\omega\varsigma \acute{\epsilon}\chi\omicron\nu\tau\alpha$ share the feature of not being $\kappa\alpha\theta'\alpha\upsilon\tau\acute{\alpha}$, but they are different because $\tau\alpha \pi\rho\acute{o}\varsigma \tau\iota$, but not $\tau\alpha \pi\rho\acute{o}\varsigma \tau\acute{\iota} \pi\omega\varsigma \acute{\epsilon}\chi\omicron\nu\tau\alpha$, are $\kappa\alpha\tau\grave{\alpha} \delta\iota\alpha\phi\omicron\rho\acute{\alpha}\nu$. It follows immediately from this that $\tau\alpha \pi\rho\acute{o}\varsigma \tau\acute{\iota} \pi\omega\varsigma \acute{\epsilon}\chi\omicron\nu\tau\alpha$ are not $\pi\rho\acute{o}\varsigma \tau\iota$. That is the same as saying that (15) is a direct consequence of (10') and (11). The argument is, of course, correct from a formal point of view. The difficulty is with its premises, namely (10') and (11), that are far from being trivial propositions. In lines 166.10–15, a very brief and rather mysterious justification of (10') and (11) is given. The statement that $\tau\alpha \pi\rho\acute{o}\varsigma \tau\acute{\iota} \pi\omega\varsigma \acute{\epsilon}\chi\omicron\nu\tau\alpha$ are not $\kappa\alpha\tau\grave{\alpha} \delta\iota\alpha\phi\omicron\rho\acute{\alpha}\nu$ is grounded on the remark that they are constituted by mere relations (166.12–13). Moreover, as an explanation of the view that $\tau\alpha \pi\rho\acute{o}\varsigma \tau\iota$ are $\kappa\alpha\tau\grave{\alpha} \delta\iota\alpha\phi\omicron\rho\acute{\alpha}\nu$, we are told that $\tau\alpha \pi\rho\acute{o}\varsigma \tau\iota$ are always conceived together with some characteristic (166.13–15). If that were all we had by way of justification for (10') and (11), we should have to give up any attempt at an interpretation. Fortunately, Simplicius

himself felt his remarks inadequate, and he added more information in lines 166.15–29, making a fresh start on the question. Therefore, we must carefully analyze this passage.¹²

VI

The key notion we have to consider is that of *κατὰ διαφοράν*. As we have already seen, a general term is *κατὰ διαφοράν* if it is a qualified term, that is, if it has a discriminating power. The natural way to interpret this view is to think that a term has a discriminating power if it makes the objects of which it is true different from the objects of which it is not true. If Leibniz's Law, (*LL*), is admitted, it is clear that every predicate that is true of some object has a discriminating power with respect to this object. With reference to one-place predicates, (*LL*) can be formulated as follows:

$$(LL) \quad a = b \rightarrow (F(a) \leftrightarrow F(b)).^{13}$$

Take things *a* and *b* and any predicate *F*. If it is true that *F(a)*, then *a = b* holds only if *F(b)*. Therefore, the fact that *a*, but not *b*, is *F* is sufficient to distinguish *a* from *b*. In the case we are considering, the discriminating power of terms belongs not to individuals but to phases of one and the same individual. For instance, it may be said that being in Assos characterizes a phase of the life of Aristotle. If (*LL*) is supposed to apply not only to individuals but also to phases of individuals, one can maintain that every phase term has a qualifying role with respect to the phases of an individual. If being in Assos is true of phase *i* of Aristotle's life, one is entitled to conclude that he was in a different phase when he was no longer living in Assos. The validity of (*LL*) in its extension to phases guarantees that every general term has a discriminating function in determining the phases of individuals: If phase *i* has a property that phase *j* fails to have, then *i* and *j* are numerically different even if both are phases of the same individual. Because the Stoics asserted that there are terms that do not have a discriminating power, to maintain the interpretation of the notion of *κατὰ διαφοράν* we are discussing involves denying that (*LL*) holds in general. But there is no reason to charge the Stoics with the denial of a logical law as basic as (*LL*).¹⁴ Therefore, I am inclined to think that the interpretation of the notion of *κατὰ διαφοράν*, and in particular of the notion of discriminating power, must be somewhat changed. The problem is: How?¹⁵

A possible way to answer this question is with the proof Simplicius offers of the statement that *τὰ πρὸς τί πῶς ἔχοντα* are not *κατὰ διαφοράν*. Simplicius first gives a characterization of the class of *P*-terms by saying:

(B) *τὰ πρὸς τί πῶς ἔχοντα* are those things which, in addition to looking outside themselves, are naturally able to occur or fail to occur to something without any change or alteration in the things themselves.

(Simplicius, *On Aristotle's Categories* 166.17–19)

This feature of *P*-terms is explicitly said to be the reason why they are not κατὰ διαφοράν in a related passage some pages after the text we are discussing:

(C) The Stoics wrongly maintain that τὰ πρὸς τί πως ἔχοντα are separated from any differentiating property (πάσης τῆς κατὰ διαφοράν ιδιότητος), because they are naturally able to occur or fail to occur to something without any change in the things.

(Simplicius, *On Aristotle's Categories* 172.1–4)

Therefore, if we want to know what a differentiating term is, we have to ask in what sense τὰ πρὸς τί πως ἔχοντα do not cause any change in the things of which they are true. Let us consider the examples that Simplicius quotes to explain the point. He says:

(D) When something is thought not in virtue of an internal difference, but in virtue of its mere relation to something, it will be a πρὸς τί πως ἔχον. For son and on the right require something external to subsist. That is why, even if no change takes place in themselves, a man may cease to be a father when his son dies and someone may cease to be on the right when what was next to him has changed position.

(Simplicius, *On Aristotle's Categories* 166.21–26; J. Annas and J. Barnes translation, slightly modified)

The main idea seems to be the following. Consider a thing *a* that is on the right of something else, for instance, *b*, and take “*H*(*x*, *b*)” to stand for “*x* is on the right of *b*.” Then, *H*(*a*, *b*) can be truly affirmed. Suppose now that *b* is moved with respect to *a* in such a way that *H*(*a*, *b*) is no longer true. Without internal alteration *a* has lost one of its properties; that is, without *a*’s being internally modified, a predicate that was formerly true of *a* is no longer true of it. It is only because of *b* that *a* is no longer on the right of it. In general, we can say that a relational term such as $\lambda x \exists y F(x, y)$ is among τὰ πρὸς τί πως ἔχοντα if, and only if, there is something of which $\lambda x \exists y F(x, y)$ can be true and become false without internal alteration of the thing itself. If we introduce the temporal predicate “*R_t*(*x*),” namely “*x* occurs (obtains) at *t*,” we can formally express the character of τὰ πρὸς τί πως ἔχοντα terms by means of the following equivalence:

$$(16) \quad \lambda x \exists y F(x, y) \in P \text{ iff} \\ \exists x M \exists t \exists t' (R_t(\exists y F(x, y)) \wedge R_{t'}(\neg \exists y F(x, y)) \wedge S(x, t) = S(x, t'))$$

where “*M*” is the modal operator of possibility and “*S*(*x*, *t*)” expresses the internal state of *x* at *t*.¹⁶ In other words, the relevant point is that a πρὸς τί πως ἔχον does not in any way alter the internal state of the things of which it is true when it ceases to be true of them. In order to make this characterization of τὰ πρὸς τί πως ἔχοντα clearer, it would perhaps be useful to make the notion of internal state more precise.¹⁷ We will try to do so later. For the time being, we rely on the rough intuition

that allows us to say that it can happen that a does not change its internal state if it loses the property of being on the right of b because b is moved. This state of affairs is known in recent literature as “Cambridge change.”¹⁸

It is with reference to this view that it is stated that τὰ πρὸς τί πῶς ἔχοντα are not qualifying terms. They do not always qualify the different internal phases of the objects of which they happen to be true. It is easy to see that this claim does not conflict with Leibniz’s Law. If a , but not c , is on the right of b at t , then, of course, a is not the same as c . In this sense, even πρὸς τί πῶς ἔχοντα terms have a discriminating power, no more and no less than any other general term, with respect to the things of which they are true. And the same is true if we consider two different phases of one and the same individual. If, in phase j , a is F , where “ F ” stands for a πρὸς τί πῶς ἔχον term with respect to a , and in phase j' , a is no longer F , phase j is different from phase j' of a . On the other hand, suppose that a is one of the individuals with respect to which we can say that F is a πρὸς τί πῶς ἔχον. Then the internal state of a does not necessarily change if F ceases to be true of it. Therefore, πρὸς τί πῶς ἔχοντα terms are nonqualifying terms in the sense that they are not always able to mark a difference in the internal histories of the individuals of which they are true.

A consequence of this is that the right-hand side of equivalence (16) entails that $\lambda x \exists y F(x, y)$ neither necessarily has, nor necessarily implies, a differentiating power and therefore does not belong to the class of τὰ κατὰ διαφοράν. Thus, we have formally:

$$(17) \quad \text{If } \exists x M \exists t \exists t' (R_t(\exists y F(x, y)) \wedge R_{t'}(\neg \exists y F(x, y)) \wedge S(x, t) = S(x, t')) \text{ then } \lambda x \exists y F(x, y) \notin D.$$

It follows immediately from (11) and (16) that if $\lambda x \exists y F(x, y)$ is a κατὰ διαφοράν term, then it must satisfy the negation of the antecedent of (17), that is,

$$(18) \quad \text{If } \lambda x \exists y F(x, y) \in D \text{ then } \forall x L \forall t \forall t' (R_t(\exists y F(x, y)) \wedge R_{t'}(\neg \exists y F(x, y)) \rightarrow S(x, t) \neq S(x, t'))$$

where “ L ” is the modal operator of necessity. Therefore, because πρὸς τί terms are κατὰ διαφοράν, they satisfy the consequent of (18). This is confirmed by the remark Simplicius makes with reference to instances of πρὸς τί terms such as the sweet and the bitter. He says that a thing cannot cease to be sweet without changing internally (166.26–27).¹⁹

VII

Up to this point, we have a possible clarification of the meaning of κατὰ διαφοράν and a characterization of τὰ πρὸς τί πῶς ἔχοντα. Even if there is something more to say about this, let us first consider πρὸς τί terms and discuss the definition of them reported by Simplicius. The problem of how to characterize R' -terms would be easily solved if we could assume that the class of relational terms is entirely

covered by the two subclasses of τὰ πρὸς τι and τὰ πρὸς τί πως ἔχοντα. If that were so, we could affirm

$$(19) \quad \lambda x \exists y F(x, y) \in R' \text{ iff } \forall x L \forall t \forall t' (R_t(\exists y F(x, y)) \wedge \\ \wedge R_{t'}(\neg \exists y F(x, y)) \rightarrow S(x, t) \neq S(x, t'))$$

by negating both members of equivalence (16) and assuming

$$(20) \quad \lambda x \exists y F(x, y) \notin P \leftrightarrow \lambda x \exists y F(x, y) \in R'.$$

One might doubt assumption (20). Equivalences such as (16) and (19) can be taken to be characterizations of special kinds of predicates, provided that they are phasal. But there are predicates that are neither phasal nor even temporal. Consider, for instance, a term such as $\lambda x 0 < x$, where x ranges over natural numbers. It is a relational term and it has nothing to do with time. Therefore, it cannot be thought that equivalences such as (16) and (19) apply to it. Because I think that, after all, (19) is a good characterization of τὰ πρὸς τι, rejection of (20) can be made consistent with the assumption of (16) and (19) only if the conditional involved in the right side of equivalence (19) is not taken as a material conditional, so that the right side of (16) cannot be identified as the negation of the right side of (19). But perhaps the problem is much more complicated. One might wonder whether the Stoics had a notion of atemporality different from that of omnitemporality. For instance, one might observe that their definitions of modal operators imply a reference to time.²⁰ Of course, this remark is not sufficient to show that they confused omnitemporality with atemporality, and we cannot investigate this problem now.

Let us turn to the definition of τὰ πρὸς τι reported by Simplicius. He says:

(E) To put it more clearly, they say that those things are πρὸς τι which, being in a certain condition by virtue of their own character, refer to something else; [. . .] consequently, when something, being in a certain condition by virtue of a difference, refers to something else, that will just be πρὸς τι [. . .]. But the sweet and the bitter could not become of another sort, unless their own power participates in the change.

(Simplicius, *On Aristotle's Categories* 166.15–27;
J. Annas and J. Barnes translation, slightly modified)

How shall we interpret the sentence according to which τὰ πρὸς τι “being in a certain condition by virtue of their own character refer to something else”? The last part is clear: A πρὸς τι term must be relational. But what about the first part? In what sense is a πρὸς τι term in a certain condition by virtue of its own character or of a difference? I take this to mean that the relational character of πρὸς τι terms is permanently tied to the internal state of the objects to which they are truly attributed. If this interpretation is admitted, it becomes reasonable to consider (19) as a possible characterization of πρὸς τι terms. The differentiating power that they exert on the objects to which they apply is exactly what makes the objects

themselves change their internal state when they lose or acquire the properties denoted by them. That is what is said by (19).

In light of these remarks it is not difficult to explain why *πρός τι* terms are said to be *κατὰ διαφοράν*. As we have seen, if something is *κατὰ διαφοράν*, it has a discriminating power, and this notion can be explained by means of the notion of internal state, which plays a central role in our characterization of *τὰ πρὸς τί πως ἔχοντα* and *τὰ πρὸς τι*. In general, the state of an object *a* at a given time *t* is determined by the set of predicates (relational or not) that are true of *a* at *t*. Then, the state of *a* at *t'* changes with respect to its state at *t* if some predicates that were true of *a* at *t* are no longer true of *a* at *t'*, or if at least one predicate that was not true of *a* at *t* becomes true of *a* at *t'*. For instance, the state of *a* at *t* is different from the state of *a* at *t'* if *b* at *t'* is no longer on the right of *a*, because at *t* the predicate being on the right of *b*, $\lambda x H(x, b)$, is true of *a* and is no longer true of it at *t'*. It is a natural restriction of this general notion to say that the internal state of *a* at *t* is given by the conjunction of the absolute predicates that are true of *a* at *t*. We call a general term “absolute” if it is nonrelational. Let “ ϕ ” be a predicate variable that ranges over absolute terms. We can define identity of internal states with respect to an object *a* and times *t* and *t'* by means of a second-order equivalence:

$$(21) \quad \forall x L \forall t \forall t' (S(x, t) = S(x, t') \leftrightarrow \forall \phi (R_t(\phi(x)) \leftrightarrow R_{t'}(\phi(x))).$$

Formulas (16) and (19) can be modified accordingly. The idea that lies behind this characterization of internal state is that for a thing *a*, what primarily marks its internal history and makes a difference for it in time is the set of its absolute properties. Relational properties have a differentiating power with respect to the internal phases of *a* if they are necessarily tied to an absolute property of *a*. If that is admitted, then the distinction between *τὰ πρὸς τί πως ἔχοντα* and *τὰ πρὸς τι* is connected with the distinction between relational properties that are dependent on absolute properties of things and relational properties that do not exhibit such a permanent link. If *a* is such that it can lose the property of being on the right of *b* without altering the set of its absolute properties, then the property of being on the right of *b* is not permanently tied to any absolute property of *a*. On the other hand, if being sweet is such that it is naturally able to alter the internal state of the objects of which it is true, then there is at least one of the absolute properties of these objects that is permanently tied to the property of being sweet. To make the point clearer, let “ ϕ ” be, as before, a variable ranging over absolute terms. Then it can be proven that

$$(22) \quad \forall x \exists \phi L(\phi(x) \leftrightarrow \exists y F(x, y))$$

together with (21), entails the right side of equivalence (19). This means that (22) is sufficient to establish that a term is an *R'*-term.

This view is confirmed by Simplicius when he states that *τὰ πρὸς τί πως ἔχοντα* are solely constituted by their relations to something else (166.13), or that they are conceived merely in virtue of their relations to something else (166.22–23).

One is prepared to admit that relations are in some sense essential to *P*-terms because they are relational. What is peculiar to *P*-relatives is that they are solely constituted or conceived with reference to something else. I take this to mean that there are relational terms that are constituted or conceived in their being true of some individuals without implying a necessary connection with any of the absolute terms that are true of those individuals. If $\lambda x \exists y R(x, y)$ is true of *a* at *t* and no longer true of *a* at *t'*, and this does not alter the internal state of *a*, then no absolute property of *a* is necessarily linked to $\lambda x \exists y R(x, y)$. On the other hand, if $\lambda x \exists y R(x, y)$ is necessarily connected to at least one of the absolute properties of *a*, then to lose this relational property entails a change in the internal state of *a*.²¹ We have now to face a rather difficult problem.

We have systematically added modal operators to formulas (16)–(22). What has led us to do this is clear. Formulas (16)–(22) are intended to capture the logical character of some terms, and it is quite reasonable to suppose that logical classifications have to do more with ideal states of affairs than with factual situations. If we decide that being on the right of something, $\lambda x \exists y H(x, y)$, is a *P*-term because, for example, *a* is on the right of something at *t* and ceases to be so at *t'* without alteration of its internal state, we do not need to carry out the operation to make the decision. It is a sort of mental experiment that gives an ideal test to classify $\lambda x \exists y H(x, y)$. This means that definitions of *P*- and *R'*-terms are intrinsically linked to modal notions.

Moreover, suppose that we drop any modal reference from (16). Then a term such as “bitter,” which according to the Stoics is put among the *R'*-terms, might, in certain possible circumstances, become a *P*-term. As we will see later, bitter is considered a relational term because it is necessarily related to experiencing subjects. Suppose, then, for the sake of simplicity, that the experiencing subjects are men and that in the interval t_0 – t_1 all men but one taste this lump of sugar *a* as sweet. The only exception tastes *a* as bitter because of an illness. At t_2 the sick person recovers and does not taste *a* as bitter any longer. On this hypothesis, the nonmodalized antecedent of (16) is satisfied as its consequent because we can easily convince ourselves that *a* has undergone no change in its internal state. Therefore, if this happens we have to classify bitter as a *P*-term. Because there is no reason to deny that such a situation can occur in the world, we cannot be sure that bitter is an *R'*-term.

But if we add the modal operator of possibility to (16), the same case of the ill man who recovers shows not that bitter can become a *P*-term, but that it *is* one. Because there is something that, possibly, is bitter at *t* and not bitter at *t'* without any internal change, then bitter is a *P*-term according to the modal version of (16).

How can we get free of these difficulties? Formulas (16) and (19), whether modalized or not, seem inadequate to express the Stoic distinction between *P*- and *R'*-terms. So either we look for another interpretation of the texts or we keep our formulas and conclude that the Stoic view is strongly inconsistent. Both horns of the dilemma are far from attractive. Perhaps there is a third, less dramatic way to get out of the blind alley that we apparently are in. It is quite clear that we cannot

admit nonmodalized versions of (16) and (19) without losing the ability to state definitely whether a term belongs to one class or the other. However, the modal versions do not seem able to distinguish adequately one class from the other. Now the counterexample to (16) we have produced was based on a notion of logical possibility. It is “logically” possible that there is a man who tastes a lump of sugar as bitter at *t*, and it is “logically” possible that after a while nobody tastes the same lump of sugar as bitter. But we can perhaps claim that this situation is not “naturally” possible. We have to be clear about the meaning of “naturally possible.” We cannot understand it in the sense of “physically possible,” that is, “consistent with the physical laws,” because the counterexample to (16) is physically possible. But we can take “naturally possible” in another sense, namely as “consistent with the standard behavior of nature.” Illness, monsters, and, in general, abnormal events in nature are not, in principle, inconsistent with natural laws. Nonetheless, they are deviant with respect to the standard of nature and in this sense they constitute exceptions. A man who does not taste a lump of sugar as sweet is not a “normal” man. Of course, it is logically and physically possible that something happens to him and as a result he behaves unnaturally. But this does not represent a natural possibility for him, namely a possibility that is in accordance with the standard development of his nature. For the natural man, let us say so, it is impossible to get ill and, similarly, to taste a lump of sugar as bitter. So with respect to this notion of natural possibility, the counterexample to (16) we have proposed and many other similar ones cannot be constructed.

A weak confirmation of this view can be found in Simplicius’s words when he says that τὰ πρός τί πῶς ἔχοντα are such that “they are naturally able to occur or to fail to occur to something (πέφυκεν συμβαίνειν τινὶ καὶ μὴ συμβαίνειν)” (166.17–18). The same words are used in a parallel passage at 172.3. Perhaps it is not too bizarre to claim that πέφυκεν συμβαίνειν contains a reference to the natural possibility of which we have tried to give a rough picture.

VIII

The notion of Cambridge change, which plays an important role in the Stoic distinction between *R*’- and *P*-relatives, was not invented or introduced for the first time by the Stoics. It was already known to Aristotle, even if he does not appear to use it to characterize the class, or a subclass, of relatives. There are at least two passages where Cambridge change is alluded to by Aristotle. At the end of *Metaphysics* 14.1 (1088a22 ff.), he blames the Academics for having taken the great and the small as a principle, because this assumption implies that a high degree of reality is conferred on a relative, but what is relative is “least of all a kind of real object” (1088a29–30). A sign that relatives occupy the lowest part of the scale of beings is, according to Aristotle,

(F) the fact that relatives alone do not come into being or pass away or change in the way that increase and diminution occur in quantity, alteration in quality, locomotion in place, sheer coming into being and passing away in the case

of a real object, There is none of this with relatives. A thing will be greater or less or equal without itself changing if another thing changes in quantity.

(Aristotle, *Metaphysics* 14.1, 1088a30–35; J. Annas translation)

The point relevant to our discussion is clear. Aristotle says that if a has a relative property F and ceases to have it by acquiring another relative property G , a does not change in itself if it is that to which a is related that changes. Cambridge change is therefore clearly recognized by Aristotle.²² What is less evident is the range of relatives to which it applies in Aristotle's view. Does he think that any relative is subject to Cambridge change? He appears here to be concerned mostly with relatives that are attributes of quantity, as he explicitly says at 1088a24–25 and as the examples he uses show. Are we entitled to say that any relative can undergo Cambridge change, or does it apply only to special kinds of relatives?

It is difficult to give a reasonable answer to this question, and it is better to move to the other passage that employs Cambridge change. Unfortunately, the passage is corrupted. If we accept the reading suggested by David Ross, Aristotle's claim that there is no proper change among the relatives would be justified in this way:

(G) There is no motion in respect to substance, because nothing is contrary to a substance. Neither is there in respect to relatives. For it is possible that if one of the two related things changes, the other <ἀληθεύεσθαι καὶ μὴ> ἀληθεύεσθαι without change. Therefore their motion is incidental.

(Aristotle, *Physics* 5.2, 225b10–13 = *Metaphysics* 11.12, 1068 a 10–13)

Schwegler and Bonitz restore the passage differently, simply adding μὴ before ἀληθεύεσθαι.²³ In both cases a difficulty remains. The usual meaning of ἀληθεύεσθαι is “be true” or “be truly predicated of,” and in this case it is the second sense that applies. On the other hand, the two occurrences of θάτερον can only refer to the elements that are said to be related, say a and b related through R . Moreover, the second θάτερον is the subject of the ἀληθεύεσθαι-sentence. How, then, can it be said that it is possible that a is true or not true of something without internal change if b is supposed to change? To avoid this difficulty, we must suppose that ἀληθεύεσθαι has a special meaning, something like “to be such that something is true of it.” I have not found any parallel Aristotelian passage, but the peculiar use of ἀληθεύεσθαι seems to me to be strongly required by the sense of this passage if Schwegler's or Ross's text is to be maintained.²⁴ If the interpretation we have given is accepted, it can be said that Cambridge change is appealed to in order to show that no proper change takes place in the category of relatives. A thing a can cease to be related to b through a relation R without undergoing an internal change, and in this sense it is said to be changed only incidentally.

Shall we conclude that the Stoics took the notion of Cambridge change from Aristotle? That is quite possible, even if we have no specific hint to confirm the hypothesis. What is clear, however, is that there is no sign that Cambridge change was used by Aristotle to determine one class of relatives distinct from other classes of relatives that are not supposed to satisfy it. In the two places where relatives are

considered *ex professo*, namely *Categories* 7 and *Metaphysics* 4.15, no reference to Cambridge change is made. Cambridge change is used by Aristotle to show that there is no movement with respect to relatives, but that does not, of course, imply that all Aristotelian relatives, or any definite subclass of them, undergo it. In addition, Cambridge change is clearly referred to by Plato:

(H) Well now, I think these three agreed statements conflict with one another in our minds when we say what we do about the dice, or when we say this about me: while being just this size, without growing or undergoing the opposite, I can within the space of a year be both larger than a young man like you, now, and smaller later on – not because I’ve lost any of my size but because you’ve grown.

(Plato, *Theaetetus* 155 B 4–C 1; J. McDowell translation)

The point is clear: Socrates has the property of being larger than Theaetetus at *t* and he no longer has this property at *t'* without having undergone an alteration in his size. Therefore, one might think that the Stoics developed this Platonic remark independently of the Aristotelian tradition.

However, one might insist that Aristotle is the source of the Stoic doctrine by pointing out, on the one hand, that Aristotle in *Categories* 7 gives two definitions of relatives, the second of which characterizes a subclass of the class characterized by the first in the following way:

(I) Now if the definition of relatives which was given above [= 6a36–37] was adequate, it is either exceedingly difficult or impossible to reach the solution that no substance is spoken of as a relative. But if it was not adequate, and if those things are relatives for which being is the same as being somehow related to something (τὸ εἶναι τὰντὸν ἐστὶ τῷ πρὸς τί πως ἔχειν: 8a32), then perhaps some answer may be found.

(Aristotle, *Categories* 7, 8a28–33; J. Ackrill translation)

Here, relatives are defined by the fact that they are the same as their being somehow related to something. But Simplicius’s τὰ πρὸς τί πως ἔχοντα are defined by the fact that they have their being in the mere relation to something (ἐν τῇ σχέσει μόνῃ τὸ εἶναι ἔχει: 166.28–29). By stressing this similarity of definition, one might think that relatives given by Aristotle’s second definition are nothing more than Simplicius’s πρὸς τί πως ἔχοντα.

This conclusion does not seem inescapable. It is true that the two definitions of relatives contain words that are similar. In both cases, a reference is made to εἶναι of relatives, and it is said that it consists in a relation. But words such as εἶναι can have different meanings, and the whole formula acquires a different sense according to the contexts in which it is placed. Actually, in Simplicius’s account their mere being in the relation is appealed to in connection with Cambridge change of τὰ πρὸς τί πως ἔχοντα. The identity of εἶναι with being somehow related to something in the Aristotle passage is invoked to explain a special epistemological

dependence of things that are so related: In order to know that *a* is a father, one must know that *a* is a father of someone.²⁵ There is no hint in Aristotle that relatives that imply identity of their εἶναι with being somehow related pass the test of Cambridge change. Nor can it be concluded from Simplicius's report that τὰ πρὸς τί πως ἔχοντα are tied to the epistemological dependence that Aristotle says holds for strict relatives. The statement that the being of τὰ πρὸς τί πως ἔχοντα consists only in their relations simply means that, for such relatives, being in some connection does not necessarily involve ownership of absolute properties and that is a necessary condition for Cambridge change. In the case of Aristotle, the identity of being with being somehow related is given as the reason why some relatives are such that they imply in their meaning a relation to something else so that they cannot be consistently used if they are not known to have a relation to something. Of course, one might see a connection between the two states of affairs and maintain that Cambridge change is possible only where the Aristotelian epistemological dependence occurs and vice versa. That may be true (I have no definite position on this), but we have no proof that either Aristotle or the Stoics were aware of the link, and without such a proof nothing can be inferred about a historical dependence of the Stoics on Aristotle.²⁶

The Stoic notion of Cambridge change, on the one hand, has a parallel in Plato and in Aristotle's doctrine of relatives. On the other hand, their way of characterizing τὰ πρὸς τί πως ἔχοντα is probably exploited in some skeptical arguments directed to show that relatives, or relations, are not real. Diogenes Laertius, in a difficult passage, maintains that relatives cannot be known in themselves, and he considers the example of being on the right. He says:

(J) Anything on the right, for instance, is not by nature (φύσει) on the right, but is thought of according to its relation to something else – if that is moved it will no longer be on the right.

(Diogenes Laertius 9.87–88; J. Annas and J. Barnes translation)

It is not our concern to discuss the whole of Diogenes's argument.²⁷ What is interesting is to remark that the example of what is on the right of something fits quite well with the view attributed by Simplicius to the Stoics. If *a* is on the right of *b* and *b* is moved in a certain way, it is no longer true that *a* is on the right of *b*. In other words, *a* has lost one of its properties without undergoing any internal change. Cambridge change is involved, and it is precisely this that allows Diogenes to conclude that *a* is not by nature on the right. By implication, one could say that a property belongs naturally to an object if the object is internally changed when it loses that property. If that is correct, and if we feel inclined to insist on the analogy between Diogenes's passage and Simplicius's report, then “by nature” (φύσει) in Diogenes can be taken to refer to the situation of properties that mark the internal state of a thing. If *a* is on the right of something, being on the right is not by nature a property of *a*, because it does not affect the internal state of *a* by being either an absolute property or necessarily connected to an absolute property of *a*. If the parallel with Simplicius's account is to be maintained, “*a* is

F by nature” in Diogenes’s report cannot mean that *F* is part of *a*’s nature if it is implied by this that *a* is necessarily *F*. According to Simplicius’s examples, both “this lump of honey is sweet” and “this cup of coffee is sweet” should be considered cases of a belonging by nature, but the second does not express a necessary predication.²⁸

For the sake of completeness, another passage, taken this time from Sextus Empiricus, should be mentioned, where once again Cambridge change is associated with the ontological state of relatives. Sextus makes the following point:

(K) Again, everything that exists (πᾶν τὸ ὑπάρχον) is incapable of admitting any change or alteration without being affected – e.g., the color white cannot become black unless it is converted and changed, and black cannot change into any other color while staying black; and similarly the sweet cannot become bitter while subsisting unaffected and unaltered. Hence everything that exists does not admit change into something else without being affected in some way. But what is relative (τὸ πρὸς τι) changes without being affected and even though no alteration occurs in it. For example, when a one-foot rule has a one-foot rule laid against it, it is said to be equal to it; but when it has a two-foot rule laid against it, it is no longer said to be equal but rather unequal.

(Sextus Empiricus, *Against the Professors* 8.455–456;
J. Annas and J. Barnes translation)

We shall not worry about the general line of the argument, which is surely mistaken.²⁹ Nor is it important to stress that the example that ought to illustrate Cambridge change is ill-chosen. From the fact that *a* is equal to *b* and different from *c*, it does not follow that *a* has changed from being equal to being unequal. What is relevant to us is only that once again Sextus’s passage goes back to a tradition that is well represented by the Stoics. It should also be noticed that Sextus tends here to identify relatives with Simplicius’s τὰ πρὸς τί πὼς ἔχοντα. He appears to maintain that all relatives can undergo Cambridge change and that properties such as the sweet and the bitter are not to be classified as relatives. That does not square at all with the definition of relatives he ascribes to the Dogmatists (probably to the Stoics) a few lines earlier (*Against the Professors* 8.454). There Sextus is perhaps betrayed by his polemical ardor.

IX

There is still a question worth considering on which some scholars have put a certain emphasis. It is customary in recent literature to set up a parallel between Simplicius’s quadripartition of τὰ καθ’ αὐτά, τὰ κατὰ διαφοράν, τὰ πρὸς τι, and τὰ πρὸς τί πὼς ἔχοντα and the four Stoic categories mentioned by Simplicius himself (*On Aristotle’s Categories* 66.32–67.8 = *SVF* II 369) and by Plotinus (*Enneads* VI 1, 25.1–5 = *SVF* II 371), namely ὑποκείμενα, ποιὰ, πὼς ἔχοντα, and πρὸς τί πὼς ἔχοντα. Some interpreters think that Simplicius’s passage on the distinction between τὰ πρὸς τι and τὰ πρὸς τί πὼς ἔχοντα can be understood to state a division

of τὰ ὄντα into καθ'αὐτά and πρὸς τι. The latter would be again divided into κατὰ διαφοράν and πρὸς τί πως ἔχοντα. All this can be summarized in the following schema, which expresses a division (διαίρεσις) of things that are (Figure 16.1).

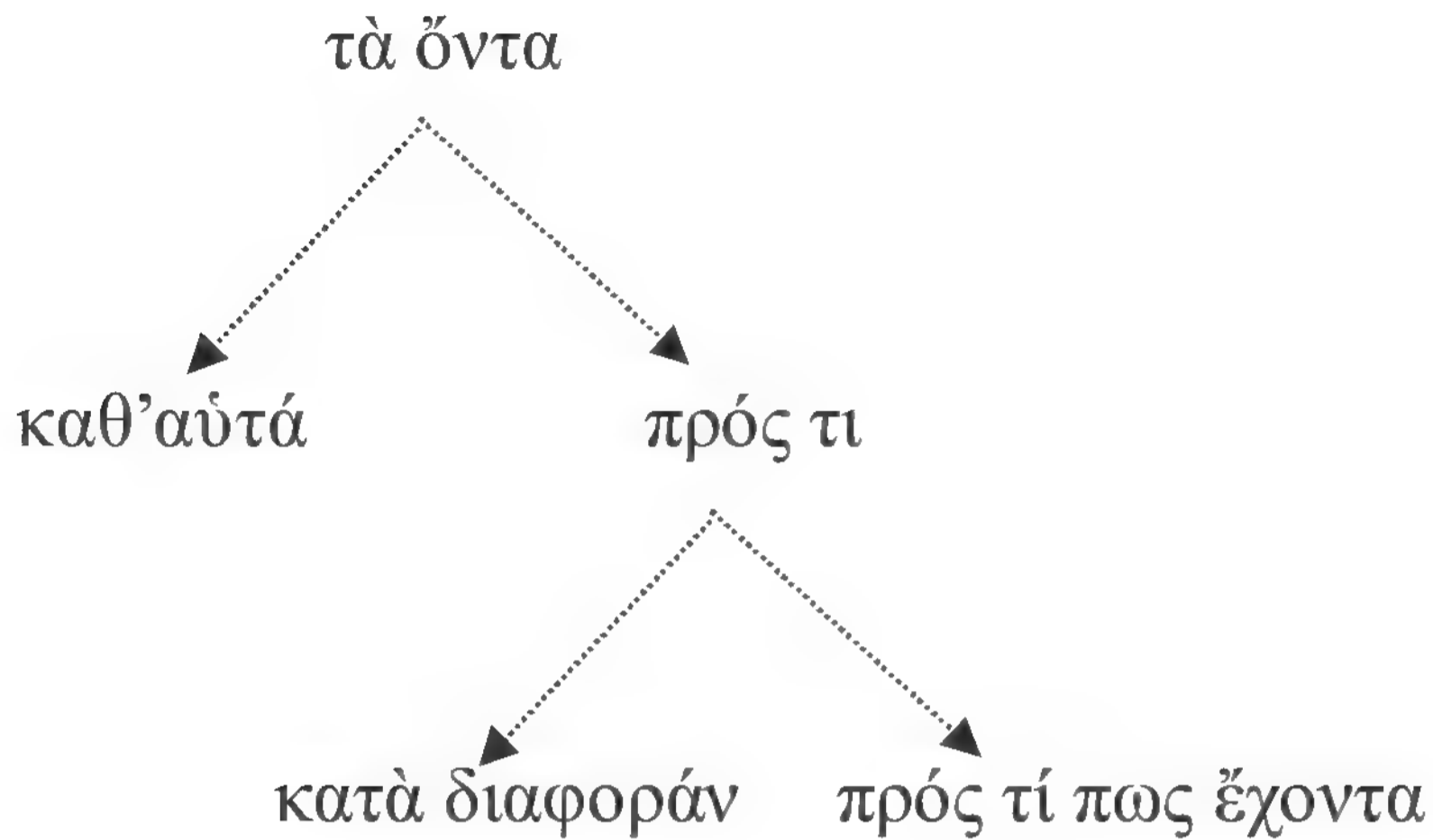


Figure 16.1

In this way an alternative system of categories would be expounded by Simplicius so that the problem of its relation to the former would raise its head.³⁰

But does Simplicius's passage really offer a new account of the doctrine of the categories? We must now discuss this question. First of all, I do not believe that the proposed schema matches Simplicius's report, and this is proven by the mere fact that (j) (see Section II) is stated in it. This is not very important, for one might agree that the schema does not work and nevertheless maintain that Simplicius speaks of a theory of the categories that is referred to by Simplicius.³¹ It is more interesting to point out that Simplicius's discussion is not introduced as an analysis of the Stoic theory of the categories, but as a view about relatives. He says that the Stoics divided the class of relatives in two subclasses (165.32–33), and, in order to prove this, as we have seen, he shows that τὰ πρὸς τί πως ἔχοντα and τὰ πρὸς τι have different contraries: τὰ κατὰ διαφοράν and τὰ καθ'αὐτά, respectively. It is not stated that τὰ πρὸς τι and τὰ πρὸς τί πως ἔχοντα are categories, or, using Simplicius's terminology, "first genera" (πρώτα γένη) (*On Aristotle's Categories* 66.32–33 = *SVF* II 369).³² Nor are τὰ καθ'αὐτά and τὰ κατὰ διαφοράν said to represent primary divisions of reality. They are simply introduced as contraries of the previous classes. Moreover, the fact that Simplicius, in order to show the failure of the Stoic distinction among the relatives, uses an argument (167.18–20) that Plotinus deployed against the Stoic doctrine of the categories (*Enneads* VI 1, 30.24–27 = *SVF* II 402) does not prove that Simplicius criticizes the doctrine Plotinus discusses, but only that both doctrines, the theory of relatives and the theory of the categories, are open to the same objection.

One might reply that this proof is not very strong, because it is simply an argumentum *ex silentio*. Even if Simplicius does not introduce the Stoic view as a view about the categories, he might refer to their doctrine of the categories in order to

explain their position about relatives. The most direct way to meet this objection is to consider the evidence we have about the Stoic categories and to see whether it is consistent with the system of the four classes introduced by Simplicius in his discussion of relatives. If the former have features that cannot be shared by the latter, it is reasonable to conclude that the two systems are not different descriptions of the same state of affairs.

The program is easy to sketch but difficult to realize, because the pieces of information we have about the Stoic categories are scattered, late, and difficult to interpret. The different interpretations that have been given of them in recent times witness the uneasiness that scholars feel when they are called upon to express their views on this part of the Stoic conception. Just to show the range of problems involved, it is sufficient to remember how many solutions have been given to the problem concerning the object of the categories. Some scholars have thought that they are meant to be methodological principles; others are inclined to say that they have to do with λεκτά; there are still others who claim that they must be conceived as classes of objects *qua denotata* or meanings of a certain type; and many defend the view that they are divisions of the physical reality.³³

I can neither discuss these opinions nor sketch an interpretation of the Stoic theory. I will just consider an aspect of it that may be relevant to our problem, even if it is difficult to reach a reliable conclusion in such a complicated matter. To make the point with Trendelenburg's words, the second category, τὸ ποιόν, expresses in a compressed way what should be fully expressed by τό πως ἔχον τὸ ὑποκείμενον ποιόν, whereas the third and the fourth, τὸ πως ἔχον and τό πρὸς τί πως ἔχον, should be expanded into τὸ ὑποκείμενον ποιόν πως ἔχον and τὸ ὑποκείμενον ποιόν πρὸς τί πως ἔχον, respectively.³⁴ It follows from this that the system of the categories can be represented as a set of classes that may overlap. More exactly, this view implies that (i) the classes of τὰ ὑποκείμενα and τὰ ποιὰ are included in the classes of τά πως ἔχοντα and τὰ πρὸς τί πως ἔχοντα and therefore that (ii) the class of τά πως ἔχοντα and the class of τὰ πρὸς τί πως ἔχοντα have a nonempty intersection. In one form or another this position is shared by many scholars, and it is customary to point to this feature of the Stoic doctrine to underline one of its differences from Aristotle's system of the categories.³⁵ I will try to defend a thesis that is slightly different from Trendelenburg's even if it has the same consequence, namely that the Stoic categories have nonempty intersections. If this consequence is admitted, it is easy to conclude that Simplicius's four classes, τὰ καθ'αὐτά, τὰ κατὰ διαφοράν, τὰ πρὸς τι, and τὰ πρὸς τί πως ἔχοντα, do not form a system of categories, because the classes of τὰ καθ'αὐτά and τὰ πρὸς τι are disjointed, and the same is true of the classes of τὰ κατὰ διαφοράν and τὰ πρὸς τί πως ἔχοντα. Let us call this consequence of Trendelenburg's thesis the "requirement of inclusion for the categories."

Is this view acceptable? The best evidence in favor of it is a passage of Plotinus where the second Stoic category, τὸ ποιόν, is considered:

(L) Qualified entities (τὰ ποιὰ) must be for them distinct from substrates. This in fact they acknowledge by counting them as the second category. If then

they form a distinct category, they must be simple; if so, they are not composite; if so, they must be devoid of matter as qualified entities; if so, they are bodiless and active; for matter is their substrate in order to make them passive. If nevertheless they hold qualified entities to be composite, it is a strange division, which first contrasts simple and composite entities, includes them in one genus, and then includes one of the two species in the other; it is as if one divided knowledge by positing grammar and grammar plus something else as its species. Again, if they identify qualified entities with matter qualified (ὅλην ποιάν), then in the first place even their reasons will be material [. . .]. Further, if they maintain that seasons are nothing but matter in a certain state (ὅλην πῶς ἔχουσιν) they evidently say that qualified entities are in a certain state (πῶς ἔχοντα) and should accordingly classify them in the third genus.

(Plotinus, *Enneads* VI 1, 29.1–17 = *SVF* II 376;
S. MacKenna translation, slightly modified)³⁶

The argument is complicated, and I do not need to analyze the whole of it. It is sufficient to consider the second horn of the dilemma, namely the case in which τὰ ποιὰ are taken to be composite, which, of course, corresponds to the Stoic position. The view that Plotinus attributes to the Stoics is that they divide the universe of things (however this universe may be conceived) by putting on one side matter, the content of the first category, and on the other side matter qualified or in a certain state, the content of the second category. This is the sense of the parallel he draws with the division of knowledge into grammar and grammar + something else. It explains why the first category is said to be included in the second. It follows immediately from this that the first and second category cannot be represented by two disjointed classes.

The requirement of inclusion we are looking for is clearly stated in this passage as far as the first and the second categories are concerned. Can it be justified and extended to the other categories? The important question is how substrate, namely matter, is to be conceived. Dexippus (*On Aristotle's Categories* 23.25–24.4 = *SVF* II 374) tells us that the Stoics distinguished two senses of ὑποκείμενον, one of which corresponds to what Aristotle would have called “potential body” and one that is τὸ ποίον. Obviously, substrates that are involved in the first category are ὑποκείμενα in the first sense. He gives “bronze” as an example of the first sense and “Socrates” as an example of the second. Shall we conclude that matter constitutes the first category in the sense that all possible pieces of matter that are going to be qualified are members of the first category, as the example of bronze suggests? Evidence for this could perhaps be found in the fact that Plotinus uses the plural ὑποκείμενα (*Enneads* VI 1, 25.2, 12 = *SVF* II 373) to indicate the elements of the first category. Suppose then that its members are unqualified pieces of matter. Members of the second category are therefore qualified pieces of matter. Thus, there is a nonempty intersection between the two classes, and the first can be said to be included in the second if we admit that all unqualified pieces of matter become qualified. But this is dubious. One might ask whether it is reasonable to speak of “unqualified pieces of matter.” Matter needs qualification to be

differentiated in any way.³⁷ Bronze is not a bare, but a qualified, piece of matter, and it is precisely its qualification that distinguishes it from another substrate, for instance, a piece of gold. Bare pieces of matter do not exist, and therefore one might say the extension of the first category is empty if they are taken to constitute its members.

Even if the conclusion of this argument is not, by itself, inconsistent with the requirement of inclusion for the first and the second category (the empty class is a subclass of any class), the Stoics would not have admitted that one of the categories may be empty. Therefore, we have to give another meaning to the phrase “unqualified pieces of matter,” which refers to the elements of the first categories. What meaning? The easiest way to answer this question is to assume that elements of the first category are things or entities (whatever the range of these words is taken to be) *qua* matter, namely pieces of qualified matter, the qualification of which we do not consider. The same things *qua* qualified pieces of matter constitute the members of the second category. Evidence for that can be found again in Plotinus, when he says that there is a difference between Aristotle and the Stoics in the way in which they consider τὰ γένη, namely the categories. Aristotle took the categories as principles of things, whereas the Stoics considered the things in the categories to be themselves the principles.³⁸ I take this statement to imply that the categories were conceived by the Stoics not as classes of principles or elements or components of things (cf. *Enneads* VI 2, 2.1–19), but as constituted by the things themselves considered from a certain point of view.

If this interpretation is accepted, it is natural to suppose that the same holds for the two remaining categories: Things that are in the first category *qua* mere pieces of matter and in the second *qua* pieces of qualified matter are also in the third and in the fourth category as pieces of qualified matter in certain states and as pieces of qualified matter in certain relations. If we also allow that there cannot be entities in a certain state that have no relation to anything and that entities that are in some relation to something must all be in a certain state, it follows that all four categories are classes that have the same extension, namely the universe to which the categories apply.³⁹

Even if this conclusion appears plausible, as far as plausibility can be appealed to in such a matter, there is at least one passage that seems *prima facie* to offer contrary evidence. With reference to the third category, Plotinus makes the same reproach against the Stoics that he had addressed to them with respect to the second category. He says:

(M) With regard to states (τοῖς πῶς ἔχουσιν) it may seem strange that states should be set up as third, or in whatever other order, since they are states of matter (περὶ τὴν ὕλην πῶς ἔχοντα). We shall be told that there is a difference among states and that this or that state of matter is different from a state which is in states, and further that qualified entities (τὰ ποιῶ) are states of matter, while states properly so called are states of qualified entities.

(Plotinus, *Enneads* VI 1, 30.1–7 = *SVF* II 400;
S. MacKenna translation, slightly modified)

Here, too, the details of the argument are not important. What is interesting is that τὰ πῶς ἔχοντα are said to be states of matter (περὶ τὴν ὕλην πῶς ἔχοντα: 2–3) or, more precisely, states of ποιὰ (6–7). Because τὰ ποιὰ are states of matter, it follows that τὰ πῶς ἔχοντα are states of states of matter. An apparently obvious interpretation of these statements is that the class of τὰ ποιὰ has as its elements not pieces of matter in a certain state, as has been suggested in the previous discussion, but the states of matter themselves. Correspondingly, one might take the class of τὰ πῶς ἔχοντα to be constituted by states of states of matter. If so, there is no reason to suppose that the second and the third classes have a nonempty intersection with each other and with the first class, namely the class of things *qua* pieces of matter. Things *qua* pieces of matter are not states of matter, and states of matter are not states of states of it. Therefore, looking at this passage, one might feel oneself entitled to represent the system of the Stoic categories as a family of disjointed classes.

Shall we infer that Plotinus does not offer sufficient evidence to confirm the requirement of inclusion for the categories? Perhaps such a skeptical conclusion can be avoided if one considers that, in the Stoic view, states of matter are nothing but matter in a certain state. There is some evidence for this Stoic view. Alexander of Aphrodisias (*On Aristotle's Topics* 360.9–13 = *SVF* II 379) reproaches the Stoics for taking air (πνεῦμα) or matter as the genus of quality by saying that quality is air (or matter) in a certain state, because qualities are in their subjects. In the same way, Galen (*On Incorporeal Qualities* 2 = XIX 466–7 Kühn = *SVF* II 384) criticizes the Stoic definition of sound as air struck in a certain way, because he thinks that this characterization depends on reducing the fact of having a property to the thing that has the property.⁴⁰ One can then assume that, according to the Stoics, to say that members of the second category are states of matter means simply that members of it are pieces of matter in a certain state. Therefore, no difficulty arises from what Plotinus says, and we are entitled to continue to represent the Stoic categories as a system of nondisjointed classes.⁴¹ If so, the four classes mentioned by Simplicius in his discussion of relatives cannot be taken to constitute another way of referring to the categories.⁴²

One might object to this conclusion that Simplicius's classes refer to a different doctrine of the categories, where the requirement of inclusion among them was no longer adopted. Nothing compels us to assume that these categories must have the same formal features as those discussed by Plotinus. The doctrine of the categories might have evolved and its evolution marked by the two theories referred to by our sources.

The answer to this objection is, in my view, simple. There is nothing in Simplicius's text about relatives that leads us to conclude that a doctrine of the categories is in question. The mere fact that the fourth category is named in the same way as the kind of relatives that are contrasted with τὰ πρὸς τι does not prove that a doctrine of the categories is involved in the Simplicius passage, but only that πρὸς τί πῶς ἔχον was used by the Stoics in different contexts and for different purposes. Moreover, members of the classes of τὰ πρὸς τι and τὰ πρὸς τί πῶς ἔχοντα are, as we have seen, general terms. Even if we consider them from the point of view of their denotations and suppose that they refer to properties, Simplicius's

classification remains a classification of properties, where individuals are out of the question. Whatever reference is attributed to the Stoic categories, it seems difficult to exclude individuals from it. Therefore, the differences between the doctrine of the categories and the partition of relatives reported by Simplicius cannot be explained by thinking of a possible evolution of the theory, because the conceptual framework of the two doctrines is different.

X

Another question must be investigated, the discussion of which will, I hope, throw further light on some of Simplicius's statements that have up to now been left in the shadows. The problem can be formulated as follows. Simplicius speaks of the Stoic division of two kinds of relatives without reporting any general definition. There are other sources, mainly the Greek and Latin grammarians, Sextus Empiricus and Aulus Gellius, who refer to material on relatives that is normally thought to come from the Stoics. From Simplicius and the other sources can we extract a consistent view about the way in which the Stoics defined relatives in general, or at least about the kind of relativity they appealed to in their conception?

In order to try to answer this question, we will first consider the Simplicius passage. With reference to it, Julia Annas and Jonathan Barnes say:

(N) What sort of relativity is being invoked by this distinction <*sc.* the distinction between τὰ πρὸς τι and τὰ πρὸς τί πῶς ἔχοντα>? We suggest – once again tentatively – that the relativity is a form of ontological or existential dependence. Fathers are relative to children in the sense that fathers exist only if children exist; right-hand things are relative to left-hand things in the sense that right-hand things exist only if left-hand things exist. (The examples may not be wholly convincing, for we may wonder if fathers really are dependent on children in this way. But the principle which the examples are designed to illustrate is nonetheless reasonably clear).⁴³

I think that, when writing these words, Julia Annas and Jonathan Barnes had in mind what Simplicius says at 166.24–26: “a man may cease to be a father when his son dies and someone may cease to be on the right when what is next to him has changed position.” Looking at this passage I have to confess that “the principle which the examples are designed to illustrate” is far from being clear to me. How must the notion of existence be conceived? It may be taken in a logical sense, in which case “*a* exists” means “*a* is one of the possible values of individual variables,” and ontological relativity can be expressed by something like

$$(23) \quad R(a, b) \rightarrow (\exists x(x = a) \rightarrow \exists y(y = b)).$$

If (23) is allowed to define ontological relativity adequately, it is plain that whatever is related to something is ontologically related to it, because (23) is a logical law at least in all systems in which existential generalization is assumed to hold.

Therefore, (23) does not help distinguish one kind of relativity from another. But “existence” can also be taken to mean “continue to exist.” In this sense, for instance, one could say that Alexander the Great does not exist, meaning that he is no longer alive.⁴⁴ The reference to dead sons in Simplicius’s passage makes it clear that “existence” is taken in this sense. It can then be interpreted as a predicate, say “ $E!(x)$,” and ontological relativity can be represented by

$$(24) \quad R(a, b) \rightarrow (E!(a) \rightarrow E!(b)).$$

We could make this stronger by stating simply

$$(25) \quad R(a, b) \rightarrow E!(b).$$

If we apply (24) or (25) to the case of fathers and sons, we get the result that if a is the father of b and b ceases but a continues to exist, then a is no longer the father of b . That sounds strange, especially if we consider the converse relation of “being the father of . . . ,” namely “being a son of” I would like to know the reaction of Golo Mann if one claimed seriously that he was no longer one of the children of Thomas.⁴⁵

One might try to meet this difficulty in the way Julia Annas and Jonathan Barnes do, namely by pointing out that the example of fathers and sons is ill-chosen. Even if that must be allowed, I am reluctant to concede that ontological relativity is in question here for at least two reasons. First, it is odd that either (24) or (25) is applied in the case of fathers and sons with the problematic conclusion that sons are no longer sons when their fathers die, but it is not applied in the case of right and left, where it would be acceptable. If a is on the right of b and b ceases to exist, then a ceases to be on the right of b . But that is not what is said in our text. What is said is that it may happen that b changes its position with respect to a in such a way that a can no longer truly be said to be on the right of b . No reference to existence is made here, and that looks strange if ontological relativity is at issue. Second, the Simplicius passage that is alleged to contain a hint at ontological relativity is meant to give examples of τὰ πρὸς τί πῶς ἔχοντα and to show that these examples behave differently from τὰ πρὸς τι. If one claims that in these examples ontological relativity is in question, it must be admitted that either the distinction between τὰ πρὸς τί πῶς ἔχοντα and τὰ πρὸς τι is a distinction between relational properties that all involve ontological dependence or, at least, that ontological dependence is entailed only by τὰ πρὸς τί πῶς ἔχοντα. Both statements are false. There are πρὸς τι properties that do not, and πρὸς τι properties that do, satisfy (25). One can continue to admire b even if b no longer exists, but one cannot perceive b after it has ceased to exist. Therefore, Simplicius’s discussion of πρὸς τι and πρὸς τί πῶς ἔχοντα does not concern itself with ontological relativity.⁴⁶

This negative conclusion does not help us solve the problem we started with. Let us consider the examples of τὰ πρὸς τι that Simplicius uses more frequently, namely the pair sweet–bitter. In what sense are the sweet and the bitter said to

be πρὸς τι, namely relational terms? Noticing that the sweet and the bitter can be taken as contraries, some scholars have thought that the sweet and the bitter are related to each other in the same way as contraries are, and they have suggested that there is an analogy between the Simplicius passage and the doctrine of relatives that Simplicius refers to and attributes to Hermodorus.⁴⁷ But Simplicius does not say that the sweet is related to the bitter, nor is this a consequence of his statement that the sweet and the bitter are among τὰ πρὸς τι. Moreover, I do not see how the claim that the sweet and the bitter must be taken as contraries can be reconciled with the fact that the internal states of the objects of which τὰ πρὸς τι are true change if they lose these relational properties. Suppose that the sweet is taken as the contrary of the bitter. Shall we say that if the sweet was no longer the contrary of the bitter, it would have changed its internal state? This is a counterfactual conditional, and even if one may think that it is true, it does not prove that the sweet and the bitter are among τὰ πρὸς τι, but simply that the property of being a contrary is. On the other hand, if the point is that nothing can cease to be sweet without changing its internal state, this, of course, shows that the sweet is among τὰ πρὸς τι, but it has nothing to do with its being contrary to the bitter.

A similar reasoning shows that the sweet and the bitter cannot be taken as epistemologically related. Roughly speaking, epistemological relativity is a relation that holds between general terms. We can say that $\lambda xF(x)$ and $\lambda yG(y)$ are epistemologically related if, for any x , the capacity to know that x is F on the part of a subject implies and is implied by the capacity to know that y is G . Formally we have:

$$(26) \quad E(\lambda xF(x), \lambda yG(y)) \text{ iff} \\ \forall x \forall y (F(x) \wedge G(y) \rightarrow (MK_n(F(x)) \leftrightarrow MK_n(G(y))))$$

where “ $E(x, y)$ ” stands for “ x is epistemologically related to y ,” “ M ” is the operator of possibility, and “ $K_n(P)$ ” stands for “ n knows that P .”⁴⁸ We need not try to determine what a capacity or possibility of knowing is. It is more interesting to remark that there is no reason to think that epistemological relativity can account for the requirement of internal states, which is said to hold for R'-relatives. Suppose that a is sweet and b is bitter, $S(a)$ and $B(b)$, and that the capacity to know that $S(a)$ entails, and is entailed by, the capacity to know that $B(b)$. It is clear that these assumptions do not justify the peculiar state of sweet things, according to which, if $S(a)$ ceases to be true, then the internal state of a must have changed. Because the relativity enjoyed by the sweet and the bitter must be such that it explains the requirement of internal states that is said to hold for the sweet and the bitter, we conclude that the sweet and the bitter are not characterized by epistemological dependence, at least not if we conceive it in the sense of (26).

Our problem may be restated as follows: Is there a sense in which the sweet and the bitter can be said to be relational terms and still pass the requirement of internal states proper to τὰ πρὸς τι? We will consider two possible ways in which the sweet and the bitter might be said to be relational. First, we might say that the sweet and the bitter are dispositions of matter. Sweetness is always sweetness of something or someone, a cup of coffee or my wife. But this kind of dependence

cannot be that which characterizes τὰ πρὸς τι, because it is a consequence of their lack of independent existence, and this feature is also true of τὰ πρὸς τί πῶς ἔχοντα. Therefore, τὰ πρὸς τι would not constitute a class that is disjointed with respect to the class of τὰ πρὸς τί πῶς ἔχοντα.

There is another sense in which the sweet and the bitter could be said to be relational terms. One might claim that a thing is sweet (or bitter) with respect to someone who tastes it. That does not necessarily imply that the meaning of “sweet” on this view becomes “sweet for someone” and that a relativistic account of sweet and bitter is introduced. It is sufficient to suppose that there is a link between these properties and an experiencing subject. In a similar way, one can sensibly say that knowledge and perception presuppose a knowing subject who knows and perceives. One might object to this view that it is easy to imagine a case in which the requirement of internal states is not met. If *a* is sweet with respect to *m*, where “*m*” stands for a person and *a* ceases to be so, it does not necessarily follow that *a* has changed its internal state. It might simply happen that *m* has changed, as the examples of normally sweet things that taste bitter in certain exceptional circumstances show. We have already given an answer to this objection by introducing natural necessity and possibility in the characterization of πρὸς τι and πρὸς τί πῶς ἔχοντα terms. It is not naturally possible that there is a man who does not taste as sweet a thing that normally tastes sweet. Another more ad hoc answer is to assume that “sweet” means not “sweet for someone,” but rather “sweet for the average man.” If we define “sweet” not with reference to the individual but the normal taste, that is, with reference to what normal or standard people taste, then if *a* ceases to be sweet, it is reasonable to conclude that *a* has changed its internal state. The problem of things that appear sweet to *m* but not to *n* does not affect our point because, in this case, we are dealing with another meaning of “sweet,” which we may express by something like “what tastes so and so to someone.” What is denoted by “sweet,” taken in this sense, is not an *R'*-relative; but this does not imply that what is denoted by “sweet” taken in the other sense is not an *R'*-relative.⁴⁹

If this interpretation is accepted, there is a confirmation of what it was reasonable to expect, namely that the sweet and the bitter are relational terms in the same sense in which the other examples of πρὸς τι mentioned by Simplicius are said to be so. Simplicius puts among τὰ πρὸς τι not only the sweet and the bitter but also possession, knowledge, and perception (166.20–21), and these cannot be classified as cases of ontological or epistemological dependence. On the view we are trying to defend, the case of sweet and bitter runs parallel to the case of knowledge and perception. One is entitled to maintain that “sweet” means “sweet for someone” (or “with respect to a standard taster”), just as “knowledge” means “knowledge of something” and “perception” means “perception of something.”⁵⁰

This remark appears to me important, because it makes it plausible to suppose that there is a homogeneous notion of relativity that constitutes the background of the distinction between τὰ πρὸς τι and τὰ πρὸς τί πῶς ἔχοντα. This claim can be illustrated as follows. The examples of τὰ πρὸς τι Simplicius offers can be interpreted as referring to the same sort of relativity, a relativity we call, after

Jonathan Barnes and Julia Annas, “semantical relativity.”⁵¹ The examples of τὰ πρὸς τί πῶς ἔχοντα Simplicius offers can be also put under this label: “*a* is on the right” means “*a* is on the right of something (or someone)” and “*a* is a father (son)” means “*a* is the father (son) of someone.” The same holds for other comparative adjectives, such as whiter, more, longer, quicker, older, and upper, which Simplicius quotes in his discussion of the Stoic view as belonging to τὰ πρὸς τί πῶς ἔχοντα (167.6–10).

Shall we conclude that the framework of the Stoic distinction is constituted by semantical relativity? For the time being, I take it as a conjecture that is plausible but not yet proven. The conclusion that the examples of relatives quoted by Simplicius can be seen as cases of semantical relativity and the remark that the Stoic distinction is consistent with this hypothesis do not prove that their object really was to distinguish πρὸς τι from πρὸς τί πῶς ἔχοντα in the field of semantical relativity. We must look for further evidence.

XI

Let us start by considering a distinction that is referred to by the Greek grammarians. In his *Ars grammatica* Dionysius Thrax distinguishes τὸ πρὸς τι ἔχον from τὸ ὡς πρὸς τι ἔχον. Again, we avoid translating these expressions and we call them “*G*-relative” (“grammarians’ relative”) and “*Q*-relative” (“quasi-relative”), respectively. Dionysius does not define them but gives us examples. Examples of *G*-relatives are father, son, and friend on the right; examples of *Q*-relatives are night and day and life and death (*GG* 1 1, 35.3–4). The scholiasts of the *Ars* explain this distinction using material that is considered by all scholars, together with the distinction itself, to be Stoic in origin.⁵² They give a definition of both items, and they try to show their differences. The most elaborate discussion of the subject can be found in the *Scholia Marciana* and has been attributed to Heliodorus, a grammarian of the eighth century AD.⁵³ He first offers a definition of τὸ πρὸς τι ἔχον by saying:

(O) A πρὸς τι ἔχον is that which has a relation to something else; if one gives one of them, an understanding of the other comes and ceases with it; for instance, when I say “father,” I understand the son too, since there will be no father if there is no son. Thus, when there is the son, there will be also the father; but if the son is destroyed, the name of the father is also destroyed.

(*GG* I 3, 387.8–12)⁵⁴

I take the expression “I say ‘father’” as meaning “I say that what is denoted by ‘father’ is true of someone” or “I say that someone is a father.” Correspondingly, I take “I understand the son too” to mean “I understand that there is someone of whom the person at issue is the father” and “the name of the father is [. . .] destroyed” as equivalent to “what is denoted by ‘father’ is not truly predicated of anything.” In other words, what characterizes a *G*-relative such as father is that we know that *a* is a father if, and only if, we know that there is a child of *a* or, what is the same, that *a* is the father of a child. If we take “*F*(*x*)” to stand for “*x* is a father”

and “ $\exists y R_F(x, y)$ ” for “ x is the father of someone” (namely “there is a child of x ”), we can express dependence in knowledge of fathers and children as follows:

$$(27) \quad \forall x (K_n(F(x)) \leftrightarrow K_n(\exists y R_F(x, y))).^{55}$$

This epistemic dependence is justified by Heliodorus by means of an ontological dependence: There is a father if, and only if, there is a child. This statement must be interpreted. Actually, when we say that there cannot be a father without a child, what we mean is not only that if there is a father there is also a child, but, more precisely, that a is a father if, and only if, there is a child of him:

$$(28) \quad \forall x (F(x) \leftrightarrow \exists y R_F(x, y)).$$

We must take (28) as a justification of (27) in a very broad and weak sense. We cannot know fathers without knowing their children, and this is confirmed by the fact that fathers cannot exist without their children. Strictly speaking, from the mere fact that fathers cannot exist without their children, it does not follow that we cannot know that a is a father if we do not know that someone is his child. However, in another *scholion* of the Marcian collection, which is also attributed to Heliodorus (*GG* I 3, 387.34 ff.), (27) and (28) are used to characterize *G*-relatives without implying that (28) entails (27). The same view can be found in the *Scholia Vaticana* (*GG* I 3, 235.12–18 and 193.23–29). Actually, in the second passage of Heliodorus and in the *Scholia Vaticana*, (28) is used to distinguish *G*- from *Q*-relatives: Both satisfy something similar to (27), whereas *G*-, but not *Q*-relatives, fulfill (28). We will return to this point later. For the time being, let us take (27) and (28) as expressing the features that characterize *G*-relatives.

We must now consider the meanings of (27) and (28) and what kind of relativity is implied by relatives so defined. The question can easily be answered if we consider what Heliodorus says of *G*-relatives in his attempt to distinguish them from *Q*-relatives. According to him, a πρὸς τι ἔχον is such that it implies in thought “of someone or something.” Thus, Heliodorus explains, if I say “son,” my addressee asks me “of whom?” (*GG* I 3, 388, 23–24). It is clear that the kind of relativity involved in terms such as “father” and “child” and, in general, by *G*-relatives is semantical relativity. Roughly speaking, we can say that semantical relativity affects pairs of general terms that are such that the meaning of one of them is expressed by the other and is given by its representing the domain of a relation. In the case of being a father, the meaning of “father” is constituted by “being the father of someone” or “having a child.” If we take “ $J(x, y)$ ” to stand for “ y represents the meaning of x ” and “ $S(x, y)$ ” for “ x is semantically related to y ,” we can state:

$$(29) \quad S(\lambda x F(x), \lambda x G(x)) \text{ iff } \lambda x G(x) = \lambda x \exists y R_F(x, y) \wedge J(\lambda x F(x), \lambda x G(x)).$$

It is not necessary to go deeply into the notion of meaning. It is sufficient to take as the meaning of a general term that which allows us to use it correctly or expresses

its truth conditions. If (29) is admitted to represent semantical relativity, (27) and (28) can be affirmed as soon as one recognizes that the terms at issue satisfy (29). If $\lambda x \exists y R_F(x, y)$ constitutes the meaning of “ $\lambda x F(x)$,” one cannot know $F(a)$ without knowing $\exists y R_F(a, y)$. On the other hand, if “ $\lambda x F(x)$ ” means that which constitutes the domain of a relation, then nothing can be truly F unless there is an image to which it is related by means of the relation that defines F – and that is precisely what (28) says.

Consider now τὰ ὡς πρὸς τι ἔχοντα (our Q -relatives). In the *Scholia Marciana* (GG I 3, 387.27–33), two definitions of Q -relatives are given. According to the first, Q -relatives are simply identified with contraries: Night and day, health and disease, death and life, and light and darkness are Q -related, because they are contraries. There is also a more detailed definition, according to which any pair of Q -related things are contraries depending on each other in knowledge but not in being, in the sense that destruction of the one does not imply destruction of the other. The passage of the *Scholia Vaticana* that comments on Dionysius’s distinction (GG I 3, 235.1936) relies on the first definition, whereas the second definition is implied by another passage of the *Scholia Vaticana* (GG I 3, 193.29–37) and by Heliodorus’s discussion of the distinction between G - and Q -relatives. I do not believe that the difference between the two definitions is very important. The first is a part of the second, and it is designed to stress the relevant feature that makes Q -relatives different from G -relatives. Indeed, one could take the first definition to be equivalent to the second if one were prepared to admit that mutual dependence in knowledge is a mere consequence of contrariety. This hypothesis is confirmed by a passage of Sextus Empiricus (*Against the Professors* 10.264) where contraries are characterized by the fact that the elements of each pair of them are linked by a mutual cognitive dependence. Although this view is attributed by Sextus to the Pythagoreans (*Against the Professors* 10.263), it might be that it also circulated among the Greek grammarians. At any rate, we will consider the second definition as the real definition of Q -relatives.⁵⁶

The meaning of this definition can be better grasped if we put it in the context of the distinction between τὰ πρὸς τι ἔχοντα and τὰ ὡς πρὸς τι ἔχοντα that the scholiasts try to illustrate. In the *Scholia Marciana* Heliodorus makes the point in the clearest way. He says that of the two conditions that G -relatives satisfy (namely, dependence in knowledge and dependence in existence) Q -relatives fulfill the first but not the second (GG I 3, 387.34–388.19).⁵⁷ In what sense do Q -relatives not satisfy existential dependence? Because they do not satisfy (28). In the case of fathers and children, if a is a father, then there must be someone of whom a is the father. Therefore, if there is nobody of whom a is the father, a is not a father. That depends on the fact that if something is in the domain of a relation, there must be something in its range. But take, for instance, life and death. From the fact that there is life with reference to a given state of affairs, it does not follow that there is death with respect to the same state of affairs, but rather the opposite follows: If there is life, there is no death. Perhaps the different way in which Q -relatives behave with respect to (28) can be made clearer if we consider a pair such as alive–dead, say $\lambda x A(x)$ and $\lambda x D(x)$. G -relatives such as $F(a)$ and $\exists y R_F(a, y)$ stand

or fall together, in the sense that if $\lambda x F(x)$ is true of a , $\lambda x \exists y R_F(x, y)$ must also be true of it, and vice versa. This does not hold in the case of contraries: $\lambda x A(x)$ and $\lambda x D(x)$ cannot be true of a at the same time, because $\lambda x D(x)$ cannot be true of a as long as $A(a)$ holds.

Let us now consider dependence in knowledge. As we have seen, Heliodorus says that both pairs of *G*-relatives and pairs of *Q*-relatives are such that they depend on each other in knowledge. Shall we conclude from this that *Q*-relatives fulfill (27) in the same sense that *G*-relatives do? We must answer “no” to this question, because Heliodorus clearly states that *Q*-relatives are dependent on knowledge differently than *G*-relatives are. He says:

(P) For the night appears to be said with respect to the day in the same way as the right is said with respect to the left and the father with respect to the son. But they differ in the sense that in the case of the relative (τὸ πρὸς τι) “of which (whom)?” must be supplied in thought. For when I say “son,” in general “of whom?” must be supplied to this expression. That does not appear to occur in the case of quasi-relatives (ὡς πρὸς τι). For when I say “night,” “of which?” is not added to this expression, in the sense that I supply “of the day,” since the night is a proper interval, which is not seen with reference to the day.
(GG I 3, 388.20–26)⁵⁸

Both πρὸς τι ἔχοντα and ὡς πρὸς τι ἔχοντα imply dependence on knowledge: If a is πρὸς τι ἔχον or ὡς πρὸς τι ἔχον with respect to b , a cannot be understood without b . But in the case of *G*-relatives this dependence on knowledge is a dependence on meaning in the sense that a sentence such as “ a is a son” cannot be understood without making it complete by stating “ a is the son of b (or someone).” That is not the kind of epistemic dependence that affects τὰ ὡς πρὸς τι ἔχοντα. In other words, “ a is alive” does not need to be completed in order to be understood. On the other hand, there is a knowledge-dependent connection between life and death, and this connection applies to any pair of contrary terms. As far as I know, the grammarians do not specify what this cognitive connection is. My conjecture is that it can be explained by what we have called “epistemological relativity” and have formalized by means of (26). If a is alive and b is dead, then n is able to know that a is alive if, and only if, n is able to know that b is dead. If this assumption is plausible, τὰ ὡς πρὸς τι ἔχοντα must be conceived as a set of pairs of properties that exclude each other and that are epistemologically related.⁵⁹

I am not particularly fond of this conjecture, and I do not feel committed to defend it beyond a certain point. For the time being, I would rather stress that *G*-relatives constitute the class of semantically linked general terms and that *Q*-relatives, whatever they actually are, do not belong to such a class. This statement is important because both Heliodorus and another anonymous *scholion* of the Marcian collection emphasize that ὡς in the formula ὡς πρὸς τι ἔχον by which *Q*-relatives are referred to expresses similarity with respect to *G*-relatives (GG I 3, 387.29; 388.12. ff.). This similarity is explained by *Q*-relatives sharing with *G*-relatives dependence on knowledge, but not on existence. A possible way to

justify this way of putting things is to assume that *G*-relatives are conceived relatives in a proper sense, whereas *Q*-relatives refer to terms that are not relative in a real way but to some extent approximate to the situation of being relative. It is difficult to think that τὰ πρὸς τι ἔχοντα and τὰ ὡς πρὸς τι ἔχοντα constitute two species of relatives if their relation is a relation of similarity. It is closer to the way in which the scholiasts express themselves to say that the former class is the class of all relatives and the elements of the latter only resemble relatives.

I am aware that this is not the only possible way to explain the terminology used by the grammarians. Nonetheless, it is one of the possible explanations, and we can perhaps find a slight confirmation of it in a passage of Sextus Empiricus, parts of which we have already looked at in the discussion of the grammarians' position. In *Against the Professors* 10.263–267, Sextus attributes to the Pythagoreans a view according to which they divided reality into things that are conceived according to a difference (κατὰ διαφορὰν νοεῖται: namely things that are conceived as absolute) contraries and relatives. Contraries and relatives are distinguished in the same ways in which τὰ ὡς πρὸς τι ἔχοντα and τὰ πρὸς τι are distinguished in the grammarians' theory (*Against the Professors* 10.266–267). It is clear that in Sextus's report, contraries do not form a subclass of relatives. They constitute a class that is on the same level as the classes of absolute things and relatives with respect to which it has similarities. Of course, the doctrine reported by Sextus is not attributed by him to the Stoics, and in this sense his passage cannot be taken as a direct confirmation of our interpretation. However, the idea that contraries do not constitute a species of relatives was not foreign to the cultural framework in which the grammarians operated, and in this sense Sextus's text can be taken to be an indirect confirmation of our interpretation.

Actually, both Sextus's Pythagoreans and Aristotle (or at least the Peripatetic tradition) testify to a sharp distinction between relatives and contraries. In *Categories* 10, 11b32–38 it is said that contraries cannot be construed as relatives. Double is relative to half, and therefore if something is said to be double, it is said to be double of something that is the half of it. The same does not hold for pairs of contraries. Consider white and black, which are contraries. If one says that *a* is white, this does not amount to saying that *a* is the white of something which is black.⁶⁰ "White" does not mean "white of something which is black," and in this sense it is not relative to the black. The same doctrine probably forms the background of divisions 67 (about relatives) and 68 (about contraries) of the so-called *Divisiones aristoteleae*.⁶¹

It would be interesting to see whether this view was a polemical statement directed against the Academic position held, for instance, by Hermodorus and attested to by Simplicius. Hermodorus divided real things into καθ'αυτὰ and πρὸς ἕτερα and distinguished the latter into contraries and relatives.⁶² Shall we conclude from this that Hermodorus considered contraries a species of relatives? If so, his view was inconsistent with the Aristotelian position, and we might consider the latter a reaction to the former. But this interpretation is not the only possible one. One might take Hermodorus's words to state that relatives and contraries constitute two different species of τὰ πρὸς ἕτερα, where τὰ πρὸς ἕτερα form the complement

of the class of τὰ καθ'αὐτά, namely the class of things that do not exist by themselves and not the class of generic relatives. Evidence in favor of this interpretation is the fact that, on the one hand, Alexander (*On Aristotle's Metaphysics* 56.13–15) uses ἀντικείμενον where one would have expected πρὸς ἕτερον and, on the other hand, Simplicius (*On Aristotle's Categories* 63.24–26) says that according to some authors, πρὸς τι that is opposed to καθ'αὐτό (Hermodorus's πρὸς ἕτερον) has to be equated with the Aristotelian κατὰ συμβεβηκός, because accidents do not exist by themselves.⁶³ Taken in this sense, Hermodorus's position becomes not very different from the view that Sextus attributes to the Pythagoreans as far as the problem that interests us is concerned, and the tradition of a clear separation between contraries and relatives is confirmed for the first followers of Plato.⁶⁴ We cannot continue with this enquiry, which would take us far from the object of our study, but I hope that what we have briefly said is sufficient to show that the idea of a sharp separation between relatives and contraries possibly held by the Stoics had a long tradition in the Academic soil in which their philosophy was rooted.

Before analyzing other texts, let us briefly consider the result of this interpretation in the light of Simplicius's account. If we rely on the conjecture we have put forward with reference to Simplicius, what the Greek grammarians called πρὸς τι ἔχον must be identified with the πρὸς τι, which in Simplicius's report denotes the genus of *R'*- and *P*-terms and which we have called “*R*-relative.” What the grammarians call τὰ ὡς πρὸς τι ἔχοντα, on the other hand, correspond neither to τὰ πρὸς τι πῶς ἔχοντα nor to τὰ πρὸς τι, which are opposed to these (our *R'*-entities), but refer to another class of things that cannot properly be called relative, being only like relatives. The comparison of Simplicius's account with the evidence coming from the grammarians allows an explanation of the controversial example of fathers and sons that Simplicius considers. As we have seen, he maintains that a father ceases to be a father when his children die, and correspondingly that a son ceases to be a son when his father dies. This view can be connected with the view expressed by the grammarians, according to which *a* is not a father if none of his children exists. We have formalized this latter view by means of (28), where “existence” was taken in its logical sense. It is quite clear that if we take “existence” in this sense, *a*'s being a father depends on the existence of his children. If no child of *a* exists, namely there is nothing in the universe of discourse that is a child of *a*, then *a* cannot be a father, because *a* cannot be in the domain of a relation whose range is empty. But from the fact that if *a* is a father, there is at least one individual who is his child, it does not follow that if *a* is a father, then at least one of his children exists in the sense that this child is alive. If “existence” is taken in its nonlogical sense, “*a* is a father” does not entail “at least one child of *a* exists.” We can refer to dead men and truly attribute properties to them as a consequence of the anaphoric power of natural language. Thus, dead men too are in the logical universe of discourse, as well as living men and whatever is an object of true predications. Therefore, the distinction between living and nonliving men, namely the distinction between existing and nonexisting things in the sense of *E*!, is a distinction that generates two subclasses of the logical universe, and what holds of the logical universe in

general applies also to the members of the two subclasses. A consequence of this way of conceiving existence is that

$$(30) \quad \exists xF(a, x) \rightarrow E!(a)$$

does not hold in general. From this point of view, it is not correct to assume that “there is no child” is equivalent to “no child is alive.” I suspect that this is precisely the confusion that Simplicius (or his source) makes when he says that if none of a ’s children is alive, then a is not a father. He (or his source) probably had in mind the doctrine referred to by the grammarians, which is sound if the logical meaning is given to “existence,” and he tried to apply it to illustrate a case of P -relatives: a ’s being a father can be made entirely dependent on the existence of his children so that a can cease to be a father without internal alteration. In doing so, “existence” must be treated as a property; but then we cannot appeal to the general doctrine about relations to justify the inference from “no children of a exist” to “ a is not a father,” which is in fact fallacious if “existence” is taken in the sense of $E!$. Notice that the confusion between the two meanings of “existence” affects Simplicius’s report but not the grammarians’s. However, it is a minor point that does not compromise the distinction between P and R' -relatives, which the ill-chosen example of fathers and sons was intended to illustrate.

XII

Let us now consider a connected piece of evidence about relatives, namely what the Latin grammarians say about them. As is well known, the Latin grammarians, with the exception of Priscian (*Inst.* II 28–29 = *GL* II 60.19–271), explain relative terms in a manner different from the one adopted by the scholiasts to Dionysius Thrax. For instance, Diomedes says:

(Q) There are some nouns, such as “*pater*” and “*frater*,” which cannot be understood without the addition of other parts of discourse. For they involve in themselves also that by means of which they are understood, as is the case with “*meus*” and “*tuus*.” These words are called by the Greeks “πρός τι,” i.e. “to something” (*ad aliquid*). They are similar to τὰ πρός τι πῶς ἔχοντα, i.e. that which turns somehow toward something (*ad aliquid quodam modo adtendentia*) or that which is in such a way that (*taliter qualiter se habentia*), such as “*dexter*,” and “*sinister*.” These words admit of a comparative form, such as “*dexterior*,” and “*sinisterior*.”

(*GL* I 322.27–33)⁶⁵

The difference between the Latin grammarians and the Greek was noticed by Barwick, who concluded that the Roman tradition is nearer to the Stoic doctrine than the Greek. This view was criticized by Pohlenz, especially on the grounds that the Latins put the pair right–left in a different category than the one to which father and brother belong, against the evidence not only of Simplicius but also of Varro

(*De lingua latina* 10.59 = *SVF* II 155).⁶⁶ I do not know whether the Roman grammarians were more or less faithful than the Greek grammarians to Stoic teaching. What must be underlined is that the class of *ad aliquid* is a class of substantives, whereas the class of *ad aliquid quodam modo adtendentia* is a class of adjectives, because they admit of a comparative form. Thus, it is not necessary to suppose that the distinction between the two classes is made because they are different from the point of view of logic. What is relevant in grammar is not always so in logic. In my view, the Latin grammarians simply made a distinction different from the distinction made by their Greek colleagues. The latter distinguished relatives that show a semantical dependence from those that exhibit an epistemological dependence, if the interpretation we have proposed is accepted, and this distinction has something to do with logic and perhaps with grammar. The former made a distinction within the class of relatives showing semantical dependence between relatives that are names and relatives that are adjectives, and this distinction affects grammar more than logic.

If this interpretation is adopted, two conclusions can be drawn. First, Greek and Latin grammarians do not offer us contrasting reports of the Stoic doctrine (if they are reporting Stoic material), but simply refer to the same theory from different points of view; and this is not, of course, a sign that the theory is in itself inconsistent. Second, if the Latin grammarians are supposed to reflect Stoic sources, they attest to a Stoic conception of relativity as semantical relativity, and that is in line with the interpretation we have proposed of the evidence coming from the Greek grammarians.

XIII

Let us move now in our review of the evidence at our disposal from the grammarians to Sextus Empiricus. In the context of a discussion in which the Stoics are probably involved, he offers a general definition of relatives that is worth quoting. He says:

(R) And that relatives (τὰ πρὸς τί πως ἔχοντα) are, in truth, only preserved by conception (τῷ ὄντι ἐπινοία μόνον σώζεται) and that they have no real existence (ὑπαρξίς), one may show by the admission of the Dogmatists. For in describing the relative (πρὸς τι) they say with one accord: “Relative (πρὸς τι) is that which is conceived with respect to something else” (πρὸς ἑτέρῳ νοούμενον); whereas if it had participated in real existence (ὑπαρξέως μετέχειν) they would not have given that account of it but rather this: “Relative (πρὸς τι) is that which exists with respect to another (τὸ πρὸς ἑτέρῳ ὑπάρχον).” Therefore the relative is not among the things which exist (ἐν τοῖς οὖσι).

(Sextus Empiricus, *Against the Professors* 8.453; part of it is printed in *SVF* II 404; R. G. Bury translation, slightly modified)

It is quite clear that a general definition of relatives is intended here and that it points to semantical relativity.⁶⁷ It would be easy to conclude that semantical

relativity is what the Stoics discussed if it could be proven that the Dogmatists mentioned by Sextus are the Stoics. There are some clues that lead one to say that this is so. First, Sextus uses the expression τὰ πρὸς τί πῶς ἔχοντα to indicate relatives, and that is a part of the Stoic terminology. Second, the context of the discussion refers to the Stoics. For instance, in the lines that follow the passage we are discussing, Sextus considers a Stoic argument.⁶⁸ Although this evidence is not conclusive, it is enough to convey the impression that we are here in the presence of Stoic material. But even so, one might be skeptical about concluding that the Stoic notion of relativity is the notion of semantical relativity. Sextus wants to prove that relations have no existential ground, and to show this he appeals to the definition we are discussing. His polemical task makes his report suspicious. We are not sure that he would have informed us about other possible definitions of relatives that the Stoics might have proposed if they had been less favorable to his thesis. This impression is reinforced by the reflection that, in a passage that immediately follows in the text we have just quoted (*Against the Professors* 8.455–456), Sextus seems to reduce relatives to what Simplicius calls τὰ πρὸς τί πῶς ἔχοντα, namely *P*-relatives, which undergo Cambridge change; and, of course, it is not true that any pair of semantically related properties passes the test of Cambridge change. However, even if it is inconsistent with other passages, his general definition of relatives is consistent with the views expressed by the Greek and the Roman grammarians, and in this sense it can be taken to be a confirmation of the conjecture we have made with reference to Simplicius's account. Although taken by themselves, none of the texts we have analyzed are sufficient to offer a definitive solution to our problem, we shall be less hesitant if we consider them all together. The resulting picture points to semantical relativity as the general framework of the discussion of the Stoic theory of relatives.

XIV

Up to now, a reasonably consistent view of the theory of relatives has emerged from the passages we have considered. To extract this view, we have made some assumptions and conjectures that are not strictly demanded by the texts but that do not force them in an intolerable way. This satisfactory conclusion, however, might be thought to be upset by a passage of Aulus Gellius. In it an argument of Chrysippus is quoted, taken from the fourth book of his *On Providence*, against people who denied the existence of providence on the ground that there is evil in the world. Gellius says:

(S) Chrysippus, arguing against such views in the fourth book of his treatise *On Providence*, says: “There is absolutely nothing more foolish than those men who think that good could exist if there were at the same time no evil. For since good is the contrary of evil, it necessarily follows that both must exist in opposition to each other, supported as it were by mutually adverse forces; since as a matter of fact no contrary is without its contrary. For how could there be an idea of justice if there were no acts of injustice? Or what else

is justice than the absence of injustice? How too can courage be understood except by contrast with cowardice? Or temperance except by contrast with intemperance? How also could there be wisdom, if folly did not exist as its opposite?" "Therefore," he said, "why do not the fools also wish that there may be truth, but no falsehood? For it is in the same way that good and evil exist, happiness and unhappiness, pain and pleasure. For, as Plato says, they are bound one to the other by their contrary extremes; if you take away one, you will have removed both."

(Aulus Gellius, *Attic Nights* 8.1.2–6 = *SVF* II 1169;
J. C. Rolfe translation, slightly modified)

The point is clear: Good things cannot exist without bad ones, because they are contraries. If a is good, namely $G(a)$, then there must be something which is bad, $\exists xB(x)$.⁶⁹ This might be taken as evidence against the view expressed by the Greek grammarians, who rejected any existential commitment for contraries. As we have seen, according to them, contraries must be known together, but they need not be together. Here an apparently different view is stated, which can be expressed formally as follows:

$$(31) \quad C(\lambda xF(x), \lambda yG(x)) \rightarrow (\exists xF(x) \leftrightarrow \exists yG(y))$$

where " $C(x, y)$ " stands for " x is contrary to y ." In the grammarians' passages, existential dependence of relatives was connected with semantical relativity. But pairs of contraries are not semantically connected if we accept the characterization of semantical relativity we have given. Shall we conclude that Chrysippus in Gellius's account considered contraries to be a kind of relative?

To answer this question, let us simply observe that (31) is not inconsistent with the grammarians's view about contraries. When they say that a pair of contraries does not entail mutual existential dependence, what they actually mean is that two contraries cannot be true of the same individual, so that if there is something that is white, it does not follow from this that the same thing is also black or that black is true of it. It is precisely this fact that makes contraries different from relatives, because for the latter, but not for the former, it holds that if a is a father, then there is something of which a is the father (see (28)). Therefore, there is no need to think that Chrysippus counted contraries among relatives. We cannot stay longer on this point. However Chrysippus may have justified (31), it must be stressed that the Gellius passage does not offer any evidence that allows us to attribute to him a view about relatives. What it allows us to say is that Chrysippus probably had a theory of contraries and that, as far as we know, this theory was not inconsistent with the statements made by the Greek grammarians.

XV

We have a further problem to consider. Is there any clue that permits us to attribute the doctrine of relatives we have sketched to any particular member of the

Stoic school? For the distinction between τὰ πρὸς τι and τὰ πρὸς τί πῶς ἔχοντα, as we have seen, we possess a *terminus ad quem*, namely the commentary on the *Categories* written by Boethus of Sidon (Simplicius, *On Aristotle's Categories* 163.6–7) in which the Stoic doctrine was discussed and criticized (167.2–4; 20–26). This means that the Stoic theory had already been developed, at least in its essential parts, by the middle of the first century BC. As far as the view reflected by the Greek grammarians is concerned, the fragments of Heliodorus we have considered appear to go back to Apollonius Dyscolus, that is, to the first half of the second century AD; but it is possible that they refer to much older Stoic views. We have no evidence at all to determine how far we may trace the theory back.

These rather weak results contrast with the optimistic statements of some scholars who are ready to attribute to Chrysippus at least a part of the doctrines about relatives we have considered. The most enthusiastic defender of this view is Rieth,⁷⁰ who maintains that the notion of πρὸς τί πῶς ἔχον was used by Chrysippus and that the distinction between πρὸς τί πῶς ἔχον and πρὸς τι is employed by Ariston and Chrysippus in their polemic about the uniqueness of virtue. In the same camp, if a little more cautious, are Max Pohlenz and Margaret Reesor. Pohlenz claims that it is likely that the definition of relatives reported by Sextus Empiricus (*Against the Professors* 8.453) goes back to Chrysippus and that the doctrine of the Greek grammarians should be attributed to him.⁷¹ Reesor thinks that Chrysippus knew and used the notion of πρὸς τί πῶς ἔχον, which he applied to contradictories.⁷²

Let us begin by considering the evidence at our disposal in which the name of Chrysippus is connected with relatives. There are two fragments to be examined. The first is taken from Varro's *On the Latin Language*. Speaking of the relation between oblique and nominative cases and between plural and singular forms, he quotes Chrysippus:

(T) For sometimes the one is seen from the other and at other times the other is seen from the one, as Chrysippus writes, as a father may be seen from his son and the son from his father, and in arches the right-hand side stands on account of the left-hand side, no less than the left on account of the right. Therefore, the oblique forms can sometimes be regained from the nominatives, and sometimes the nominatives from the oblique forms; sometimes the plural from the singular forms, and sometimes the singular forms from the plural.

(Varro, *On the Latin Language* 10.59 = *SVF* II 155;
R. G. Kent translation, slightly modified)

I am not convinced that here the traditional doctrine of semantical relativity is hinted at by Chrysippus. His point does not seem to be that *n* cannot know that *a* is a father without knowing that *a* is the father of a child. Chrysippus's claim is rather that we can recognize the features of a father by looking at his children and vice versa, because there is a family resemblance between them. In a similar way, because the stones of an arch stand because they are tied together and propped up

on each other, from the existence or standing of one of them, which is on the right (left) side of the arch, one can infer that another stone exists or stands on the left (right) side of the arch. Here the reference to right and left is unimportant, being only a way of indicating different stones in an arch. What makes the inference possible is the special link that is supposed to hold between the parts of the arch. If we interpret the two examples of fathers and sons and of the arch stones in this way, the sense of the comparison and the final part of the passage become clear. We can reach nominative from oblique cases and vice versa or plural from singular forms and vice versa. In addition, we can learn to recognize fathers by being acquainted with their children and vice versa and infer the existence of a stone on the left side of an arch by knowing that there is one on its right side and vice versa.

If this is the meaning of the Varro passage, it is rash to infer from it that Chrysippus defined relatives in the way that Sextus ascribes to the dogmatists or that the theory of relatives hinted at by the Greek grammarians goes back to Chrysippus. Unfortunately, our passage says nothing about relatives in general, but only exploits some particular relations to explain a grammatical doctrine.

The second passage is found in Plutarch, who quotes the following statement from the second book of Chrysippus's *Περὶ κινήσεως*:

(U) for after remarking that the universe is a perfect body whereas the parts of the universe are not perfect, since their existence is not independent but is their particular relation to the whole (τῷ πρὸς τὸ ὅλον πως ἔχειν καὶ μὴ καθ' αὐτὰ εἶναι), [. . .] Chrysippus has added this statement.

(Plutarch, *On the Self-contradictions of the Stoics*
1054 E–F = *SVF* I 550; H. Cherniss translation)

Even if the expression *πρὸς τὸ ὅλον πως ἔχειν* could be taken as an instance of the more general expression *πρὸς τί πως ἔχον* by which relatives are sometimes referred to, Plutarch's text does not show that Chrysippus had in mind a doctrine of relatives when he stated that the existence of parts depends on the whole of which they are parts. To be generous, we can only suppose that a theory of the relation between parts and whole forms the background of Chrysippus's statement, and we know that the Stoics had a view about that.⁷³

Let us now consider the discussion between Ariston and Chrysippus on which Rieth relies to find the ultimate source of the doctrine reported by Simplicius. As we have said, Rieth thinks that Ariston, in defending his claim that virtue is unique, used the same notion of *πρὸς τί πως ἔχον* we have met in Simplicius's discussion of the Stoic position. If that were true, we could conclude that the distinction between *τὰ πρὸς τι* and *τὰ πρὸς τί πως ἔχοντα* belongs to the earliest core of Stoic philosophy. Unfortunately, Rieth's assumption has no basis. If we ask whether in Ariston's argument *πρὸς τί πως ἔχον* is used in the sense described by Simplicius, independently of what Rieth thinks it must be, there is no reason for a positive conclusion.

To prove this claim we must go into it in detail. Ariston's argument is preserved by Plutarch (*On Moral Virtue* 440 E ff. = *SVF* I 375) and Galen (*On the*

Doctrines of Hippocrates and Plato 434.31 ff. De Lacy = *SVF* I 374). The differences between our two sources do not affect the problem we are concerned with,⁷⁴ and therefore we can summarize the argument as follows. In Plutarch's account, Ariston's thesis was that virtue is essentially unique (*On Moral Virtue* 440 E), and Galen (*On the Doctrines of Hippocrates and Plato* 434.32–33 De Lacy) reports that he defined it as “knowledge of things good and evil.” The differences between the virtues, namely the differences by which we are entitled to say, for instance, that justice is different from prudence or temperance, depend on the objects to which ethical knowledge is related. Thus, knowledge of what one must do and avoid doing is prudence, and justice is knowledge of what should be distributed to each according to his worth (*On the Doctrines of Hippocrates and Plato* 436.1–3 De Lacy). The idea seems to be that knowledge of the *A*'s is different from knowledge of the *B*'s if the *A*'s are different from the *B*'s. The mere difference between the *A*'s and the *B*'s is sufficient to account for the difference between knowledge of the *A*'s and knowledge of the *B*'s. Therefore, because there are reasons to think that knowledge, as a power of the mind, is one and the same in all circumstances (*On the Doctrines of Hippocrates and Plato* 434.31–32 De Lacy), one can conclude that knowing the *A*'s and the *B*'s does not imply a difference in knowing or, consequently, in the notion of virtue.⁷⁵

I do not see how this argument can be connected to the distinction between *πρός τί πως ἔχοντα* and *πρός τι*. First of all, the relational term that is considered in Ariston's argument is knowledge, and knowledge is listed among *τὰ πρός τι* in Simplicius's account (165.21). Therefore, Ariston would be considering here a *πρός τι*, but not a *πρός τί πως ἔχοντα*, term. One might reply that Ariston could have believed that knowledge must be put among *τὰ πρός τί πως ἔχοντα*, whereas later Stoics, on whom Simplicius relies, would have put it among *τὰ πρός τι*. The important point, one might continue, is that the notion of *πρός τί πως ἔχοντα* is used by Ariston. Plutarch says that, according to Ariston, virtue is one of *τῇ οὐσίᾳ*, and the virtues are many and in some sense different *τῷ πρός τι* (*On Moral Virtue* 440 E–F), and Galen (*On the Doctrines of Hippocrates and Plato* 430.32–34 De Lacy) reports that in Ariston's view, virtue takes different names: *κατὰ πρός τι σχέσιν*. Then one might say that knowledge as such is not affected by its being knowledge of the *A*'s and knowledge of the *B*'s. The relation of knowledge to the *A*'s or to the *B*'s does not modify its nature, namely its internal state, and in this sense knowledge may be considered a *πρός τί πως ἔχον*.⁷⁶ But this does not correspond to the definition of *πρός τί πως ἔχον* that Simplicius relates (at least on the interpretation we have given). This definition concerns the cases in which an individual *a* does not modify its internal state when a *πρός τί πως ἔχον* term ceases to be true of it. Consequently, if knowledge were a *πρός τί πως ἔχον*, it should follow that if *a* has knowledge of the *A*'s and loses this property, then this could happen without changes in his internal state. Therefore, the notion of *πρός τί πως ἔχον* might come into question if a knower were at issue. But that is not the case in Ariston's argument. His claim is about knowledge in the sense that he maintains that knowledge remains the same whether it is directed to the *A*'s or to the *B*'s.

Moreover, even if knowledge were a *πρός τί πως ἔχον* according to Ariston, there is no evidence that he had in mind a theory of *P*-relatives when he defended his view about virtue. Just as one can correctly use a particular kind of inference without possessing a theory of this kind of inference, so one might discover that a certain thing has a feature without having a general theory about things that have that feature. The mere observation that its being relative does not affect the nature of knowledge does not imply that a general theory of relatives is at work. Ariston's claim may simply depend on an analysis of the way in which knowledge behaves.

In his critical answer to Ariston, Chrysippus does not appear to be aware of Simplicius's distinction. Galen clearly states the point on which Chrysippus disagreed with Ariston when he remarks:

(V) That man <sc. Ariston> holds that virtue, being one, is given a number of names according to its relationships (*κατὰ πρὸς τι σχέσιν*). Chrysippus, however, shows that the plurality of virtues and vices is not a matter of relationships (*οὐκ ἐν τῇ πρὸς τι σχέσει γινόμενον τὸ πλήθος τῶν ἀρετῶν τε καὶ κακιῶν*) but of the qualitative differences in their own essences, as the account of the ancients intended.

(Galen, *On the Doctrines of Hippocrates and Plato* 430.32–432.2 De Lacy = *SVF* III 259; Ph. De Lacy translation)

Chrysippus's view is clear. According to Ariston, virtue *a* is different from virtue *b* merely because the object to which *a* is related is different from the object to which *b* is related. If this were so, Ariston would be right to maintain that *a* and *b* differ only in name. But the premise from which Ariston derives his conclusion is not correct, because *a* and *b* differ substantially. One might say that an admission of Simplician *πρός τί πως ἔχοντα* is implied by Chrysippus when he concedes the conditional according to which, if *a* and *b* differ only with respect to the objects to which they are related, then *a* and *b* differ merely in name. The objects to which *a* and *b* are related, one might continue, are not able by themselves to make the virtues different in more than name. Therefore, *a* and *b* are not affected in their internal states by the objects to which they are related, and consequently the relations at issue are of the kind that are involved in *P*-terms. But once again it is easy to observe that Cambridge change, at least as we have defined it, plays no role in making the possible distinction between *a* and *b*, just as it played no role in the case of knowledge. In the case of knowledge, it is quite reasonable to maintain that knowledge of *P* differs from knowledge of *Q* not because of a difference in knowledge, but merely because *P* and *Q* are different. The opposite view could be expressed by pointing out that knowledge of *P* and knowledge of *Q* differ not only because *P* and *Q* are different but also because knowledge itself has different features in the two cases. Shall we conclude that on the former hypothesis knowledge has to be conceived as a *P*-relative and on the latter as an *R'*-relative? I do not think so. One might consistently maintain that knowing *P* and not knowing *P* makes a difference in the internal state of a knower and, at the same time, that knowing *P* is different from knowing *Q* if,

and only if, P and Q are different, so that the nature of knowing is not changed if we pass from P to Q . On the other hand, one might equally claim both that the passage from not knowing P to knowing P does not imply an alteration of the internal state of a knower and that the difference between knowing P and knowing Q is determined not only by the difference between P and Q but also by different features of knowing. So the distinction between τὰ πρὸς τί πως ἔχοντα and τὰ πρὸς τι in the Simplicius passage seems to be quite irrelevant to the discussion between Ariston and Chrysippus.

A more positive conclusion cannot be reached, even if we look at the terminology used by Galen. One might argue as follows: σχέσις πρὸς τι must be taken as the nominalization of πως ἔχειν πρὸς τι. Therefore, where the former expression is used the latter is hinted at, and because we know from Simplicius what meaning the Stoics attributed to the latter, we may give the same meaning to the former. We do not know whether σχέσις πρὸς τι was really used by Chrysippus or if it depends on Galen's way of reporting him. If the latter is the case, we must see how Galen uses the expression in order to determine its meaning. And there is no reason to conclude that he takes it in the Simplician sense. The expression "σχέσις πρὸς τι" is nontechnical Greek for "relation to something," and in this plain meaning it is used by Galen, for instance, at *On the Doctrines of Hippocrates and Plato* 434.9 and 602.2 De Lacy. On the other hand, even if we generously assume that Galen was exactly repeating Chrysippus's words, we cannot draw any conclusion. There is another passage in Galen where Chrysippus is quoted almost verbatim and where πως ἔχειν πρὸς τι does not bear the Simplician meaning.⁷⁷

Finally, even if it is admitted that Chrysippus sometimes uses the so-called Stoic system of the categories,⁷⁸ we cannot conclude from this that he knew, adopted, or was aware of the distinction between P - and R' -terms, because, as we have seen, τὰ πρὸς τί πως ἔχοντα was considered by Simplicius have little to do with τὰ πρὸς τί πως ἔχοντα, which are said to form the fourth Stoic category.

By claiming that there is no proof that Chrysippus was the creator of the Stoic doctrine referred to by Simplicius, I do not imply that we must conclude that it should be attributed to the Stoics of the second century BC, as August Schmekel many years ago and Margherita Isnardi Parente quite recently have contended.⁷⁹ Schmekel goes so far as to propose the name of Antipater of Tarsus as a possible author of the doctrine, but his argument is very weak. His main point is that Simplicius (167.18–20), in refuting the Stoic view, exploits an argument used by Plotinus to reject their theory of the categories (*Enneads* VI 1, 30.24–27). Because, according to Schmekel, Plotinus directed his objections against Antipater, it must be concluded that Simplicius also attacked Antipater's view. I do not want to discuss Schmekel's claim that Antipater is the target of Plotinus's blows – although his view seems to me hard to prove. It is sufficient to notice that the similarity of the Plotinus and the Simplicius passages does not even prove that they were discussing the same subject. If the interpretation of Simplicius's text we have proposed is correct, it might at best be said that Simplicius or his source took from Plotinus (perhaps through an intermediate source) the argument directed against the theory of the categories and applied it to the distinction between the two kinds of relatives. The similarity

certainly does not prove that both Simplicius and Plotinus were discussing the view of the same author. The general principle involved in Plotinus's and Simplicius's argument is that there cannot be an order of priority among the species of the same genus, and we know that Aristotle was already aware of this.⁸⁰ This shows that we cannot even be sure that it was from Plotinus that Simplicius took (directly or indirectly) his objection to the Stoic theory of relatives.

Margherita Isnardi Parente's proposal is not very convincing either, although she is more cautious than Schmekel and does not commit herself to any particular representative of the school. Her conclusion is vitiated in so far she adopts Krämer's diairetic interpretation of the distinction between *τα τὰ πρὸς τί πως ἔχοντα* and *τὰ πρὸς τι* and has not clearly understood the meaning of *κατὰ διαφοράν* in Simplicius's account. She takes *κατὰ διαφοράν* as synonymous with (or at least as implying) *καθ' αὐτό*. But the Simplicius passage does not warrant this view, as we have seen, and if this premise is taken away, her conclusion fails.

If we were asked to draw a conclusion from the whole of the previous discussion, we would be obliged to give a somewhat disappointing answer. We do not possess any evidence that warrants attributing Simplicius's distinction to Chrysippus or any other of the older Stoics. What we know is that the doctrine was held by some Stoics earlier than, or contemporary with, Boethus of Sidon (second half of the first century BC). We may also imagine that because this doctrine prompted objections from an important Peripatetic philosopher, it cannot have been a marginal part of Stoic philosophy. If the *terminus ad quem* is clear and precise, we have no idea about the *terminus a quo*. As far as the doctrine reflected by the Greek grammarians is concerned, our sources are not very informative. Their view is consistent with Simplicius's account and with Sextus's general definition of relatives, but we are not sure to what extent they reflect Stoic positions. At any rate, no name of any member of the school is mentioned. On the other hand, the texts where Chrysippus's view is reported, namely the passages in Gellius, Varro, and Plutarch, have nothing to do with the doctrine of relatives, although they are often quoted in connection with it. I hope that interpreters more gifted and learned than I may be able to find a satisfactory solution to the problem.

Notes

- 1 These texts are not found in *SVF*. They will be discussed in Section VIII.
- 2 These passages too are not in *SVF*. We will examine them later (see Sections XI–XII).
- 3 I am thinking especially of three passages we will consider later, namely Varro, *De lingua latina* 10.59 (= *SVF* II 155); Plutarch, *On the Self-contradictions of the Stoics* 1054 E–F (= *SVF* II 550); and Aulus Gellius, *Attic Nights* 7.1, 1–6 (= *SVF* II 1169). See Sections XIV and XV.
- 4 At 1669–10, I follow Rieth 1933: 70n1 in adopting the reading: *ὁ γὰρ δεξιὸς καὶ πατὴρ μετὰ τοῦ <πρὸς τί> πως ἔχειν καὶ πρὸς τί εἰσιν*. I am not translating the four technical expressions, as they will be discussed in detail.
- 5 Zeno considered colors the first configurations of matter (*SVF* I 91).
- 6 Pace Baldassarri 1984: 118.
- 7 The right interpretation is given by Rieth 1933: 71. Against it, Krämer 1971: 85n331 says that “*der Text spricht jedoch nicht davon, daß die Ansichseienden Differenzen*

sind, sondern lediglich, daß sie Differenzen haben, die ihren anhaften, nicht aber umgekehrt.” But from the fact that τὰ καθ’αὐτά have differences, it *does* follow that they are κατὰ διαφοράν. Evidence for this is that the denial of τὰ καθ’αὐτά συνυπάρχει τοῖς κατὰ διαφοράν is said to be entailed by the statement that there are κατὰ διαφοράν entities that are not καθ’αὐτά.

- 8 David Sedley (in Sedley 1985: 87–92) has elucidated some of the complicated issues involved in the Stoic notion of universals.
- 9 I am grateful to Jonathan Barnes and Michael Frede, who helped me get free of a wrong interpretation I had formerly maintained.
- 10 If (9) is taken to represent the conclusion of this part of the Stoic argument, one might find it surprising that an Aristotelian commentator such as Simplicius found anything to object to. As is well known, one might argue, Aristotle, too, in *Categories* 7 distinguishes two kinds of relatives, of which two distinct definitions are given (6a36–37 and 8a31–32), and he says that the relatives of the second definition are included in those of the first (8a33–35). Two observations can be made against this view. First, Simplicius tries not to refute (9), but rather a stronger thesis that will be formulated in due course. Second, in revising his first definition of relatives, Aristotle does not claim that there are two kinds of relatives. He simply says that the second definition is more adequate than the first at characterizing them. Therefore, something that satisfies the first but not the second definition cannot be properly considered a relative. This point is clearly implied by 8b19.
- 11 Rieth 1933: 71 adds to the schema reported in Section II a double arrow, “ \leftrightarrow ,” connecting τὰ πρὸς τι with τὰ κατὰ διαφοράν. If that means reciprocal inclusion, as one is entitled to think, two difficulties arise. First, Simplicius does not say that τὰ κατὰ διαφοράν are included in τὰ πρὸς τι, and he could not have said this without contradicting himself. For suppose that it were so. Because inclusion is transitive, it would follow that τὰ καθ’αὐτά are included among τὰ πρὸς τι, and that is absurd because they are said to be contrary to each other. Second, the inclusion of τὰ πρὸς τι among τὰ κατὰ διαφοράν cannot be taken together with the statement that τὰ πρὸς τί πὼς ἔχοντα are πρὸς τι and contrary to τὰ κατὰ διαφοράν; otherwise, the implausible situation we have seen with respect to (13) would come about. That is why we have not represented relations (g)–(k) in the schema offered earlier in Section II. Rieth appears to be somehow aware of this problem when he points out (76) that the meaning of πρὸς τι changes in the progress of the argument, but he does not draw the right consequence from this fact.
- 12 One might think that the way in which this new section opens (εἰ δὲ δεῖ σαφέστερον μεταλαβεῖν τὰ λεγόμενα: 166.15–16) suggests that here Simplicius is not reporting the Stoic doctrine, but rather giving his own interpretation of it. Therefore, the whole passage should be regarded as suspect. Against this it could be replied that at 172.1–10 (*SVF* II 403), a summary of the doctrine is attributed to the Stoics that substantially repeats what is said in the alleged interpretation of their view.
- 13 For a discussion of its philosophical relevance with respect to identity, see Wiggins 1980: 15 ff.
- 14 (*LL*) is explicitly stated by Arius Didymus (Stobaeus, *Anthology* I 20, 179.6–8 = *DG* 46.5–6) with reference to a view maintained by Mnesarchus, a Stoic of the second century BC who was a pupil of Panaetius and Diogenes of Babylon. That does not necessarily imply that the supporters of the doctrine referred to by Simplicius held the same position concerning identity, but it is quite plausible to attribute (*LL*) to them, because rejecting (*LL*) runs contrary to our intuitions about identity.
- 15 Sextus Empiricus contrasts τὰ πρὸς τί πὼς ἔχοντα with τὰ κατὰ διαφοράν (*Against the Professors* 8.161), but in a completely different sense. According to his definitions, τὰ κατὰ διαφοράν can be conceived independently, whereas τὰ πρὸς τί πὼς ἔχοντα can only be conceived with reference to something else. This distinction is exploited by him to show that everything is relative (*Outlines of Pyrrhonism* 1.137) by means of a quite unconvincing argument (see Annas and Barnes 1985: 140–141). This cannot be

the meaning that τὰ κατὰ διαφοράν has in Simplicius's passage; otherwise, it would be a nonsense to say that τὰ πρὸς τι are κατὰ διαφοράν.

- 16 When the predicate R_t is applied to a formula such as $\exists y F(x, y)$, one should not take the formula itself as its argument; otherwise, we would obtain an ill-formed expression. This difficulty can be overcome, but we prefer to avoid complications here.
- 17 For instance, we must allow that there are things that can be at least twice in the same internal state if there are terms that can be truly said to be πρὸς τί πως ἔχοντα. This assumption is far from obvious, but it seems to be consistent with the physical doctrine of the Stoics.
- 18 See Geach 1969: 71–72; Geach 1972: 321–322.
- 19 John Rist (in Rist 1971: 54) offers a different characterization of Simplicius's τὰ πρὸς τί πως ἔχοντα, which he refers to as “relative dispositions.” He says that “relative dispositions are the relations of an individual thing to other individual things that are associated with it in the world, but on which its continuing existence as an entity does not depend.” This way of approaching τὰ πρὸς τί πως ἔχοντα seems to imply that they include phasal relational properties of things. If a is related through R to b at t , and it loses such a relation to b at t' , it also loses the relational property $\lambda x R(x, b)$, which is consequently a phasal property of a . Because it happens that a loses it without ceasing to exist, $\lambda x R(x, b)$ is a πρὸς τί πως ἔχον property according to Rist's definition. I do not believe that this view is the right one. The class of τὰ πρὸς τι (namely R' -terms) is disjointed with respect to the class of τὰ πρὸς τί πως ἔχοντα. Therefore, if terms for phasal relational properties are among τὰ πρὸς τί πως ἔχοντα, they cannot be included among τὰ πρὸς τι. Now one of the examples of πρὸς τι terms quoted by Simplicius is perception (166.21). We may suppose that if perception is a πρὸς τι, perceiving is so too. In particular, perceiving something, say b (namely $\lambda x P(x, b)$), may be thought to be πρὸς τι. But it can surely happen that $\lambda x P(x, b)$ is true of a at t but not at t' . Therefore, “ $\lambda x P(x, b)$ ” denotes a phasal relational property and must be included among τὰ πρὸς τί πως ἔχοντα if Rist's definition is admitted. The conclusion is that the class of τὰ πρὸς τί πως ἔχοντα and the class of τὰ πρὸς τι are not disjointed, contrary to Simplicius's claim. If Rist's definition must be rejected, one might be doubtful about the idea that led him to characterize τὰ πρὸς τί πως ἔχοντα in the way he did and that he shares with Reesor 1954: 45. His assumption is that Simplicius's τὰ πρὸς τί πως ἔχοντα are the πρὸς τί πως ἔχοντα that are said by Simplicius himself in another passage of his commentary on the *Categories* (66.32–67.8 = *SVF* II 369) and by Plotinus (*Enneads* VI 1. 25.1–5 = *SVF* II 371) to constitute the fourth of the Stoic categories. We will return to this point later (Section IX). If the view that terms for phasal properties can be included among τὰ πρὸς τι is accepted, the explanation that Rieth (Rieth 1933: 83) gives of the statement that τὰ πρὸς τί πως ἔχοντα are not κατὰ διαφοράν must be rejected. According to him, being on the right is not κατὰ διαφοράν because it does not permanently differentiate the things of which it is true. Because τὰ πρὸς τι are said to be κατὰ διαφοράν, it should follow that only permanent properties of things are πρὸς τι. As we have seen, however, that is not the case. Therefore, Rieth's interpretation must be abandoned.
- 20 The Stoic definitions of modal operators are reported by Diogenes Laertius 7.75 (= *SVF* II 201) and Boethius, *On Aristotle's On Interpretation* 2a, 234.27–235.4 (= *SVF* II 201). An analysis of these passages can be found in Frede 1974a: 107–117; Mignucci 1978: 325–330; and, more recently, in Bobzien 1986: 40 ff.
- 21 The explanations normally given of the statement that τὰ πρὸς τί πως ἔχοντα depend for their being merely on the relations they have to something else are far from illuminating. For instance, some think that Simplicius's words constitute evidence for interpreting τὰ πρὸς τί πως ἔχοντα as correlatives (Krämer 1971: 85; Isnardi Parente 1986: 4, 7; Baldassarri 1984: 94). But every relative is correlative, because every relation has its converse. Therefore, being simply correlative does not distinguish τὰ πρὸς τί πως ἔχοντα from τὰ πρὸς τι. One might insist that τὰ πρὸς τί πως ἔχοντα are only correlative, whereas τὰ πρὸς τι are correlative and something else. But this does not explain the point. What one would like to know is not what both categories of

relatives have in common, but what makes them different. Even more mysterious is what Elorduy (Elorduy 1972, I: 254) says: “*El πρὸς τί πῶς ἔχον, que traducimos como lo ‘cuasirrelativo’, no tiene actividad alguna; un hombre que está a la derecha de otro es lo mismo que si estuviera a su izquierda.*” But does the same remark hold also for the case of father and son? De Lacy 1945: 252 does not seem to have caught the point either, when he says that “the relative disposition of things to each other, according to the Stoics, determines the nature of the things related,” where it is clear by the context that relative dispositions are Simplicius’s τὰ πρὸς τί πῶς ἔχοντα (cf. 252n33). Much better is Graeser’s interpretation (Graeser 1978: 211–212), although I do not agree with his view that τὰ πρὸς τί πῶς ἔχοντα do not denote “genuine property-entities.” Reesor 1954: 77 thinks that τὰ πρὸς τί πῶς ἔχοντα involve internal relations, because “they depend entirely upon their relation to something else.” But her general definition of internal relation, that is, a relation “which is such that a change in that to which the entity is related would affect the entity itself” (64n4), seems to suggest the opposite, namely that τὰ πρὸς τί πῶς ἔχοντα do not imply internal relations. In general, it seems dangerous to use notions as complicated as those of internal and external relation to elucidate the Stoic position – there is a risk of explaining *obscurum per obscurius*. The interpretations offered by Rieth and Rist have already been considered in endnotes 11 and 19.

- 22 David Sedley has drawn my attention to this passage. Sillitti 1985: 371–372, in her analysis of this passage, does not seem to have recognized that Aristotle here uses the notion of Cambridge change.
- 23 See Schwegler 1847–1848: IV, 232; Bonitz 1862: 217; Jaeger 1957: *ad locum*. David Ross, in his edition of the *Metaphysics*, adopts Schwegler’s emendation (Ross 1953³: II, 340), but in the *Physics* he prefers the longer emendation ἀληθεύεσθαι καὶ μὴ (Ross 1966⁴: 620–621), which seems to be supported by Alexander (*apud* Simplicius, *On Aristotle’s Physics* 835.1–2) and Themistius (*On Aristotle’s Physics* 170.23–24). Wagner 1967: 597–598 does not find Ross’s proposal persuasive and prefers Schwegler’s emendation, although he defends the possibility of the reading of the manuscripts.
- 24 It is interesting to notice that when Ross (Ross 1966⁴: 620–621) illustrates Aristotle’s point by means of an example, he says that if *a* is double the size of *b*, then “double” can cease to be applicable to *a*, though *a* has not changed in its size. Wagner (Wagner 1967: 131) in his translation renders the Greek quite arbitrarily by “*es ist sehr wohl möglich, daß die Bestimmung des einen Gliedes, obwohl es sich nicht im geringsten selbst verändert, nicht mehr wahr bleibt, weil nämlich das andere Glied einen Prozeß durchmacht.*” Both authors seem to be unaware of the difficulty Aristotle’s text presents.
- 25 See Aristotle, *Categories* 7, 8a35 ff. For a discussion of this difficult passage, I refer the reader to Chapter 15 in this volume.
- 26 I have to confess that I would have been much more confident in positing a dependence of the Stoic doctrine on Aristotle had I not read Sandbach’s book (Sandbach 1985) and participated in the workshop on Aristotle and the Stoics held in Cambridge in May 1986. At this conference, David Sedley gave a paper on the Stoic doctrine of relations in which he argued convincingly that the Stoic view does not depend on Aristotle. It seems to me now that a considerable skepticism about the possibility of our proving that the Stoics read and reacted to Aristotle is the minimum that one can learn from Sandbach’s work. Richard Sorabji has pointed out to me a passage of Simplicius (*On Aristotle’s Physics* 604.9–11) where Theophrastus is reported to have exploited the notion of Cambridge change in his criticism of Aristotle’s definition of place. Even if it is perhaps less probable, the hypothesis that Theophrastus was the main source of the Stoic doctrine cannot be in principle be ruled out.
- 27 See Annas and Barnes 1985: 135–137.
- 28 The way in which Julia Annas and Jonathan Barnes explain Diogenes’s natural belonging is perhaps too strong. They say “if some property, *F*, is part of a thing’s nature, then

the thing cannot cease to be *F* without changing in its nature or ceasing to exist” (Annas and Barnes 1985: 137). It seems to me that their statement entails that if *a* is *F* by nature, then *a* is necessarily *F*.

- 29 I refer the reader to Annas and Barnes 1985: 137.
- 30 See Krämer 1971: 85–86; Isnardi Parente 1986: 4n2. Slightly different is the division proposed by Graeser 1978: 208. I am not concerned with a discussion of these differences.
- 31 This is, for instance, the position defended by Rieth 1933: 70 ff.
- 32 Actually, the word “category” (κατηγορία) is not used by our sources in this sense; γένος is the standard name adopted by Plotinus in his discussion of the Stoic theory of the categories. I continue to use “category” (and “categories”) here for convenience.
- 33 I here give a representative of each view: De Lacy 1945: 246–263 for the first; Lloyd 1971a: 58–74 for the second; Christensen 1962: 48–52 for the third; Pohlenz 1955–1959: II, 69 for the fourth.
- 34 Trendelenburg 1979²: 220–221.
- 35 See, for instance, Zeller 1963⁶: III.1: 104–105; Mates 1961²: 18; De Lacy 1945: 252; Christensen 1962: 51; Rist 1971: 54–55, criticizes this view, but he seems to admit that the Stoic categories are not disjointed classes.
- 36 I follow the text proposed by Henry and Schwyzer, who read ἐν τῷ τρίτῳ γένει τακτέον (lines 16–17), instead of retaining the reading of the manuscripts (ἐν τῷ τετάρτῳ γένει τακτέον) and emending τὰ ποια δηλονότι πως ἔχοντα at 15–16 into τὰ ποια δηλονότι πρὸς τί πως ἔχοντα, as suggested by Rieth 1933: 80n1.
- 37 This problem is in the background of Plutarch’s discussion in *On the Common Notions against the Stoics* 1086 A (= *SVF* II 380).
- 38 See Plotinus, *Enneads* VI 1, 1.12–14, ἔστι δὲ καὶ ἐν τοῖς γένεσι διαφορά, οἱ μὲν γὰρ τὰ γένη ἀρχάς, οἱ δὲ αὐτὰ τὰ ὄντα τῷ γένει τοσαῦτα. The words “οἱ δὲ” refer to the Stoics; cf. *Enneads* VI 1, 25.1–3 and 23–25 and Graeser 1972: 88.
- 39 Prantl 1927²: I, 428.
- 40 The same criticism is directed by Simplicius against Diogenes of Babylon (*On Aristotle’s Physics* 426.1 ff. = *SVF* III, Diogenes 19). In a similar way, knowledge is said to be τὸ ἡγεμονικόν in a certain state (Sextus Empiricus, *Against the Professors* 7.38 = *SVF* II 132).
- 41 Lloyd 1971a: 69–70, who first noticed the differences in Plotinus’s ways of referring to the categories, does not attribute to them any great importance.
- 42 Rieth 1933: 190–191, thinks that Trendelenburg’s interpretation of the Stoic doctrine of the categories is wrong because the requirement of inclusion defended by him is inconsistent with Simplicius’s words. Because the requirement of inclusion is based on Plotinus’s authority, it is reasonable to conclude that Trendelenburg’s view is correct and that Simplicius’s text does not refer to the doctrine of the categories.
- 43 Annas and Barnes 1985: 135.
- 44 For this notion of existence, see Liske 1985: 76n34, where a short bibliography on the subject is offered.
- 45 To overcome this difficulty one might think that “son” implies in its meaning not only “being born from . . .” but also “having a living father.” Similarly, the meaning of “father” should correspond to something like “having a living son.” If such meanings are given to “father” and “son,” it becomes plausible to say that if *a* loses his father, *a* is no longer a son and if the sons of *b* die, *b* is no longer a father. But it is difficult to avoid an impression of an ad hoc solution. The most natural meanings of “son” and “father,” as well as of their Greek counterparts, are such that they do not imply that a man ceases to be a son when his father dies. It is said by Simplicius that the pair father–son satisfies (25), and that is far from obvious. We will return to this question when we consider the evidence from the grammarians (Section XI).
- 46 The problem of the example of fathers and sons still remains.

- 47 Cf. Simplicius, *On Aristotle's Physics* 248.2–5 (= fr. 7 Isnardi Parente). Strong supporters of the view that the sweet and the bitter are related as contraries are Krämer 1971: 87–88; Isnardi Parente 1986: 9–10. The link between contraries is also considered in Sextus Empiricus, *Against the Professors* 10.263 ff., with reference to a doctrine attributed to the Pythagoreans and, in a Stoic context, by the scholiasts of Dionysius Thrax. These passages will be considered in due course (Sections XI and XIII).
- 48 To be exact, (26) needs refining, because it has some undesirable consequences. For instance, empty terms are epistemologically related to each other and to any other term. A possible way to avoid this and other similar consequences is to make the conditional on the right side of the formula stronger than material implication. But we can ignore such complications here.
- 49 Rieth 1933: 78, thinks that sweet and bitter are relative because they are dispositions of matter, although he seems also to admit the possibility that they are relative with respect to a perceiving subject. He does not consider the difficulties that arise from these assumptions, and the passage to which he appeals to confirm his interpretation (165.35–36) is not good evidence for it, as Reesor 1954: 76–77, has remarked.
- 50 Pohlenz 1955–1959: I, 40 (followed by Krämer 1971: 87n337) thinks that the examples of 166.21 are not Stoic, but come from Simplicius, who would here be recalling Aristotle's *Categories* 7 (6a2–3). But they do not justify their view, and the mere analogy with *Categories* 7 is not sufficient to support it. Moreover, even if Simplicius took the examples from Aristotle, it would have to be explained why he considered them examples of $\pi\rho\acute{o}\varsigma$ $\tau\iota$ $\mu\acute{o}\nu\omicron\nu$, that is, as R' - but not P -relatives.
- 51 I take from these authors the terminology of relativity, namely the expressions “ontological relativity,” “epistemological relativity,” and “semantical relativity.” See Annas and Barnes 1985: 130–140.
- 52 Pohlenz 1939: 185–188; Barwick 1967²: 103–107.
- 53 Heliodorus's explanation of relatives probably goes back to Apollonius Dyscolus: cf. $\pi\epsilon\rho\iota$ $\delta\nu\omicron\mu\acute{\alpha}\tau\omega\nu$, *GG* II 3, 49.37–50.10, and Pohlenz 1939: 185–186.
- 54 The example of fathers and sons is repeated *verbatim* in the *Scholia Londinensia* (*GG* I 3, 553, 29–30) and in a slightly different way in the *Scholia Vaticana* (*GG* I 3, 235.24–26). Priscianus, *Grammatical Foundations* [*Inst.*] II 28 (*GL* II 60.19–22) has a similar characterization of what he calls *ad aliquid*.
- 55 I take “ $\lambda x\exists yR_F(x, y)$ ” to express the relational term “ $\lambda xF(x)$.” In this sense, “being a father” is constituted by “having a child.”
- 56 It is easy to point out that Sextus's Pythagoreans were not very far from the position of the old Academy, as the similarity of their doctrine with the view attributed to Hermodorus (Simplicius, *On Aristotle's Physics* 247.30 ff. = fr. 7, Isnardi-Parente) shows. See Heinze 1965²: 38; Wilpert 1941: 230. Krämer 1982 (Appendix III 12); Gaiser 1968² (*Testimonia platonica* 32) put the whole passage of Sextus among Plato's *agrapha dogmata*.
- 57 The same is said in the *Scholia Vaticana* (*GG* I 3, 193.23–37), and a similar view is attributed to the Pythagoreans by Sextus (*Against the Professors* 10.266–267).
- 58 The same point is also made more briefly in the *Scholia Londinensia* (*GG* I 3, 554.10–14) but not in the *Scholia Vaticana* (*GG* I 3, 193.15.37; 2359–36). See also Priscianus, *Grammatical Foundations* [*Inst.*] II 29 (*GL* II 6023–27).
- 59 The distinction between $\pi\rho\acute{o}\varsigma$ $\tau\iota$ $\epsilon\chi\omicron\nu$ and $\omega\varsigma$ $\pi\rho\acute{o}\varsigma$ $\tau\iota$ $\epsilon\chi\omicron\nu$ found in the grammarians has been misunderstood by Reesor 1954: 45n15. She says: “the same examples [i.e., father-son, right-left], however, are found in a passage of Dionysius Thrax where they are relations ($\pi\rho\acute{o}\varsigma$ $\tau\iota$ $\epsilon\chi\omicron\nu$), but not relative dispositions ($\tau\acute{o}$ $\omega\varsigma$ $\pi\rho\acute{o}\varsigma$ $\tau\iota$ $\epsilon\chi\omicron\nu$).”
- 60 This view is not inconsistent with the statements of *Categories* 7, 6b15–19 (*pace* Krämer 1967: 284n90). In this passage, Aristotle does not say that contraries are relatives, but simply that some relatives admit contraries. For instance, virtue and knowledge are relative terms – virtue is virtue *of* something and knowledge is knowledge *of*

something – and they also have contraries, namely vice and ignorance. From this it does not follow that virtue is related to vice and knowledge to ignorance.

61 Rossitto 1984: 368.

62 Simplicius, *On Aristotle's Physics* 248.2–5 (= Hermodorus, fr. 7, Isnardi Parente).

63 Fritz 1978: II, 26–27; Krämer 1971: 82–83.

64 For a discussion of the Hermodorus fragment in connection with the Sextus passage, see Isnardi Parente 1982: 439–444.

65 On the same line are Charisius (*GL* I 156.4–9), the *Excerpta Bobiensia* of Charisius's *Ars grammatica* (*GL* I 536.6–10), Probus (*GL* IV 119,36–120.1), Donatus (*GL* IV 374.8–11), and Dositheus (*GL* VII 397.10–398.2).

66 Barwick 1967²: 107; Pohlenz 1939: 186–187. I am not sure that the passage of Varro referred to by Pohlenz can be used as evidence for the thesis that Chrysippus put the pair right–left in the same class or category in which fathers and sons are (I will discuss the Varro passage in Section XV). However, that is what Simplicius says with reference not to Chrysippus but to the Stoics in general.

67 Isnardi Parente's contention (Isnardi Parente 1986: 16) that Sextus's general definition of relatives refers only to τὰ πρὸς τί πῶς ἔχοντα of Simplicius has no basis in Sextus's words and is against the sense of his argument.

68 Sextus Empiricus, *Against the Professors* 8.455–456. See Annas and Barnes 1985: 137.

69 Without explicit mention of contrariety, there are many passages in which the Stoics are committed to maintaining that vice is useful, because there would be no virtue or good without it. See Plutarch, *On the Common Notions against the Stoics* 1065 A–B (= *SVF* II 1181); *On the Self-contradictions of the Stoics* 1050 F (= *SVF* II 1181); and, with reference to Posidonius, Diogenes Laertius 7.91 (= fr. 29 Edelstein-Kidd). Philo, *Allegorical Interpretation* [*Legum all.*] 3.23, 128.22–24, probably also reflects the Stoic view. Aulus Gellius himself refers to an illustrious tradition, namely to Plato's *Phaedo* (60 B–C), where pleasure and pain are said to be connected, and *Theaetetus* 176 A 5–8 could be added to this passage (an interesting analysis of the *Phaedo* text, which is far from plain and clear, can be found in Gallop's commentary (Oxford 1975: 76–8)). It does not matter here whether this claim of paternity is correct (see Cherniss 1954: 24n7), nor shall I analyze in detail the reasons that may have led the Stoics to put forward their thesis. In a debated passage of Plutarch, it is said that the Stoics naively maintained that “the generation of evil things and good came about for the sake of prudence” (*On the Common Notions against the Stoics* 1066 F; Cherniss translation). Some scholars think that the view attributed to the Stoics by Plutarch is not their own (Cherniss 1976: 717 note *e*, and Baldassarri 1976: II, 44n54), whereas others take it as genuinely Stoic (Babut 1969: 298). Plutarch's point seems to be that if prudence exists, namely if there are prudent men, then there are things that are good and things that are bad. This statement can be explained by pointing out that prudence is defined as knowledge of good and bad things and that a thing must exist to be knowable; and it implies that, under the condition that there are prudent men, the existence of good things entails the existence of bad things and vice versa. Even if this view cannot be surely attributed to the Stoics, it remains true that it is quite near to the point made by Gellius in the passage we are discussing.

70 Rieth 1933: 83–87.

71 Pohlenz 1939: 188 note *i*; Pohlenz 1955–1959: II, 40.

72 Reesor 1954: 68–69.

73 Cf. Sextus Empiricus, *Against the Professors* 9.332 and 9.336 (= *SVF* II 524); 11.24 (= *SVF* III 75).

74 See Ioppolo 1980: 215 ff.

75 Schofield 1984: 88–89.

76 That is the line of reasoning on which Rieth 1933: 85 ultimately bases his view.

- 77 Galen, *On the Doctrines of Hippocrates and Plato* 254.18–19 De Lacy: τὸ ἔχειν ἐναντίως πρὸς τὸ εὐλόγως. I am grateful to Jonathan Barnes for having drawn my attention to these Galenian passages.
- 78 See, for instance, Sedley 1982: 272n21.
- 79 Schmekel 1938: I, 630–631 (629n2); Isnardi Parente 1986: 12 ff.
- 80 See, for instance, Aristotle, *Politics* 3.1, 1275a34–38. The question has been studied by Lloyd 1962: 67–90. Plotinus also applies the principle in *Enneads* VI 1, 25.15 ff.

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